12$^{\text{TH}}$ CONFERENCE
on
DYNAMICAL SYSTEMS THEORY AND APPLICATIONS
December 2-5, 2013. Łódź, Poland

Energy efficient walking and running - impact dynamics based on varying geometric constraints

Ambrus Zelei, László Bencsik, László L. Kovács, Gábor Stépán

Abstract: The mechanics of human walking and especially running is an actively researched field. To find the perfect running form is an eternal problem both for hobby runners and professionals. The essential problem in the dynamic investigation of such biomechanical systems is the changing of the topology and the modeling of the foot impact with the ground. Besides, human body is a complex multibody system. The main source of energy loss during legged locomotion is generated by foot impact which is in focus in this work. We assume inelastic collisions and the ground-foot contact is handled by impulsively arising geometric constraints. The energy loss and the impulsive forces of each foot impact can be determined by a projection technique well known from the literature of multibody dynamics. These work as indicators of the intensity of each foot impact. Besides the energy efficiency, these indicators are important in the aspect of possible sport injuries. The presented method permits the investigation of running characteristics, e.g. foot strike patterns (fore-foot, mid-foot or rear-foot strike), stride length, head and torso position.

1. Introduction

Nowadays passive dynamic walking [1] and human walking and running [2, 3, 4] have been actively researched areas. The study of the working principle of passive walkers contributes to the thorough understanding of the dynamics of human motion. Besides, passive walkers show how limited is the energy which is demanded for legged locomotion. Human walking and running also utilizes the same advantages as passive walkers. Humans and passive or underactuated walking systems swing through from one configuration to another instead of always keeping themselves in the sequence of static equilibrium states.

The energy efficiency is a crucial question in endurance sports like long distance running. Some tribes are naturally talented in running like the Kalenjin runners from Kenya in [3] or the native American Tarahumara people from northwestern Mexico [5] who are capable to run unbelievably long distances. Members of both people usually wear only thin sole shoes that minimally influence their body motion. There is no flexible or damping part in their shoes, therefore they can rely only on the flexibility of their legs. In contrast, urbanized people usually use sometimes even too expensive or overdeveloped shoes for running with several conveniency features like excessive flexibility or artificial stabilization of the ankle, etc. These features make it impossible to develop a good running form. However, all children instinctively know how to run. A proper running form can also help to avoid injuries as much as possible, like in the case of the above mentioned naturally talented runners.

Energy efficiency is a crucial question in long distance running. The sources of energy loss are the aerodynamical forces, damping and vibrations at every step and foot impact. The analysis of energy loss due to foot impact can be done by exact mechanical techniques, which will help to improve running forms. In this work we are focusing on this analysis.

The most important parameter in endurance sports is the mechanical power required: at different intensity levels the ways of refueling the body can be very different [6]. However there is no exact method yet how to make a good choice of heart rate (HR) in relation with the distance one wishes to run. One can only rely on approximations and experience, which takes long time to measure and develop; the mechanical explanation is not perfectly complete for the question: why a certain HR is needed for a certain speed.

Figure 1 shows the relation between the variables HR, mechanical power, running form, speed and the distance to run. The relation between HR and mechanical power can be determined by an EKG test. The curve that shows the coherence between average speed and distance takes a long time to draw by recording the individual data, e.g. one has to go training and record the data day by day for months. If running form is known, than speed is given by the stride length l_s and the cadence c as $v = l_s c$. Our goal is to find the relation between running form and required mechanical power by pure mechanical calculations. Thus the arrow now drawn by the dashed line completes the circle.

