Antropomorfic parameters of a nonlinear dynamic model of self sustained hopping

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Abstract: In this work, we study the dynamics of legged locomotion adopting a combination of experiments and mathematical models. Our recently developed planar model is capable of self-sustained human-like hopping motion. However, the biomechanical indicators, such as joint angle ranges, joint velocities and joint torques which provide stable motions are not compatible with realistic parameter values. In general, parameters can be optimized with respect to a variety of cost functions, such as energy efficiency, impact reduction, locomotion speed or hopping height. In contrast to these performance measures, our current objective is to minimize the gap between the measurement and simulation results of human locomotion in terms of the aforementioned biomechanical indicators. This will finally provide realistic anthropomorphic motion, reducing the gap between real life legged locomotion and locomotion based on mathematical models.

1. Introduction

The dynamic analysis of legged locomotion via mathematical models is common in the literature [2, 10]. One of the most simple models is the spring loaded inverted pendulum (SLIP) model of running [3, 10, 11]. Some other models were developed which catch the geometry of the human leg more precisely, e.g., the segmented leg model in [20] provides stable locomotion. Although, our ultimate goal is the analysis of the biomechanics of running (we started in [26]), we believe that the dynamic analysis of hopping locomotion is a proper preliminary study. The dynamic model of a hopping locomotor is less complex than running motion, because only one leg (or two legs moving together) is modeled. We developed a self-stable model of human hopping locomotion and published the main properties in [28].

Prior to the model extension, our goal is to tune physical parameters in order to reach human-like and feasible motion both in terms of kinematic and dynamic characteristics. This is achieved by tuning the control parameters with which the simulated motion is as close as possible to the experimentally obtained motion. In the present paper, we collect the experimental data from the literature and introduce the cost function that we use for the parameter optimization.

Having a well-tuned (human-like) model, we will be able to perform the sensitivity analysis of the parameters and to discover the optimality principles (similarly as in [5])
in future studies. This is especially advantageous, when biomechanical experiments cannot clearly uncover the parameter effects. For instance we will be able to analyse the effect of the foot strike pattern on the impulsive ground reaction forces and energetic costs. Although, there are uncertainties in the experimental studies, as it is mentioned in [1].

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2. The hopping model
The sagittal plane model of human hopping locomotion consists of the equations of motion of the multibody system shown in Fig. 1. Besides, a controller is an inseparable part of the mathematical model. The model published in [6] was further developed in [27, 28, 25]:

- The strike index (see $s$ in Fig. 1) can be varied.
- The locomotion can be analysed on uphill or downhill terrain too (see $\psi$ in Fig. 1).
- It is checked, whether the ground-foot friction force exceed the limiting value.
- A multi-level (partial) feedback controller guarantees that the desired locomotion speed and the desired hopping elevation is accurately maintained regardless the possible perturbations coming from the environment.

The main characteristics of the model are:

- It contains similar geometric nonlinearities as the human body, i.e., the geometry is not oversimplified such as in case of the SLIP model.
- The impulsive forces at the initial ground-foot contact are considered in the model and therefore the impact induced kinetic energy loss (constrained motion space kinetic energy CMSKE [14, 12, 13]) is taken into account.
- An active controller maintains the energy level and the stable periodic motion.
- The model and the calculations are formulated in such way that they can be easily applied for more complex multibody models of locomotion.

2.1. Mechanical part of the model
As Fig. 1 shows, the mechanical model consist of the foot, the shank and the thigh (masses $m_i$ and lengths $l_i; i = 1 \ldots 3$ respectively). Rotational springs with stiffness $k_B$ and $k_C$ maintain the joint angles. Furthermore, a reaction wheel (with mass $m_r$ and mas moment of inertia $J_r$), which is connected in the hip, prevents fallover. One period of locomotion consists of the flight (airborne) phase and the ground (stance) phase. These continuous phases are separated by the events respectively, when the point A reaches the ground, and when the ground-foot contact force becomes zero. For more details and model assumptions (frictionless joints, no rebound or slip of the foot, smooth terrain), see [6, 28, 25].
2.2. Control part of the model

