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Computed torque control of a constrained manipulator considering the actuator saturation

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Abstract: The saturation of the actuators causes serious nonlinearity, and the consideration of the bounded actuator torques in the computed torque control (CTC) method is a challenging task. In this work we handle the saturation of the actuator torques as the decrease of the number of independent control inputs. If the number of the independent control inputs is less than the degrees of freedom of the manipulator than the system is underactuated. Thus, because of the saturation of some actuators a fully actuated manipulator becomes underactuated. In this work we apply the combination of two CTC algorithms. In the case of saturation we switch from the classical CTC method to one which is generalized for underactuated systems. Furthermore, in case of complex structured constrained manipulators the dynamical model of the system is subjected to geometric constraints, resulting a differential algebraic equation (DAE). Thus the applied control methods are extended for DAE modelled systems.

1. Introduction

Controlled dynamical systems with limited actuator performance form a special class of nonlinear systems. The actuator saturation may cause essential problems when a manipulator performs trajectory tracking. Computed torque control (CTC) method also known as inverse dynamics control is an efficient approach for trajectory tracking control problems, although the presence of the physical limits of the actuators are challenging to be handled in CTC method. Several control approaches that are based on CTC method and take into account the limited actuator torques can be found in the literature.

In [1] a continuous-time predictive control approach is used to derive the nonlinear constrained control law for tracking control in the presence of actuator saturation. The proposed method is limited for those systems that are input-output feedback-linearizable after a specific dynamic expansion. The control minimizes the tracking errors even with saturated actuators. In reference [2] an adaptive, full-state feedback controller and in parallel an exact model knowledge, output feedback controller are designed and a comparative numerical analysis is fulfilled to demonstrate the benefits of the two proposed controller methods. On the basis of the classical computed torque control method, a composite nonlinear feedback design method for robot manipulators with bounded torques is presented in [3]. The resulting controller consists of two loops. The inner loop is for the full compensation for manipulators nonlinear dynamics and the outer loop is the composite nonlinear feedback controller for stabilisation and performance enhancement.

The above mentioned control approaches handle the actuator saturation as a nonlinear behaviour of the system. As an alternative approach, actuator saturation can be considered as the decrement of the number of independent control inputs. Let us consider a fully actuated system, where the number of independent control inputs equals to the degrees of freedom (DoF) of the system. When some of the actuators saturate the number of independent control inputs becomes less than the DoFs thus the system can be handled underactuated. In this work we apply a computed torque control (CTC) method developed for underactuated systems to handle the actuator saturation. Additionally the present method is proposed for complex constrained systems which need to be modelled by redundant descriptor coordinates.

2. CTC method for underactuated and geometrically constrained systems

In the case of a fully actuated robot manipulator an independent control input is possible to be defined for each DOF. Thus the classical CTC method can easily be applied for such systems, especially when they are modelled in the classical way using minimum set of generalized coordinates and equation of motion in ODE form [4].

In contrast, the control problems are more difficult for underactuated robot manipulators in general. The computed torque control method was generalized for underactuated systems [5] and was called computed desired computed torque control (CDCTC) method where the term "desired" refers to the fact that the desired value of a set of uncontrolled coordinates has to be calculated first, thus after the calculation of the desired zero dynamics, the control inputs are determined. This method requires the separation of the generalized coordinates into controlled and uncontrolled ones.

The CTC method for underactuated systems can be further generalized for systems modelled by non-minimum set of descriptor coordinates, when geometric constraint equations are introduced. In such case the dynamical model can be written in the form of a differential algebraic equation, which has the following general form [6]:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{q})\boldsymbol{\lambda} = \mathbf{Q}(\mathbf{q}) + \mathbf{H}(\mathbf{q})\mathbf{u}, \tag{1}$$

$$\boldsymbol{\phi}(\mathbf{q}) = \mathbf{0}, \tag{2}$$

where $\mathbf{M} \in \mathbb{R}^{n\times n}$ is the constant massmatrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) \in \mathbb{R}^n$ is the vector of the forces arise from the dynamics of the system, and $\mathbf{\Phi}_{\mathbf{q}}(\mathbf{q}) = \partial\boldsymbol{\phi}(\mathbf{q})/\partial\mathbf{q} \in \mathbb{R}^{m\times n}$ is the constraint Jacobian associated with the geometric constraints $\boldsymbol{\phi}(\mathbf{q}) \in \mathbb{R}^m$. $\mathbf{Q}(\mathbf{q}) \in \mathbb{R}^n$ is the vector of gravitational forces. $\mathbf{H}(\mathbf{q}) \in \mathbb{R}^{n\times l}$ is the control input matrix and $\mathbf{u} \in \mathbb{R}^l$ is the control input vector. In the fully actuated case the dimension of the control input l is equal to the degrees of freedom, so $l = n - m$. For underactuated systems $l < n - m$ stands.