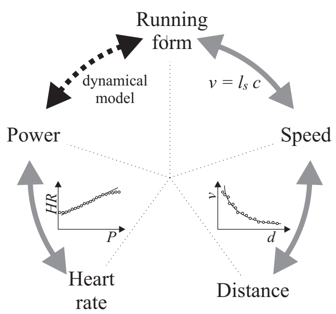

Figure 1. Connection of heart rate, running form and running speed

The energy efficiency and injury free running motion is examined widely in the world. Several different school developed for the investigation of the healthy, efficient and natural way of running. Our work aims to contribute to this field by using tools of analytical mechanics. In [3] different human running forms were studied. The ground reaction force impact was analyzed with respect to the foot strike patterns: fore-foot, mid-foot or rear-foot strike. For the analysis, a simple one degree of freedom model was introduced with vertical shank. The authors drew the conclusion that fore-foot landing is the most appropriate one to avoid injury and to be energy efficient. For the same problem a more complex dynamical model for foot strike was introduced in [7]. This model also contains only the foot and the shank, but the angle of the shank is a new parameter. The calculations are based on the consideration that the motion of the runner is constrained via the ground contact. The kinetic energy content associated with the constrained motion, which serves as an indicator to represent the intensity of foot impact, was calculated. In this paper we present some results drawn from an even more complex mechanical model which contains the full leg and the trunk of the human body as well.

2. Modeling issues and indicators for foot impact intensity

The essential problem of dynamical modeling in the case of legged locomotion systems is the changing of the topology. A common challenge in mechanical modeling of walking and running systems is the modeling of the foot impact with the ground.

In the mechanical description of legged locomotion the finite (continuous) dynamics and the discrete collision event (impulsive dynamics) are distinguished. Let the finite-time dynamics be described by the following equation of motion:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{Q}, \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ contains the Coriolis and centrifugal terms. Vector $\mathbf{Q}$ represents the generalized forces applied on the system, e.g. gravity or muscle forces. For the description of the impulsive dynamics (1) can be rewritten in the form [7]:

$$\mathbf{M}(\mathbf{q})(\dot{\mathbf{q}}^{+} - \dot{\mathbf{q}}^{-}) = \hat{\mathbf{F}}, \tag{2}$$

where $\dot{\mathbf{q}}^{-} = \dot{\mathbf{q}}(t^{-})$ and $\dot{\mathbf{q}}^{+} = \dot{\mathbf{q}}(t^{+})$ are the generalized velocities before and after the collision respectively. $\hat{\mathbf{F}}$ is the impulse of the impact forces, all other forces are neglected. Using (2) the post impact generalized velocities can be theoretically determined.

2.1. Impact modeling - conservation of the angular momentum

When $\dot{\mathbf{q}}^+$ is to be determined, the typical approach in the literature of passive walkers is the conservation of the angular momentum during the impulsive dynamics of foot impact. The simplest model of walking is demonstrated in [8], and the impulsive dynamics is detailed there. As it is explained, there is an impulse at the swing foot contact point and the former stance leg has no impulsive reaction with the ground when it leaves it. A more general explanation of the angular momentum conservation is detailed in [9] for determination of the post impact velocities. Neglecting non-impulsive forces at foot strike, angular momentum is conserved during the collision for the full system and also for its subsystems. Figure 2.a shows an open kinematic chain. When it collides with the ground, the angular momentum of system "a" for point A is reserved, and also for subsystem "b" and "c" for point B and C respectively. With this, the system of algebraic equations for the unknown velocities are:

$$\mathbf{D}(\mathbf{q})\dot{\mathbf{q}}^- = \mathbf{A}(\mathbf{q})\dot{\mathbf{q}}^+, \tag{3}$$

from which the post impact generalized velocity is determined as: $\dot{\mathbf{q}}^+ = \mathbf{A}^{-1}\mathbf{D}\dot{\mathbf{q}}^-$. This matrix formalization appears in [10]. This approach requires a kinematical description using the minimum set of generalized coordinates, where the stance leg is connected to the ground by a revolute joint as showed in Figure 2.b.

