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# TRAJECTORY GENERATION OF AN UNDERACTUATED AND REDUNDANT SERVICE ROBOT PLATFORM EQUIPPED WITH DUCTED FAN ACTUATORS

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*Abstract:* The paper investigates the motion planning of a suspended service robot platform equipped with ducted fan actuators. The platform consists of an RRT robot and a cable suspended swinging actuator with ducted fans that form a subsequent parallel kinematic chain. The system is redundant and, in spite of the complementary ducted fan actuators, it is underactuated. The paper discusses the inverse dynamics problem and proposes trajectory generation methods for the motion planning of the complex hybrid robotic system. The closed form results include the desired actuator forces as well as the nominal swing angle corresponding to the desired motion of the carried payload. Numerical simulation and experiments demonstrate the applicability of the presented concepts.

# 1. Introduction

Obstacle avoidance is an important problem in service and mobile robotics, especially when the robots have to move in an everyday indoor environment. Static obstacles on the floor of a room include various objects such as stairs, doorsteps, chairs, tables and even the edge of carpets. In addition, floor based domestic robots need to have strategies to overcome randomly placed objects, e.g., children toys left on the floor. Hence, these robots need improved sensing capabilities and control, or the use of auxiliary devices, e.g., virtual walls to avoid obstacles.

A new direction in the development of indoor service robots is the use of robotic structures that can move on the almost obstacle free ceiling of a room, while transport the payload or an actuator mechanism similarly to gantry cranes. For example, the Flora ceiling absorbed service robot [1] utilizes permanent magnets to keep and move its mobile cart on the ceiling, and its working unit is actuated by three telescopic arms in the vertical direction. In reference [2], a similar concept is applied to direct a webcam towards objects in an online laboratory, where the ceiling based robots are commanded by distant users. A cable suspended parallel robot with four arms that can dispense cables by shooting them towards a steel ceiling is disclosed in [3]. The attachment mechanism is based on permanent magnets, but due to the fixed anchoring of the suspending cables the workspace of the robot is limited.

A design of a tethered aerial robot is presented by [4], in which the working unit is suspended on a single cable and equipped with two ducted fan actuators. The system is transported by an unmanned aerial vehicle, and the fans are used to stabilize the motion of the working unit that carries tiny robotic agents, which are used for exploring tasks in rescue operations. Although the described service robotic application is outdoor by itself, the concept can be realized in indoor environments by substituting the aerial vehicle with a ceiling based transporting unit.

The above solutions solve the problem of avoiding obstacles on the floor, while they are able to roam over almost the whole inner space of a room, and compared to gantry cranes, they enable the use of co-operating multiple units. Cable suspended structures provide larger vertical workspace and they are lightweight, however, the under-actuation of the suspended payload is a critical issue.

The present paper describes a new indoor service robot locomotion technology developed within the ACROBOTER (IST-2006-045530) project [5]. The suspended payload platform is equipped with windable complementary orienting cables and ducted fan actuators in order to resolve the underactuation of the payload.

### 2. The ACROBOTER platform

Structure and the first prototype of the ACROBOTER service robotic platform [5] are presented in Figure 1. The system is attached to the ceiling by using anchor points that are simple and passive elements providing fixation and through which the power is supplied. The climber or crawling unit (CU) is an RRT robot which can move in a grid of anchor points and transport the working unit of the system in the horizontal direction. The used planar RRT structure is redundant and provides smooth change of anchor points with respect to the motion of its end effector, i.e., the widing mechanism of the main suspending cable (MC). The lower end of the MC is attached to the cable connector (CC), which clamps three additional windable cables that orient the base plate of the swinging unit (SU). The SU is equipped with six ducted fan actuators which generate thrust forces parallel to the plane of the base plate and a torque that can rotate the SU at the same time. A mechanical interface is attached to the bottom of the SU that serves as a tool changer with electrical connections.

