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Impact models for walking and running systems - angular moment conservation versus varying geometric constraints

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Abstract

Passive dynamic walking [1] and human walking and running [2] is an actively researched area. The essential problem in case of these systems is the changing of the topology. A common challenge in mechanical modeling of walking and running systems is the modeling of the foot impact with the ground. We propose an approach for handling the impulsive dynamics of the step, and at the same time this approach is also able to determine the vanishing and remaining part of the kinetic energy. This is useful when passive walkers are studied and besides, it is helpful when different running styles are analyzed.

In the mechanical description of legged locomotion the finite (continuous) dynamics and the discrete collision event (impulsive dynamics) are distinguished. Let the finite-time dynamics be described by the following equation of motion:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{F}, \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ contains the Coriolis and centrifugal terms. Vector $\mathbf{F}$ represents the generalized forces applied on the system, e.g. gravity. For the description of the impulsive dynamics (1) can be rewritten in the form [3]:

$$\mathbf{M}(\mathbf{q})(\dot{\mathbf{q}}^+ - \dot{\mathbf{q}}^-) = \hat{\mathbf{F}}, \tag{2}$$

where $\dot{\mathbf{q}}^- = \dot{\mathbf{q}}(t^-)$, $\dot{\mathbf{q}}^+ = \dot{\mathbf{q}}(t^+)$ are the coordinate velocities before and after the collision respectively. $\hat{\mathbf{F}}$ is the impulse of the impact forces, all other forces are neglected. Using (2) the coordinate velocities after the foot impact can be theoretically determined.

When $\dot{\mathbf{q}}^+$ is to be determined, the typical approach in the literature of passive walkers is the conservation of the angular momentum during the impulsive dynamics of foot impact. The simplest model of walking is demonstrated in [4], and the infinite dynamics is detailed there. As it is explained, there is an impulse at the swing foot contact point and the former stance leg has no impulsive reaction with the ground when leaves it. A more general explanation of the angular momentum conservation is detailed in [5] for determination of the velocities after the collision. Neglecting non-impulsive forces at foot strike, angular momentum is conserved through the collision for the full system and also for its subsystems. Figure 1.a shows an open kinematic chain. When it collides with the ground, the angular momentum of system "a" for point A is reserved, and also for subsystem "b" and "c" for point B and C respectively. With this, a system of algebraic equations for the unknown velocities arises in the form:

$$\mathbf{D}(\mathbf{q})\dot{\mathbf{q}}^- = \mathbf{A}(\mathbf{q})\dot{\mathbf{q}}^+, \tag{3}$$

from which the new coordinate velocities are determined as: $\dot{\mathbf{q}}^+ = \mathbf{A}^{-1}\mathbf{D}\dot{\mathbf{q}}^-$. This formalization appears in [6]. This approach requires a kinematical description using the minimum set of generalized coordinates, where the stance leg is connected to the ground by a revolute joint as showed in Figure 1.b.

In [2] different gaits for human running were studied. The ground reaction force impact was analyzed with respect to the foot strike patterns: fore-foot, mid-foot or rear-foot strike. For this, a simple one degree of freedom model was introduced. For the same problem more thorough dynamical model for foot strike is introduced in [3]. The calculations are based on the consideration that the motion of the runner is constrained via the ground contact. The kinetic energy content associated with the constrained motion serves as an indicator to represent the intensity of foot impact. A similar concept of variable constraints are also used in [7].

Thus, instead of the conservation of the angular momentum of the subsystems, the foot impact can also be considered as an impulsively arising holonomic constraint as in [7, 3]. In this modeling approach, the joint constraint of the stance leg is canceled, thus two more descriptor coordinates (x and y) are needed as it is illustrated on Figure 1.c. It is straightforward that holonomic constraint equations are introduced when one of the legs is in contact with the ground. Before the foot strike, the system is moving under the control of the constraints related to the previous stance leg, thus at the time instant of the collision $\mathbf{q}$ and $\dot{\mathbf{q}}$ satisfies them. The new constraints related to the previously swing-leg arise and with the projection to the space of the admissible motion the new velocities can be determined as the admissible velocities:

$$\dot{\mathbf{q}}^{+} = \mathbf{P}_a \dot{\mathbf{q}}^{-}, \tag{4}$$

where $\mathbf{P}_a$ is the null space of the constraint Jacobian: $\mathbf{P}_a = \mathbf{1} - \mathbf{P}_c$, where $\mathbf{P}_c = \boldsymbol{\phi}_{\mathbf{q}}^{\dagger}\boldsymbol{\phi}_{\mathbf{q}}$ and the generalized inverse of the constraint Jacobian can be calculated according to [8] as: $\boldsymbol{\phi}_{\mathbf{q}}^{\dagger} = \mathbf{M}^{-1}\boldsymbol{\phi}_{\mathbf{q}}^{\mathrm{T}}(\boldsymbol{\phi}_{\mathbf{q}}\mathbf{M}^{-1}\boldsymbol{\phi}_{\mathbf{q}}^{\mathrm{T}})^{-1}$.

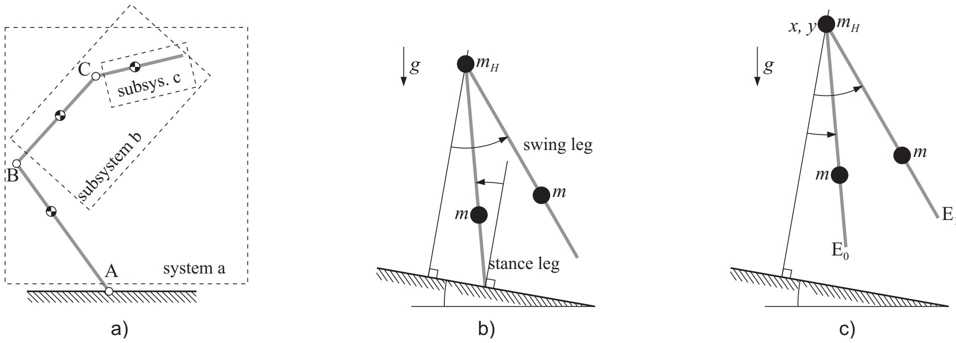

Figure 1. a: collision of an open kinematic chain with the ground, b: minimum coordinate model, c: dependent coordinate model

The comparison of the two methods highlighted advantages and drawbacks of the two methods. The main strength of the method using the projection related to the constraints is the applicability for closed kinematic loops, which is not true for the approach using conservation of angular momentum. Besides the projection method is more algorithmic. It can be applied not only for walking but also for running, when none of the legs are in connection with the ground. Furthermore, the decrement and the remaining part of the kinetic energy can also be easily determined via the kinetic energy related to the constrained and the admissible part of the motion respectively:

$$T_c = \frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{P}_c^{\mathrm{T}}\mathbf{M}\mathbf{P}_c\dot{\mathbf{q}}, \qquad T_a = \frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{P}_a^{\mathrm{T}}\mathbf{M}\mathbf{P}_a\dot{\mathbf{q}}. \tag{5}$$

With the knowledge of the energy T_c which disappear in every step, we are allowed to explain why the different running styles are suitable for long distance running and for short distance sprinting.

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