2.2. Impact modeling - projection technique

Instead of the conservation of the angular momentum of the subsystems, the foot impact can also be considered as an impulsively arising holonomic constraint as in [11, 7]. This kind of approach for modeling the foot contact also can be found in [12]. This modeling approach is illustrated in Figure 2.c, where the permanent constraint of the stance leg is canceled, thus two more descriptor coordinates (x and y) are needed. It is straightforward that holonomic constraint equations are introduced when the legs get in contact with the ground. Before the foot strike, the system is moving under the control of the constraints related to the previous stance leg, thus at the time instant of the collision $\mathbf{q}$ and $\dot{\mathbf{q}}$ satisfy these constraints but the system is considered to be moving free. The new constraints related to the previously swing-leg arise and with the projection to the space of the admissible motion the new coordinate velocities can be determined as the admissible velocities:

$$\dot{\mathbf{q}}^+ = \mathbf{P}_a\dot{\mathbf{q}}^-, \tag{4}$$

where $\mathbf{P}_a$ is the null space of the constraint Jacobian: $\mathbf{P}_a = \mathbf{I} - \mathbf{P}_c$, where $\mathbf{P}_c = \mathbf{\Phi}_\mathbf{q}^\dagger\mathbf{\Phi}_\mathbf{q}$ and the generalized inverse of the constraint Jacobian can be calculated according to [13] as: $\mathbf{\Phi}_\mathbf{q}^\dagger = \mathbf{M}^{-1}\mathbf{\Phi}_\mathbf{q}^\mathrm{T}(\mathbf{\Phi}_\mathbf{q}\mathbf{M}^{-1}\mathbf{\Phi}_\mathbf{q}^\mathrm{T})^{-1}$.

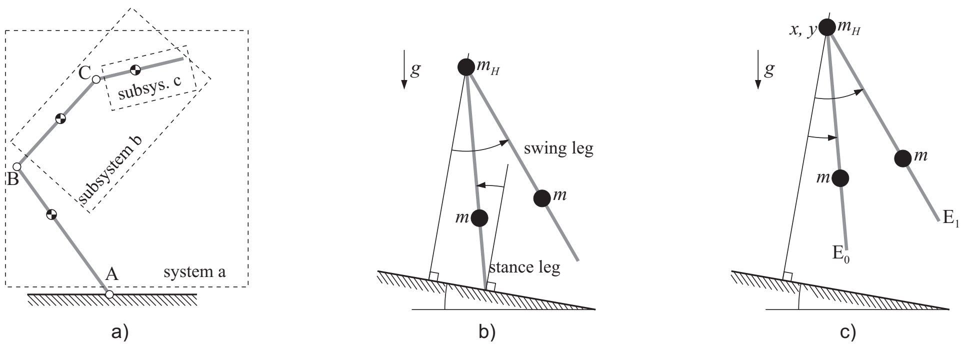

Figure 2. a: collision of an open kinematic chain with the ground, b: minimum coordinate model of a biped passive walker, c: dependent coordinate model

A similar approach can be found in [14] where the problem of foot impact was avoided by smooth steps. Machines lack many of the features that natural walking systems have and allow them to produce smooth locomotion: elastic body, superior mobility and agility, damping abilities. Impacts due to the collisions of legs with the ground may destabilize the walking cycle of artificial walking systems, thus they are to be avoided. [14] aims to minimize these phenomena via the design of a control scheme for walking without impacts.

The comparison of the two impact modeling methods highlighted advantages and drawbacks of them [15]. The main strength of the method using the projection is the applicability for closed kinematic loops (e.g. walking, when both legs are connection with the ground for finite time), which is not true for the approach using conservation of angular momentum. Besides the projection method is more algorithmic, which is very important in case of complex multibody models, e.g. full body human model with legs, arms, trunk and head. Furthermore, the decrement and the remaining part of the kinetic energy can also be easily determined via the kinetic energy related to the constrained and the admissible part of the motion respectively:

$$T_c = \frac{1}{2}(\dot{\mathbf{q}}^-)^{\mathrm{T}}\mathbf{P}_c^{\mathrm{T}}\mathbf{M}\mathbf{P}_c\dot{\mathbf{q}}^-, \tag{5}$$

$$T_a = \frac{1}{2}(\dot{\mathbf{q}}^-)^{\mathrm{T}}\mathbf{P}_a^{\mathrm{T}}\mathbf{M}\mathbf{P}_a\dot{\mathbf{q}}^-. \tag{6}$$

T_c is *called constrained motion space kinetic energy* (CMSKE) in the literature. Papers [7] and [16] showed that foot strike intensity can be characterised by the CMSKE which depends on the pre-impact configuration and velocity and the effective mass matrix $\mathbf{P_c^{\mathrm{T}}MP_c}$. The effective mass concept for foot impact is introduced in [17] for a one degree of freedom model. In this work the CMSKE is used in the same form for characterise the foot impact intensity.