From the dynamics modeling point of view, the small sized CC can be considered as point mass and the weight of the cables can be neglected. Then the degrees-of-freedom (DoFs) of the whole system is 12 in total containing the 3 plus 3 DoFs of the CU and the CC, and the 6 DoFs of the SU, respectively. The actuators of the system is the 3 joint drives of the CU, the 4 cable winches and the



Figure 1: Structure (left) and the first prototype (right) of the ACROBOTER platform

effective thrust forces (a coplanar and perpendicular pair) and torque provided by the ducted fans. Consequently, despite of these complementary actuators, the system is still underactuated. This analysis refers to the fact that the CC can only be actuated by the windable cables and its horizontal position cannot directly be actuated. Moreover, the vertical position of the SU is determined by both the lengths of the secondary cables and the main cables, which makes the system kinematically redundant.

#### 2. Trajectory generation and inverse dynamics

In case of underactuated systems the solution of the inverse kinematics problem naturally involves dynamic calculations. In addition, the redundancy of ACROBOTER may be resolved by optimization. However, applying some reasonable simplifications the complex problem of the trajectory generation of the underactuated and redundant robot can be converted into the trajectory generation problem of a double pendulum system. For brevity the method is presented for the spatial double pendulum, but the results are compared with the planar model of ACROBOTER only. The investigated pendulum models and the complete planar model are presented in Figure 2.

# 2.1 Single pendulum model

The ducted fan actuators generate typically low thrusts compared to the possible total weight of the robot including the payload. Thus they can be excluded from the dynamical model during trajectory generation. The prescribed trajectories of the CU and the windable cables have to be determined to provide the desired pendulum-like motion of the platform during which the orientation



Figure 2: Single pendulum (left), double pendulum (middle) and complete planar models (right)

of the SU is kept constant (e.g., horizontal) by the secondary cables (see Figure 1). In the control of the system this trajectory can be used for calculating feedforward torques for the corresponding actuators, while the ducted fans can be used for stabilizing the motion of the SU along its prescribed trajectory. In addition, since the CC has much lower weight than the SU, the CC might be seen as a source of high frequency but small disturbances, only.

These considerations lead to the single pendulum model presented left in Figure 2, where  $x = x^d$ ,  $y = y^d$ ,  $z = z^d$  are the desired positions of the centre of mass of the SU. The goal of the trajectory generation and inverse dynamics tasks is to define the corresponding desired trajectory of the CU denoted by  $x_{CU}^{d}$  and  $y_{CU}^{d}$ , the desired forces  $F_{x}^{d}$  and  $F_{y}^{d}$  that has to be applied at the upper end of the MC, and the nominal cable length  $l^d$  in closed form.

Considering the cable length as a time varying (but not explicitly time dependent) constraint the equation of motion of the single pendulum model can be expressed in the convenient form

$$\lambda_1 (x_{CU} - x) - F_x = 0 , \quad \lambda_1 (y_{CU} - y) - F_y = 0 , \qquad (1)$$

$$m_{SU}\ddot{x} - \lambda_1(x_{CU} - x) = 0$$
,  $m_{SU}\ddot{y} - \lambda_1(y_{CU} - y) = 0$ ,  $m_{SU}\ddot{z} + \lambda_1 z + m_{SU}g = 0$  and (2)

$$(x_{CU} - x)^{2} + (y_{CU} - y)^{2} + z^{2} - l^{2} = 0 , \qquad (3)$$

where eq. (3) is the constraint equation and  $\lambda_1$  is the corresponding Lagrangian multiplier. This multiplier can directly be calculated from the third expression of eq. (2) and its substitution into the first two expressions of the same equation leads to the desired CU trajectories as follows

$$x_{CU}^{d} = x^{d} - \frac{\ddot{x}^{d} z^{d}}{\ddot{z}^{d} + g} \quad , \quad y_{CU}^{d} = y^{d} - \frac{\ddot{y}^{d} z^{d}}{\ddot{z}^{d} + g} \quad .$$
(4)

Then it is straightforward to calculate the desired length of the MC from eq. (3), and the corresponding winding torque can also be expressed by knowing the dynamics of the winding actuator. The lateral forces that acting upon the suspension point of the MC are defined by eq. (1).

