THE INFLUENCE OF PARAMETRIC EXCITATION ON FLOATING BODIES

A. Zelei

Ph.D. student, Department of Applied Mechanics,
Budapest University of Technology and Economics, Hungary
H-1111. Budapest, Műegyetem rkp. 5., e-mail: zelei[at]mm.bme.hu

G. Stépán

Professor, Department of Applied Mechanics,
Budapest University of Technology and Economics, Hungary
H-1111. Budapest, Műegyetem rkp. 5., e-mail: stepan[at]mm.bme.hu

Abstract: *In the present study, the influence of the parametric excitation is considered with respect to the stability of floating bodies. We investigate the possibility of stabilizing a floating body that is unstable without parametric excitation. We also inspect the possibility of making a floating body unstable that is stable by itself.*

It is well known that the vertical position of a symmetric floating body is an equilibrium position. First, we identify the other possible equilibria of the floating body. The stability of the equilibria is examined by the analysis of the potential function of the system.

The dynamics of a three-degree-of-freedom planar model of the floating body is investigated. We reduce the problem to a one-degree-of-freedom model by separating a cyclic coordinate in the system, and by prescribing a non-stationary geometric constraint, that is, by moving the centre of gravity periodically in vertical direction. This periodic movement causes parametric excitation. It means that the equation of motion is a non-autonomous differential equation and its structure is in correspondence with the so-called Mathieu equation. By using the stability chart of the Mathieu equation we can find sets of parameters where the stabilization of the floating body is possible. This can have an effect that helps in stabilization in case of a canoeist. But it can cause normally stable ships to lose their stability as well, that is, they can capsize due to the parametric excitation.

Numerical simulations were also accomplished and they confirmed that the stability chart derived for the case of the floating body is correct.

An experiment verified the practical validity of the mechanical model and its analytical and numerical study, and it showed that the ship stabilization is possible via parametric excitation.

Keywords: *parametric excitation, Mathieu-equation, ship motion*

1 INTRODUCTION

In this work we investigate the possibility of the stabilization of a floating body by the parametric excitation.

2 FLOQUET THEORY

Parametric excitation means that some of the parameters of the system change as a periodic function of time. The most general form of a parametrically excited system is given by the Floguet equation,

$$\mathbf{y}'(\tau) + \mathbf{A}(\tau)\mathbf{y}(\tau) = \mathbf{0}, \quad (1)$$

where matrix A is two pi periodic.

$$\mathbf{A}(\tau) = \mathbf{A}(\tau + 2\pi). \quad (1)$$

This system can be reduced to the Hill equation, which is only one dimensional.

$$x''(\tau) + p(\tau)x(\tau) = 0, \quad (1)$$

where the time dependent parameter is periodic:

$$p(\tau) = p(\tau + 2\pi). \quad (1)$$

If the coefficient of x is harmonic, we gain the so-called Mathieu equation.

$$x''(\tau) + (\delta + \varepsilon\cos(\tau)) = 0. \quad (1)$$

This phenomenon can be used for stabilizing unstable processes. The main idea is to eliminate an oscillation with the help of another oscillation. The more general example is the inverse pendulum which can be stabilized by parametric excitation []. So if the amplitude is large enough the upper equilibria can be balanced by parametric excitation. In this work, we examine how a normally unstable floating body can be stabilized with parametric excitation.

The stability chart of the Mathieu equation was derived in nineteen twenty eight. It is known as Incze-Strutt diagram.

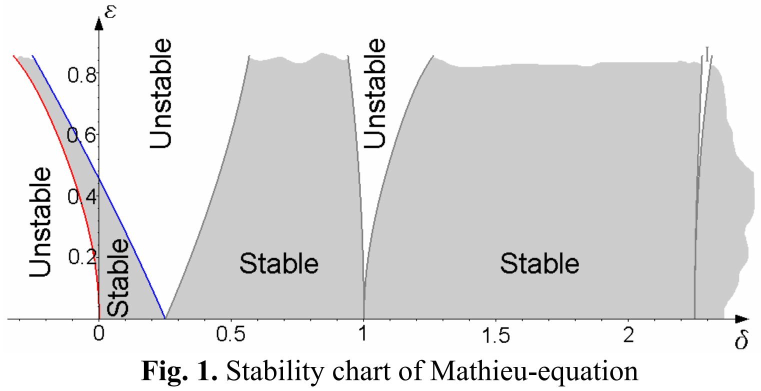

Fig. 1. Stability chart of Mathieu-equation

The basic idea was to search the boundaries in series expansion of δ and x.