The only inconsistency with [16] is that the constraints related to ground contact are slightly different. This issue is detailed in subsection 2.3.

2.3. Ground contact and friction

The ground contact is handled by holonomic constraints in our work and the related literature [7] and [16]. However a significant contradiction with [16] is that the CMSKE is considered only in connection with the vertical (normal) direction constraint represented by the ground. The horizontal (tangent) ground force is modeled by a friction force in [16]. Thus T_c derived from only the constraint that constrains the vertical motion of the foot.

The approach of this work is different. We assume that there is no slip when touchdown, than the friction force can be replaced by a constraint force. In our modeling approach both the vertical and the horizontal motion of the foot contact point is constrained immediately when touches the ground. So the projection matrix P_c in our calculations and thus the kinetic energy T_c obviously will differ from P_c and T_c considered in [16] for same configurations. However in [16] the horizontal motion is also prevented by a friction force, so that part of kinetic energy will be absorbed as well.

Our approach indicates a question whether our assumption about the slip is true or not. In general this drives to a combinatorial problem if there is more than one contact point at the same time. We avoid this problem by making this check after the simulations and not during the simulation. So we calculate the critical value of the friction coefficient along the time propagation. If the initial assumption did not stand, we would throw away the simulation results, but in a normal situation, when the running form is not extremely bad, the assumption is true. It is also proved by simulation results, which is presented below.

The critical value of the friction coefficient μ_c is introduced in [16] as another indicator for foot impact quality. Instead of using the formula adopted from [16]:

$$\mu_c = \frac{\mathbf{\Phi}_{\mathbf{q},t}\mathbf{M}^{-1}\mathbf{\Phi}_{\mathbf{q},n}^{\mathrm{T}}}{\mathbf{\Phi}_{\mathbf{q},t}\mathbf{M}^{-1}\mathbf{\Phi}_{\mathbf{q},t}^{\mathrm{T}}} \tag{7}$$

we calculate the critical value of the friction coefficient as:

$$\mu_c = \frac{f_t}{f_n}, \tag{8}$$

where f_n and f_t are the normal and tangent direction ground contact force respectively. They are calculated from the corresponding Lagrange multipliers. In equation (7) $\mathbf{\Phi}_{\mathbf{q},n} = \partial\boldsymbol{\varphi}_n/\partial\mathbf{q}$ and $\mathbf{\Phi}_{\mathbf{q},t} = \partial\boldsymbol{\varphi}_t/\partial\mathbf{q}$ are the constraint Jacobians with respect to the normal and tangent direction constraints $\boldsymbol{\varphi}_n$ and $\boldsymbol{\varphi}_t$ respectively. Another possible representation of the same constraint Jacobians are adopted from [16]:

$$\mathbf{v}_c = \begin{bmatrix} v_{c,n} \\ v_{c,t} \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{\mathbf{q},n} \\ \mathbf{\Phi}_{\mathbf{q},t} \end{bmatrix}\dot{\mathbf{q}}, \tag{9}$$

where $\mathbf{v}_c$ is the velocity of the contact point of the foot with normal $v_{c,n}$ and tangent direction component $v_{c,t}$.

2.4. Another possible indicator: change in the velocity

We propose to use the change of the velocity vector of the trunk as an indicator for the quality of stride, because it is a very important, how the next flying phase starts. Obviously the velocity vector should have a positive vertical component. The exact definition of this indicator is not easy to make unique, because the velocity can be different at every point of the body. Since, the trunk is modeled by a single point mass, here it was obvious to use the change of its velocity components (Δv_x, Δv_z). However, for example the center of mass of the full body could be chosen.