An important result of the above calculation is that the use of the single pendulum model in trajectory generation of the CU requires  $C^4$  continuity of the desired SU trajectory. This is because the CU trajectories need to be two times continuously differentiable to have smooth desired accelerations.

#### 2.2 Double pendulum model

A further step in modeling the ACROBOTER platform could be the double pendulum model presented in the middle of Figure 2. Beyond enabling a more accurate calculation of the desired trajectory of the CU this model makes it possible to consider the redundant actuation of the windable cables in the vertical direction. In the following the vertical distance  $z_{CC} - z$  between the SU and the CC (see middle in Figure 2) is considered as a desired parameter that can be obtained as a fixed value by experiments or it can be calculated based on some optimization method.

Then the inverse kinematics and dynamics of the double pendulum model can be solved in two steps. First the SU can be considered as a single pendulum attached to the CC which is interpreted as a floating suspension point above the SU at a desired elevation. Then the resulting trajectory of the CC plays the role of the desired trajectory of the bob of a single pendulum system composed of the CC and the CU.

Consequently, the desired trajectories of the CC projected into the horizontal plane can be expressed similarly to eq (4) in the form

$$x_{CC}^{d} = x^{d} + \frac{\ddot{x}^{d}(z_{CC}^{d} - z^{d})}{\ddot{z}^{d} + g} , \quad y_{CC}^{d} = y^{d} + \frac{\ddot{y}^{d}(z_{CC}^{d} - z^{d})}{\ddot{z}^{d} + g} ,$$
(5)

where  $z_{CC}{}^d - z^d$  is the desired elevation of the CC above the SU. In part, the desired motion of the CC is provided by the force delivered through its suspending cable, the Cartesian components of which are

$$F_{Cx}^{d} = m_{SU}\ddot{x}^{d} + m_{CC}\ddot{x}_{cc}^{d} , F_{Cy}^{d} = m_{SU}\ddot{y}^{d} + m_{CC}\ddot{y}_{CC}^{d} , F_{Cz}^{d} = m_{SU}\ddot{z}^{d} + m_{CC}\ddot{z}_{CC}^{d} + (m_{SU} + m_{CC})g$$
(6)

where  $m_{SU}$  and  $m_{CC}$  are the masses of the SU and the CC respectively.

Taking into consideration the opposite of the forces in eq. (6) and repeating the procedure presented in subsection 2.1, the desired trajectory of the CC can be calculated as

$$x_{CU}^{d} = x_{CC}^{d} + \frac{m_{CC}\ddot{x}_{CC}^{d} + F_{Cx}^{d}}{\lambda_{1}^{d}}, \quad y_{CU}^{d} = y_{CC}^{d} + \frac{m_{CC}\ddot{y}_{CC}^{d} + F_{Cy}^{d}}{\lambda_{1}^{d}}, \quad \lambda_{1}^{d} = m_{CC}\frac{\ddot{z}_{CC}^{d} + g}{z_{CC}^{d}} + \frac{F_{Cz}^{d}}{z_{CC}^{d}}.$$
(7)

#### 2.3 Complete planar model

In orther to present the applicability of the results obtained for the single and double pendulum models, the complete planar model of ACROBOTER (see right in Figure 2) is considered which also includes the secondary (orienting) cables of the SU.

The dimension of the SU is characterized by the variables *a* and *b*, which together with the vertical distance *c* desribes the location of the centre of mass of the unit. In addition, coordinate  $\xi$  denotes the relative distance of the CC to the rod that represent the SU, while *h* is the elevation of the CC above the SU.

The desired orientation of the SU is considered to be fixed, and without the loss of generality it is assumed that the SU is kept always horizontal by the secondary orienting cables. This restriction is reasonable in a wide range of possible applications, such as pick and place objects in indoor environments.