The inverse dynamical calculations have unique solution if the number of control inputs and the dimension of the task is equal [7]. This statement stands both for underactuated and fully actuated cases. Thus we assume that the task is defined by l number of algebraic equations. This set of additional constraint equations are the so-called servo-constraints (control-constraints) $\boldsymbol{\phi}_s(\mathbf{q},t) \in \mathbb{R}^l$:

$$\boldsymbol{\phi}_s(\mathbf{q},t) = \mathbf{0}. \tag{3}$$

After the introduction of servo-constraint equations the n number of independent descriptor coordinates are constrained by $n = m + l$ constraint equations in fully actuated case. In underactuated systems $n > m + l$ which means that a part of the dynamics is independent from the geometric and servo-constraints.

In this work we do not assume that the servo-constraints can be satisfied with bounded control forces. On the contrary the control method handles the saturation effect. In the simplest concept the control algorithm calculates the desired control input vector $\mathbf{u}$ and than check if each control input value exceeds the limiting value. The number of the unsaturated actuators is l_c. The algorithm calculates again the desired control input $\mathbf{u}$ with $l_c < l$ which is related to an underactuated system. For the $l - l_c$ number of saturated actuators the limit value is commanded by the controller. This operation is executed until every control input value in $\mathbf{u}$ is under the limiting value or every actuator saturates.

The dimension reduction of the servo-constraint vector is a critical issue, because its dimension has to be equal with the number of independent actuators l_c. In the saturated cases we use $\boldsymbol{\phi}_{sc} \in \mathbb{R}^{l_c}$ instead of $\boldsymbol{\phi}_s \in \mathbb{R}^l$. The transformation between $\boldsymbol{\phi}_{sc}$ and $\boldsymbol{\phi}_s$ is not unique, several optimization approach is possible to use.

In the present CTC method we apply the backward Euler discretization of the DAE system (1)-(3) directly and the resulting set of nonlinear algebraic equations are solved by the Newton-Raphson method for the desired control inputs $\mathbf{u}_i$, descriptor coordinates $\mathbf{q}_i$, velocities $\mathbf{y}_i$ and Lagrange multipliers $\boldsymbol{\lambda}_i$ for the upcoming time instant [7, 8].

The control law is formulated by introducing $\mathbf{y}^d = \dot{\mathbf{q}}^d$ and deriving the first order form of the dynamical equation (1). The geometric constraint equation (2) is also involved into the control law. The servo-constraint equation (3) is considered in the level of acceleration and similarly to the Baumgarte stabilization method [9] a stable dynamics is provided by the properly chosen P and D parameters.

$$\mathbf{q}_i - \mathbf{q}_{i-1} = h\mathbf{y}_i, \tag{4}$$

$$\mathbf{y}_i - \mathbf{y}_{i-1} = h\mathbf{M}^{-1}\left[-\mathbf{C}(\mathbf{q}_i, \mathbf{y}_i) - \mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{q}_i)\boldsymbol{\lambda}_i + \mathbf{Q}(\mathbf{q}_i) + \mathbf{H}(\mathbf{q}_i)\mathbf{u}_i\right], \tag{5}$$

$$\mathbf{0} = \boldsymbol{\phi}(\mathbf{q}_i), \tag{6}$$

$$\mathbf{G}_{\mathbf{q}}(\mathbf{q}_i)[\mathbf{y}_i - \mathbf{y}_{i-1}] = -h\left[\dot{\mathbf{G}}_{\mathbf{q}}(\mathbf{q}_i, \mathbf{y}_i)\mathbf{y}_i + \dot{\mathbf{c}}(t_i) + D[\mathbf{G}_{\mathbf{q}}(\mathbf{q}_i)\mathbf{y}_i + \mathbf{c}(t_i)] + P\boldsymbol{\phi}_s(\mathbf{q}_i)\right], \tag{7}$$

where $\mathbf{G}_{\mathbf{q}}(\mathbf{q}) = \partial\boldsymbol{\phi}_s(\mathbf{q}, t)/\partial\mathbf{q}$ and $\mathbf{c} = \partial\boldsymbol{\phi}_s(\mathbf{q}, t)/\partial t$. One advantage is that the coordinates do not have to be separated into controlled and uncontrolled part, which is not always possible.

3. Numerical simulation

A numerical simulation was accomplished for a two-link manipulator (shown in figure 1) that consist of two homogeneous prismatic bars with parameters $m_1 = 0.2[kg]$, $L_1 = 0.4[m]$, $m_2 = 0.2[kg]$, $L_2 = 0.4[m]$ respectively. The Cartesian coordinates of the endpoints of the

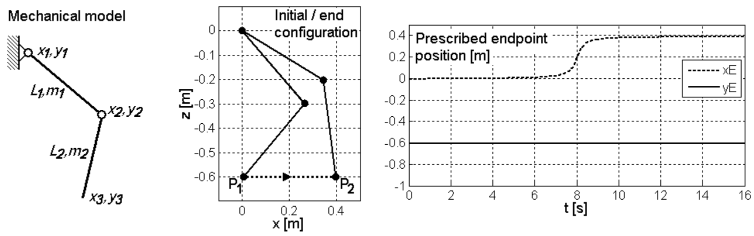

Figure 1. The prescribed trajectory of the endpoint

bars $\mathbf{q} = [x_1, y_1, x_2, y_2, x_3, y_3]^T$ are used as $n = 6$ number of descriptor coordinates, thus 4 geometric constraints arise in the form:

$$\boldsymbol{\phi} = \left[x_1,\ y_1,\ (x_2 - x_1)^2 + (y_2 - y_1)^2 - L_1^2,\ (x_3 - x_2)^2 + (y_3 - y_2)^2 - L_2^2\right]^T. \tag{8}$$