3. A simple mechanical model of running

As it was mentioned above, two main phases of walking or running can be distinguished, one is the continuous phase governed by equation (1), the other is the stride which is governed by impulsive dynamics (2). This work aims to focus on the landing phase of running, hence a one legged model is introduced. We will show that the consideration of the full body weight is needed to obtain a good insight of the dynamics of foot landing. The aim of the investigations is to identify the most important parameters of running form, e.g. strike pattern, hip, knee and ankle angle, the angle of the trunk and the neck ect., and to identify their optimal value. The model and its parameters shown in Figure 3.a is used to analyze the effect of ankle angle α_A and two different strike patterns: heel strike and forefoot strike. Weight and geometric data used in the simulations are collected in Table 1 and adopted from [18, 19]. Segmental center of gravity (c.o.g.) locations are measured from proximal ends of each segment. Since we have a one legged model, the mass of the missing body parts (head, arms and other leg) are collected in the trunk weight. The moment of inertia of each segment of the leg is estimated by homogeneous rod model. The data correspond to an average 24 years old male person with 73[kg] bodyweight and 173.1[cm] height [18, 19]. The mechanical

Table 1. Inertial and geometric data of body segments

	weight [kg]	moment of inertia [kgm^2]	length [m]	c.o.g. [m]
trunk	58.5	-	-	-
thigh	10.3	0.557	0.402	0.164
shank	3.16	0.193	0.428	0.188
foot	1.00	0.0018	0.074	0.032

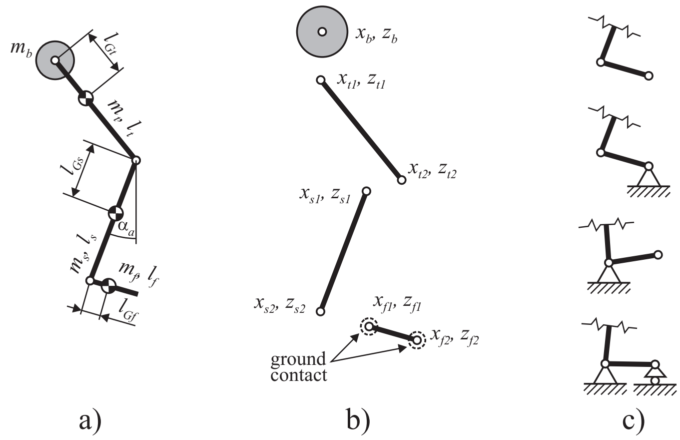

Figure 3. **a)** one legged mechanical model and its parameters, **b)** descriptor coordinates of the system, **c)** constraints of the foot in different configurations: free motion, forefoot contact, heel contact, full sole contact (midfoot contact)

model uses the concept of the so called natural coordinates explained in [20]. The number of descriptor coordinates (n=14), which are shown in Figure 3.b, is higher than the number of degrees of freedom thus geometric constraints collected in φ_b are used.

$$\boldsymbol{\varphi}_b = \begin{bmatrix} x_b - x_{t1} \\ z_b - z_{t1} \\ x_{t2} - x_{l1} \\ z_{t2} - z_{l1} \\ x_{l2} - x_{f1} \\ z_{l2} - z_{f1} \\ (x_{t1} - x_{t2})^2 + (z_{t1} - z_{t2})^2 - L_t^2 \\ (x_{l1} - x_{l2})^2 + (z_{l1} - z_{l2})^2 - L_l^2 \\ (x_{f1} - x_{f2})^2 + (z_{f1} - z_{f2})^2 - L_f^2 \end{bmatrix}, \quad \boldsymbol{\varphi}_c = \begin{bmatrix} x_{f1} - x_{f1c} \\ z_{f1} - 0 \\ x_{f2} - x_{f2c} \\ z_{f2} - 0 \end{bmatrix}. \tag{10}$$

$\boldsymbol{\varphi}_b$ contains the $m_b = 9$ number of geometric constraints of the body model itself while $\boldsymbol{\varphi}_c$ represents the $m_c = 4$ number of constraints related to the ground contact. The contact position of the heel and the forefoot is represented by x_{f1c} and x_{f1c}. The redundant set of constraints have to be avoided, thus depending on the configuration the constraints in $\boldsymbol{\varphi}_c$ can be activated or released. The possible situations are illustrated in Figure 3.c.