$$\delta(\varepsilon) = \delta_0 + \delta_1\varepsilon + \delta_2\varepsilon^2 + \ldots, \quad (1)$$

$$x(\tau,\varepsilon) = u_0(\tau) + u_1(\tau)\varepsilon + u_2(\tau)\varepsilon^2 + \ldots. \quad (1)$$

In this work we are interested in the negative delta region, were the epsilon equals to zero case is unstable.

$$\delta = -\frac{1}{2}\varepsilon^2, \tag{1}$$

$$\delta = \frac{1}{4} - \frac{1}{2}\varepsilon - \frac{1}{8}\varepsilon^2. \tag{1}$$

Let's see the phase space of a two dimensional case. The two pi periodic coefficient matrix means that the motion started two pi later (red) is exactly the same (blue) if the initial contitions are in correspondence. Thus we can fold back the phase space.

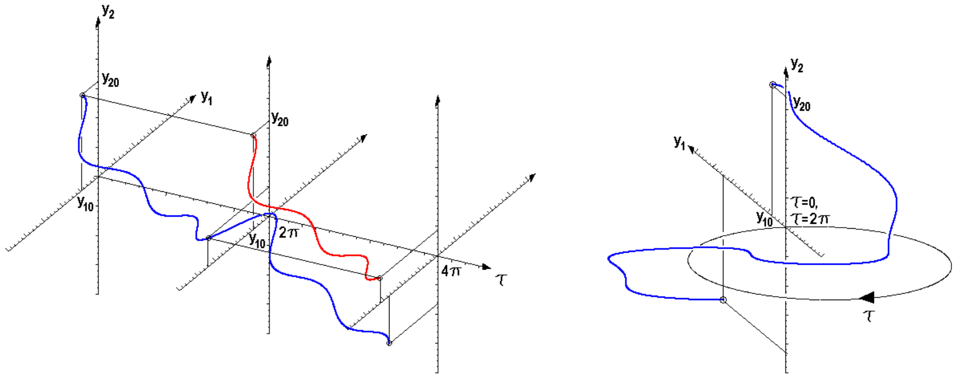

Fig. 1. Phase space of a 2 DoF dynamical system, folded phase space

In the numerical simulations we will investigate the $\tau = 0$ plane. The stability boundaries are defined by the two pi and the four pi periodic solutions of the Mathieu equation. The tau equals to zero plane shows the periodicity of the solutions.

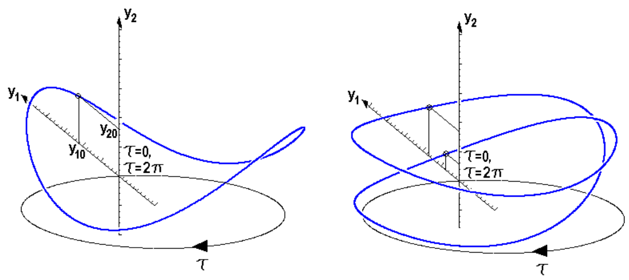

Fig. 1. 2π and 4π periodic solutions

We investigated the application of the Floquet theory for floating bodies. The fundamental question is whether the parametric excitation helps the canoeist to balance himself. Parametric excitation may help the canoeists to balance themselves because they are told to keep a good rhythm of rowing, and not to row faster but row more powerfully when they want to go faster. In this case the parametric excitation is generated by the vertical periodic motion of the sportsman's body.

3 PLANAR MODEL OF A FLOATING BODY

Now we derive a reduced planar ship model. A ship has six degrees of freedom, namely roll, pitch and yaw. These are the rotations. And surge sway heave are the transversal motions. Roll sway and heave will be the three degrees of freedom of the planar mechanical model. In order to gain as simple model as possible for the analytic calculations we investigate planar motion.