Both of the two joints are actuated, so the $l = 2$ dimension control input vector is $\mathbf{u} = [T_1, T_2]^T$. The actuators saturate when the actuator torques reach $|T_i|_{max} = 0.04[Nm]$ value. The task of the manipulator is to go from point P1 to P2 on a prescribed trajectory as shown in figure 1 middle. The prescribed time history of the endpoint coordinates is shown in figure 1 right. The task was defined by the following servo-constraint vector:

$$\boldsymbol{\phi}_s = \left[x_3 - x^d,\ y_3 + 0.6\right]^T, \tag{9}$$

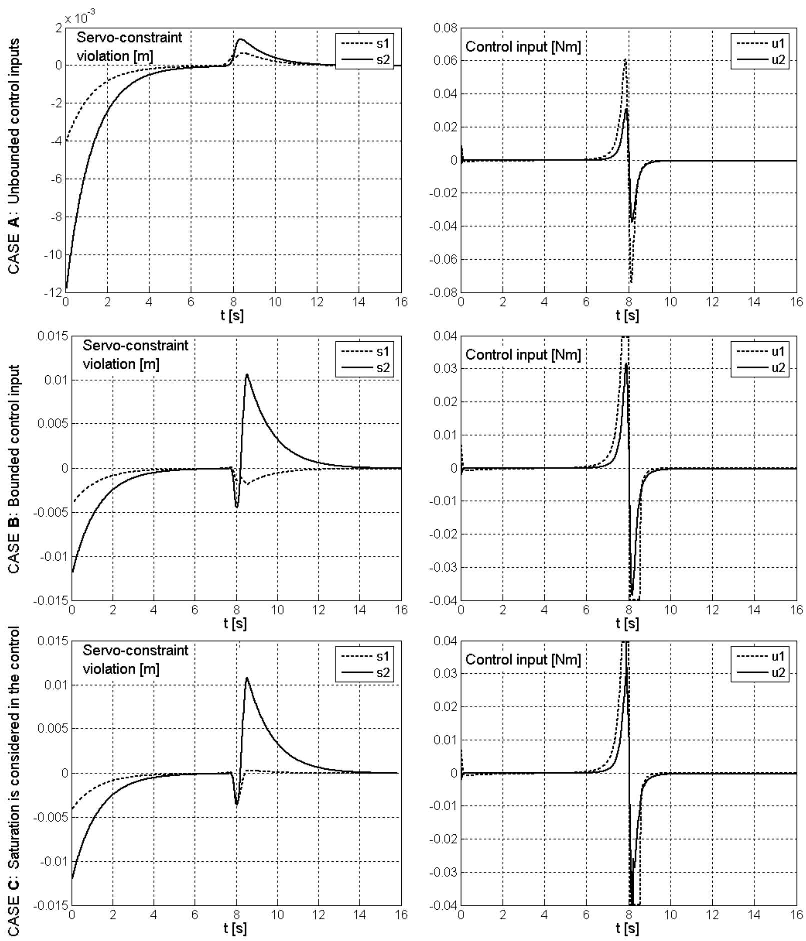

Figure 2. Numerical results for case "A", "B" and "C"

where x^d was described by an arc tangent function.

The numerical results shown in figure 2 were obtained in three different cases. In case "A" the actuator saturation was not considered and the value of the control torques reached 0.08[Nm] absolute value as it can be seen on the right side in the first row of figure 2. The actuator torques T_1 and T_2 are denoted by "u1" and "u2" respectively on the graphs. The 4 [mm] and 12[mm] initial servo-constraint errors went to zero in a short time, and the servo-constraint violation was kept under 0.2[mm] as shown in the left side. In the graphs "s1" and "s2" denotes the first and the second servo-constraint respectively. The fluctuation at $t = 8$[s] was caused by the numerical errors in the backward Euler discretization based control algorithm.

Case “B” shows the effect of the actuator saturation while the saturation was not handled in the control algorithm. Significant increment occurred in the servo-constraints when the actuator torque T_1 reached the critical 0.04[Nm] absolute value, as it can be seen on the graphs in the second row of figure 2. Note that the second control input T_2 stayed under the saturation value.

In case “C” the controller took into consideration the actuators' limitations. When the first actuator saturated the second control input was recalculated by the underactuated control algorithm, and this caused the higher value of the T_2 torque. As a main result the first servo-constrain violation denoted by “s1” went to zero much faster than in case “B”. One can observe that the second actuator also saturated for a very short time in case “C”. It shows that the power of the actuators are utilized more efficiently than in case “B”.

4. Conclusions

The computed torque control algorithm developed for underactuated systems was successfully applied for handling actuator saturation. Numerical simulations showed the efficiency of the proposed algorithm. A general and efficient method for the dimension reduction of the servo-constraints is required in further research work.

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