Since two base point on the foot can be connection with the ground, the strike index used in [4] and [7] can be $s = 0$ or $s = 1$ which corresponds to heel strike and forefoot strike respectively. Therefore the presented mechanical model is not appropriate for investigating fractional values of s. In fact in reality these two values are the most relevant.

4. Results

Numerical simulations were done with the model presented in section 3. All simulations started a few hundredths of a second before the strike event. We simulate the whole process of foot landing, it means that we do the simulation until both heel and forefoot touches the ground. Usually it happens at different time instances. The indicators T_c and μ_c of the heel and forefoot impact intensity were calculated, and we also calculated the change of the velocity Δv_x and Δv_z of the trunk.

4.1. Basic cases of foot strikes

It is known from experience that overstride and heel strike is in connection. Usually overstriding impels heel strike, and it is natural that forefoot landing happens if the landing point is almost under the center of gravity of the body. However it worth to investigate all possible situations. The results are summarized in Figure 4. On the time history of the kinetic energy (Figure 4.a) the impulsive decrements of the kinetic energy can be seen due to heel and forefoot touchdown. The dashed line on the first diagram show the case when there is no ground-heel contact at all, [4] also distinguishes toe strike and forefoot strike. The sum of these decrements and the impulsive velocity changes are shown in Figure 4.b. One can conclude that T_c related to the forefoot touchdown is always smaller than T_c from heel strike, and it is the same for the velocities. As another important result we can see that the effect of the ankle angle α_A (see Figure 3) is more significant than strike pattern. Simulations also

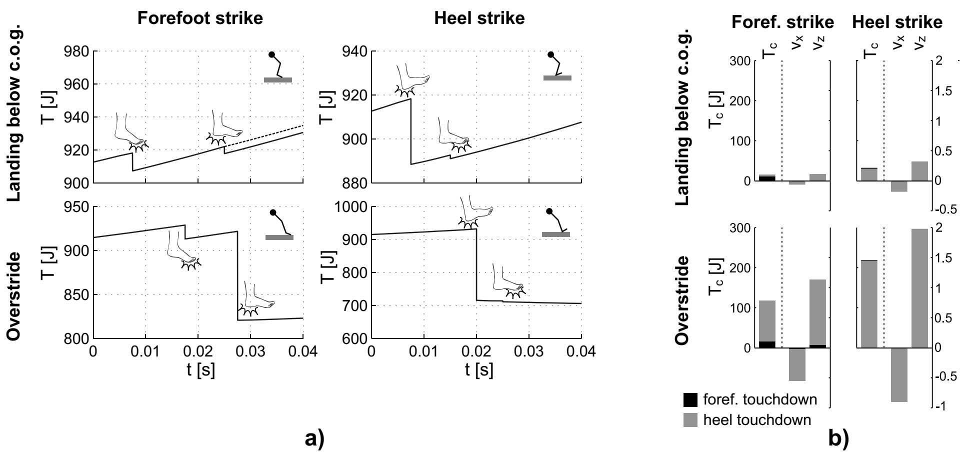

Figure 4. Different landings: a) time history of the kinetic energy b) absorbed part of the kinetic energy, and changes in the velocity of the trunk in horizontal and vertical direction

showed that if the forefoot touches the ground first, than the shank is pushed back, and the ankle angle becomes almost ideal before the heel reaches the ground. As a practical result we can conclude that overstriding should be avoided and forefoot landing is better than heel strike both from the viewpoint of energy efficiency and injury free running.

4.2. Effect of ankle angle

The importance of ankle angle was pointed out in subsection 4.1, thus a more thorough analysis of its effects were done by a sequence of simulations with different ankle angles in case of heel strike and also forefoot strike. In [7] the model shows that in higher strike index values the effect of the changes in the ankle angle is not significant. This contradiction is because there is no body mass in the models investigated in [4] and [7]. We can conclude that the leg cannot be analyzed in itself, just only together with the inertia of other body parts. We also have to notify that a correct ankle angle imposes right strike index, so the conclusion in [4] and [7] that strike index is so important is indirectly confirmed. Figure 5 also confirms that T_c can be considered as a proportional function of the contact force denoted by f_t and f_n. It is in coincidence with [21].