Separate control law is defined for the ground phase and the flight phase (see (1) and (2) respectively):

\[
\begin{bmatrix}
M^F_{B} \\
M^C_{B} \\
M^D_{B}
\end{bmatrix} =
\begin{bmatrix}
-D_B \dot{\theta}_{12} \\
-D_C \dot{\theta}_{23} \\
P(x_A - (x_G + x_\Delta)) + D(x_A - x_G)
\end{bmatrix} \tag{1}
\]

\[
\begin{bmatrix}
M^F_{B} \\
M^C_{B} \\
M^D_{B}
\end{bmatrix} =
\begin{bmatrix}
P_E(E - E_0) \text{sgn}(\dot{\theta}_{12}) \\
P_E(E - E_0) \text{sgn}(-\dot{\theta}_{23}) \\
-P_r \dot{\theta}_r - D_r \dot{\theta}_r
\end{bmatrix} \tag{2}
\]

**Flight phase**: The vibrations are suppressed by torques \(M^F_{B}\) and \(M^C_{B}\). The tip-toe position \(x_A\) is stabilized about the nominal horizontal position \(x_G + x_\Delta\) by the proportional-derivative (PD) controller realized by \(M^D_{B}\). Parameters \(D_B\), \(D_C\), \(P\) and \(D\) in (1) are control gains. In the foot landing preparation, zero moment pole (ZMP) control \([10, 24]\), is achieved by \(x_\Delta = P_\Pi \Pi_A - K_v\), where \(\Pi_A\) is the angular momentum about the point A, \(P_\Pi\) is a control gain and \(K_v\) is a control parameter that affects the locomotion speed.

**Ground phase**: The overall mechanical energy is stabilized around the target energy level \(E_0\) by means of the control torques \(M^F_{B}\) and \(M^C_{B}\). To prevent the continuous growth of the angular velocity \(\dot{\theta}_r\), the torque \(M^D_{B}\) stabilizes the position of the reaction wheel. Variables \(P_E\), \(P_r\) and \(D_r\) are control gains.
2.3. Parameters to tune

The control law defined in (1) and (2) guarantees hopping-like periodic motion in a certain range of the parameters [28]. The goal of the present study is to tune the parameters in such way that the hopping motion is not only stable, but the achieved motion characteristics are human-like and realistic, i.e., the cost function in Section 4 is minimal. The variable control parameters are: \( D_B, D_C, P, D, P_H, K_v, P_E, P_r, D_r, E_0 \). Some physical parameters can be also tuned: the spring stiffnesses \( k_B \) and \( k_C \) and the ankle and knee joint angles \( \alpha_{12} \) and \( \alpha_{23} \) which belongs to the unstretched springs. The goal in Section 4 is to find the point in this \( p = 14 \) dimensional parameter space \( p = [D_B, D_C, P, D, P_H, K_v, P_E, P_r, D_r, E_0, k_B, k_C, \alpha_{12}, \alpha_{23}] \), where the motion characteristics are the closest to the curves which are presented in Section 3.

3. Collection of biomechanical data from the literature

In order to tune the hopping model (presented in Fig. 1) in such way that the simulated time history of the joint angles, the joint torques and the joint powers are realistic and feasible, we have acquired measurement data from the literature. Because of the controversies found, we analysed the data statistically.

Measurement data from articles by Ahn et al [1], Caekenberghe et al [23], Goss [8], Liu [15], Mann [16], and Novacheck [18] were used. The original datasets are not available, therefore they have to be extracted from the presented graphs in articles. After setting the origo and the scales of the axes, 20 points on each graph were pointed manually. It is important to note, that the obtained datasets have small errors compared to the original measurement data, because the points were chosen manually, and the resolution of the screen and the accuracy of the human eye and the mouse have limitations. Spline-interpolation was applied on the sets of 20 points and the average and variance were calculated for each set.