The desired horizontal orientation of the SU implies that the corresponding angular acceleration is zero, which yields the equation of motion of the planar model in the simplified form

$$m\ddot{x} = F_1 \cos \varphi_1 - F_2 \cos \varphi_2 , \quad m\ddot{z} = F_1 \sin \varphi_1 + F_2 \sin \varphi_2 - mg \text{ and}$$
(8)

$$0 = -F_1(a\sin\varphi_1 + c\cos\varphi_1) + F_2(b\sin\varphi_2 + c\cos\varphi_2), \qquad (9)$$

where  $F_1$  and  $F_2$  are the cable forces of the orienting cables and  $\varphi_1$  and  $\varphi_2$  are their angles at the SU, respectively.

In equations (8) and (9) the number of unknown parameters is 5 in total, since the trajectories x(t) and y(t) of the centre of mass of the SU are prescribed. When the elevation h of the CC above the SU is also a desired parameter, the necessary independent constraint equations can be formulated for the cable angles as

$$\cos \varphi_1 = \frac{\xi}{\sqrt{\xi^2 + h^2}} , \quad \cos \varphi_2 = \frac{a + b - \xi}{\sqrt{(a + b - \xi)^2 + h^2}} . \tag{10}$$

Then the solution of the system of equations (8-10) results in the desired relative position of the CC in the form

$$\xi^{d} = \frac{(c+h^{d})\ddot{x}^{d} + a(\ddot{z}^{d} + g)}{\ddot{z}^{d} + g} , \qquad (11)$$

which reduces exactly to eq. (5) with the substitutions of expressions  $x_{CC}^{\ d} = x^d - a + \xi^d$  and  $z_{CC}^{\ d} = z^d + c + h^d$ .

The above heuristic derivations demonstrates that due to the assumption of the horizontality of the SU the complete planar model and the simplified double pendulum model result in the same desired trajectory of the CC.

# 4. Numerical simulation and experiment

Simulation and experimental results corresponding to the single pendulum model of ACROBOTER are presented in Figure 3. The required path of the robot (centre of gravity of the SU) was set to a slanted rectangle with 300mm by 400mm projected size onto the horizontal plane and with 130mm elevation in the *z* direction. These dimensions correspond to maximum workspace of the prototype experimental setup, in which the CU and the winding mechanism of the MC were emulated by a Hirata MB-H230/ MB-H180 3D Cartesian robot (see right in Figure 1.). The single pendulum model is characterized by the estimated cable length of 820mm and the weight 2.8kg of the SU, while the elevation of the CC with respect to the SU was set to 500mm.



Figure 3: Simulation (left) and experimental results (right) based on the single pendulum model

The desired trajectories were calculated by using  $6^{th}$  order polynomial approximation of the required rectangular path. The top left panel in Figure 3 presents one segment of this trajectory in the *x* direction, where the crosses denotes the data used for fitting the polynomial. The bottom left panel summarizes the results of the numerical simulations applied for the same desired trajectory segment. It shows that the calculated trajectories of the CU are close to each other in case of both the single and double pendulum models. During experiments the HIRATA robot were commanded to move along the simulated CU trajectories providing an open loop continuous path control for realizing the desired motion of the SU. In the right panels of Figure 3, the experimental results show that even the applied open loop controller can provide tolerable errors, since the maximum deviation (abs error) from the desired path remained below 60mm that may be compensated by the ducted fans of the SU.

# 5. Conclusions

By using single and double pendulum based dynamical models, the trajectory generation problem of the underactuated and redundant service robot platform ACROBOTER is solved in closed form. The applicability of these concepts is demonstrated by numerical simulation and experiments. Within the limitations of reasonable/practical restrictions the presented results can be generalized for a class of ceiling based service robots including the complex 3D model of the ACROBOTER system.

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# 7. Patents pending

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