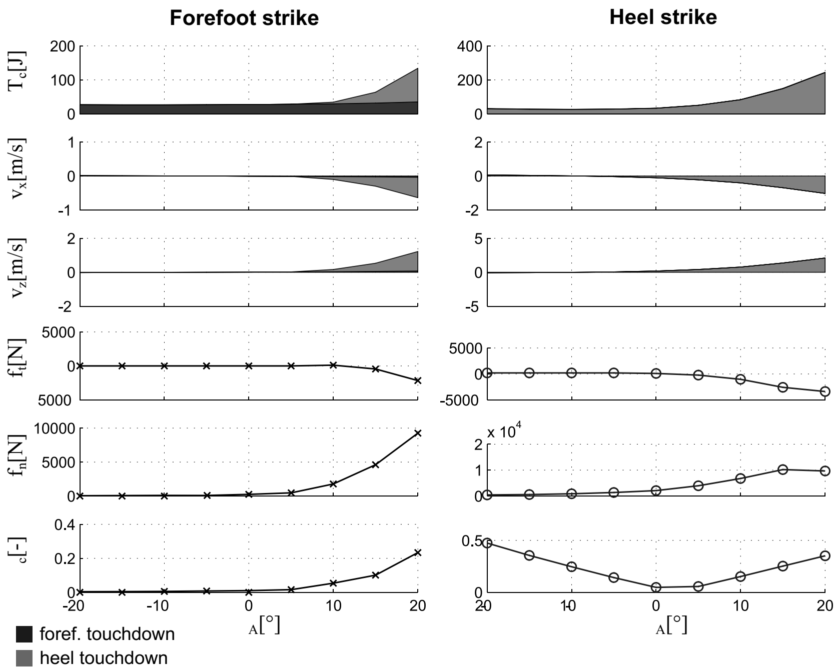

Figure 5. Indicators of impact intensity as functions of ankle angle α_A

Figure 6. Different landing strategies are used when running downhill or running flat

Figure 6 shows the evidence how important the ankle angle is. When running downhill one tries to control the speed and not to accelerate too much because that may cause accident. The breaking technique is basically the overstriding, because it results in the largest energy absorption. In contrast, a professional marathon runner, whose aim is to run fast and at the same time to save energy, keeps his ankle in negative angle (see Figure 6 right).

5. Conclusions

This work showed that it is not possible to achieve realistic results without modeling the motion of thigh and the weight of the trunk. We also showed that the ankle angle is a very important parameter, even more important than strike index. However, the practical meaning of this statement is in total correspondence with the referred literatures [3, 4, 7] because forefoot strike impels correct ankle angle. The calculations confirmed that knowing exactly the energy T_c (CMSKE) which absorbed at every step and the critical friction value μ_c, we are able to analyze and compare different running styles. Besides we proposed another indicator for foot impact quality, which is related with the velocity vector of the body center of mass.

References

1. T. McGeer: Passive dynamic walking, *Int. J. of Robotics Research*, 9 (2), 1990, 62–82.
2. T. F. Novacheck: The biomechanics of running, *Gait and Posture*, vol. 7, 1998, 77–95.
3. W. L. Jungers: Barefoot running strikes back, *Nature*, 463(7280), 2010, 433–434.
4. D. E. Lieberman, M. Venkadesan, W. A. Werbel, A. I. Daoud, S. DAndrea, I. S. Davis, R. O. MangEni and Y. Pitsiladis: Foot strike patterns and collision forces in habitually