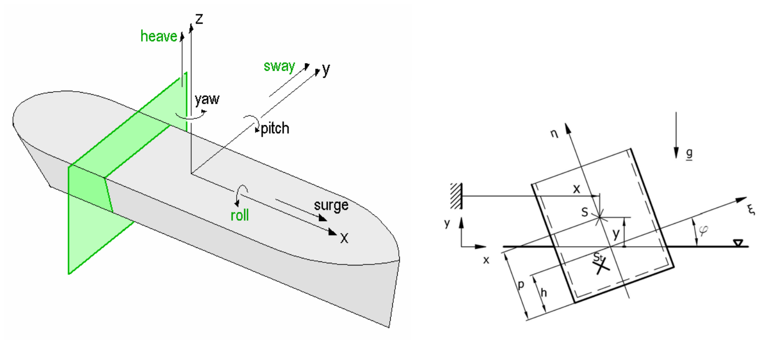

Fig. 1. 2π and 4π periodic solutios

So if we investigate the planar motion of the floating body. The generalized coordinates can be the roll angle φ and the position coordinates x and y of the centre of gravity. The Lagrangian is given by the kinetic and the potential energy. The damping effect of the water is completely neglected. The full Lagrangian can bee seen here.

$$L = \frac{1}{2} m\left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2} J_C \dot{\varphi}^2 - mgy - \mu g\left(\frac{p^2}{2}\cos\varphi - py + \frac{y^2}{2\cos\varphi} + \frac{a^2}{24} tg\varphi \sin\varphi\right). \quad (1)$$

$$\mu = \rho a l . \quad (1)$$

The general coordinate x does not appear in the Lagrangian, so x is a cyclic coordinate. Therefore we can fix the horizontal motion of the body.

4 PARAMETRICALLY EXCITED MODEL – MODEL OF A KAYAK

Now we go on to derive the 1DoF model of the parametrically excited ship model. We prescribe the height of the centre of gravity by a harmonic function. It means that one more generalized coordinate is prescribed, thus we have an only one degree of freedom model. The only generalized coordinate is the roll angle.

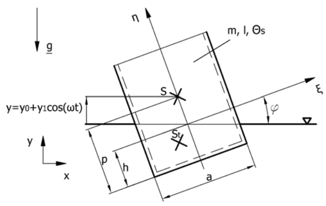

Fig. 1. 2π and 4π periodic solutios

Y0 is also determined by the roll angle. Where h comes from the geometry and the water density. The to parameters of the excitation is the amplitude and the angular frequency.

$$y_0 = (p - h)\frac{1}{\cos\varphi} . \quad (1)$$

$$h = \frac{m}{\mu} . \quad (1)$$

We use the Lagrange equation to derive the equation of motion of the shaken body.

$$J_C\ddot{\varphi}+mg\left(\frac{1}{2}\frac{m}{\mu}-p\right)\sin\varphi+\mu g\left(\frac{{y_1}^2\cos^2(\omega t)tg\varphi}{2\cos\varphi}+\frac{a^2}{24}\left(\sin\varphi+\frac{tg\varphi}{\cos\varphi}\right)\right)=0\,. \qquad (1)$$

Here is the linearized equation of motion. Since we only investigate the stability of the vertical position where phi is zero, the linearized equation of motion can be used. We use the dimensionless time tau, and substitute it into our differential equation. Finally the equation is in complete correspondence with the Mathieu equation. If epsilon is zero and delta is negative than the vertical position is unstable and the ship model capsizes, But it can be stable if the amplitude y1 increases ,that is, the epsilon is non zero.

$$\varphi''+\left\{\frac{\mu g}{4\omega^2 J_C}\left[\frac{m}{\mu}\left(\frac{1}{2}\frac{m}{\mu}-p\right)+\frac{{y_1}^2}{4}+\frac{a^2}{12}\right]+\frac{\mu g}{4\omega^2 J_C}\frac{{y_1}^2}{4}\cos(\tau)\right\}\varphi=0\,. \qquad (1)$$

We can apply the stability chart of the mathieu equation for the floating body. We can transform the original stability chart to a chart that shows the physical parameters, namely the angular frequency and the amplitude of the oscillation. This delta and epsilon are substituted into the equations of the boundaries.

ACKNOWLEDGEMENTS

This research was supported in part by the Hungarian National Science Foundation under grant no. OTKA.

REFERENCES

[] Insperger, T., Horváth R.: *Pendulum with harmonic variation of the suspension point.* Periodica Polytechnika Ser. Mech. Eng. vol. 44, no. 1, pp. 39–46, 2000.

[] van de Pol, F., and Strutt, M. J. O.: *On the Stability of the Solutions of Mathieu's Equation.* Philos. Mag., 5, pp. 18–38., 1928.

[] Floquet, M. G.: *Équations Différentielles Linéaires a Coefficients Péridiques.* Ann. Sci. Ec. Normale Super., 12, pp. 47–89., 1883.

[] Farkas, M.: *Periodic Motions*. Springer, New York, 1994.