barefoot versus shod runners, *Nature, Biomechanics*, 463(7280), 2010, 531–535.
5. C. McDougall: *Born to Run: A Hidden Tribe, Superathletes, and the Greatest Race the World Has Never Seen*, New York, published by Alfred A. Knopf, 2009.
6. W. D. McArdle, F. I. Katch and V. L. Katch: *Sports and Excercise Nutrition*, 3rd ed. Lippincott Williams & Wilkins, Philadelphia, 2009.
7. J. Kövecses and L.L. Kovács: Foot impact in different modes of running: mechanisms and energy transfer, *Procedia IUTAM*, vol. 2, 2011, 101–108.
8. M. Garcia., A. Chatterjee., A. Ruina and M. Coleman: The simplest walking model: stability, complexity, and scaling, *J. of Biomech. Engineering*, vol. 120, 1998, 281–288.
9. V.F. Hsu Chen: *Passive dynamic walking with knees: a point foot model*, MSc thesis, Massachusetts Institute of Technology, Supervisor: Russ Tedrake, 2007.
10. J. Goswami and B. Espiau and A. Keramane: Limit Cycles in a Passive Compass Gait Biped and Passivity-Mimicking Control Laws, *Autonomous Robots*, 4 (3), 1997, 273–286.
11. M. Amakita and F. Asano: Extended passive velocity field control with variable velocity fields for a kneed biped, *Advanced Robotics*, 15 (2), 2001, 139–168.
12. D.G. Alciatore, L.D. Abraham and R.E. Barr: Matrix Solution of Digitized Planar Human Body Dynamics for Biomechanics Laboratory Instruction, *12th Annual ASME International Computers in Engineering Conference*, 1992.
13. J. Kövecses, J.-C. Piedoboeuf and C. Lange: Dynamic Modeling and Simulation of Constrained Robotic Systems, *Transactions on Mechatronics*, 2 (2), 2003, 165–177.
14. W. Blajer and W. Schiehlen: Walking without impacts as a motion/force control problem, *Journal of dynamic systems, measurement, and control*, 114 (4), 1992, 660–665.
15. A. Zelei, L. Bencsik, L. L. Kovács, G. Stépán: Impact models for walking and running systems - angular moment conservation versus varying geometric constraints, *Eccomas, Multibody Dynamics 2013*, Book of Abstracts, 1-4 July, Zagreb, Croatia, pp 47–48.
16. J. M. Font-Llagunes, R. Pamies-Vila, J. Kövecses: Configuration-dependent performance indicators for the analysis of foot impact in running gait, *Eccomas, Multibody Dynamics 2013*, Book of Abstracts, 1-4 July, Zagreb, Croatia, pp 31–32.
17. K.-J.Chi and D. Schmitt: Mechanical energy and effective foot mass during impact loading of walking and running, *Journal of Biomechanics*, VOL. 38, 2005, 1387–1395.
18. S. Plagenhoef, F.G. Evans and T. Abdelnour: Anatomical data for analyzing human motion, *Research Quarterly for Exercise and Sport*, vol. 54, 1983, 169–178.
19. P. de Leva: Adjustments to Zatsiorsky-Seluyanov's Segment Inertia Parameters, *Journal of Biomechanics*, 29 (9), 1996, 1223–1230.
20. J.G. de Jalón and E. Bayo: *Kinematic and dynamic simulation of multibody systems: the real-time challenge*, Springer-Verlag, New York, NY, 1994.
21. J. Kövecses and J. M. Font-Llagunes: An Eigenvalue Problem for the Analysis of Variable Topology Mechanical Systems, *ASME Journal of Computational and Nonlinear Dynamics*, 4(3), 2009, (9 pages).

Ambrus Zelei, M.Sc., Research assistant: HAS-BME Research Group on Dynamics of Machines and Vehicles, Muegyetem rkp. 3, Budapest, H-1111, Hungary. The author gave a presentation of this paper during one of the conference sessions (*zelei@mm.bme.hu*).

László Bencsik, M.Sc., Ph.D. student: Budapest Univ. of Technology and Economics, Dept. of Applied Mechanics, Muegyetem rkp. 3, Budapest, H-1111, Hungary (*bencsik@mm.bme.hu*).

László L. Kovács, Ph.D., Senior researcher: HAS-BME Research Group on Dynamics of Machines and Vehicles, Muegyetem rkp. 3, Budapest, H-1111, Hungary (*kovacs@mm.bme.hu*).

Gábor Stépán, DSc., Professor: Budapest Univ. of Technology and Economics, Dept. of Applied Mechanics, Muegyetem rkp. 3, Budapest, H-1111, Hungary (*stepan@mm.bme.hu*).