Due to lack of measurement data about the full running cycle, only the stance phase was concerned. Therefore, the initial contact was considered to \( t_0 = 0\% \), and the end, the "toe off" was set to \( t_e = 100\% \). The proportion of the stance phase during running varies strongly, depending on personal attributes. According to [18], it is generally below 50\%, and its minimal length can be 22\% of the running cycle for the best sprinters. In our case, its length was set to 40\%, estimating an average value.

The results can be seen in the Fig. 2-10. The black thick line without markers represents the average and the grey area shows the variance. The differences of the obtained data from the different resousces can be caused by several factors, including individual ones, e.g., age [17], sex [22], and ethnicity-race [21] can cause differences in the anatomy, which affect the gait [19, 7, 4]. Some of the runners were professional and trained, specialized in long-term running or short-term sprinting, while other subjects were not necessarily expe-
rienced athletes; however, the participants were healthy, without any injury related to the musculoskeletal or nervous system. The used measurement devices and the environmental conditions were also different. The participants were running at different speed for each measurement which causes changes in the kinematics, and the stiffness of the surface may also influence the running strategy [9]. Some measurements were taken on treadmill. Still, these data can be applied for model tuning. We collected the ankle angle $\alpha_a$, the ankle torque $\tau_a$, the ankle power $P_a$, the knee angle $\alpha_k$, the knee torque $\tau_k$, the knee power $P_k$, 

Figure 2. Ankle angle during the stance phase

Figure 3. Ankle torque during the stance phase
the hip angle $\alpha_h$, the hip torque $\tau_h$ and the hip power $P_h$, as it is detailed below.

- Ankle angle (shown in Fig 2): For this quantity, relatively large difference was expected among the different resources, because the different running strategies (forefoot and rearfoot strike) highly influence the ankle angle. As it can be seen, the average of the ankle angle values is not the best way to approximate a real movement, because it stays between -10 and 0 degrees. The datasets from each measurement have fair
difference, the variance is high compared to other datasets’ variances.

- **Ankle torque (shown in Fig 3):** The obtained graphs have similar characteristics and relatively small variance, therefore using the average as estimation of the human running is fair. The graphs show that the maximum magnitude ankle torque is exerted in the middle of the stance phase.

- **Ankle power (shown in Fig 4):** The obtained graphs have similar characteristics and
relatively small variance compared to other datasets, therefore using the average as estimation of the human running is fair. The graphs show that negative power (absorption of mechanical energy) is realized in the first half of the stance phase. Positive power (acceleration of the segments) is observable in the second half of the stance phase. Note that the positive peak is three times higher than the negative peak. It refers to the fact that the ankle torque contributes in the propulsion of the body
Figure 10. Hip power during the stance phase

rather than in the damping of the ground-foot collision.

- Knee angle (shown in Fig 5): For this quantity, a quite large amount of data was available. They all have similar characteristics, therefore the average can be considered as a good estimation.

- Knee torque (shown in Fig 6): The obtained graphs have similar characteristics and relatively small variance, therefore using the average as estimation of the human running is fair. The shape of the knee torque is in correlation with the ankle torque: the maximum is in the middle.

- Knee power (shown in Fig 7): The variance of the data is fair. The negative peak in the first part of the stance phase is much larger than the positive peak in the second half of the stance phase. We can drive the conclusion that knee contributes more in the energy absorption at the ground-foot collision than in the propulsion of the body. This is the opposite compared to the ankle behaviour.

- Hip angle (shown in Fig 8): The obtained graphs have similar characteristics and relatively small variance, therefore using the average as estimation of the human running is fair.

- Hip torque (shown in Fig 9): Only three datasets were available, and two of them are from the same paper. All have similar, ascending characteristics. For future investigations, acquiring more data could improve the quality.

- Hip power (shown in Fig 10): According to the literature, running on a treadmill does not cause any difference in the kinetic parameters. However, the power of the
hip definitely changes. The cause of this phenomena should be investigated in the future.

Despite of the several possibilities to improve the data set, the current method and acquired data can be considered suitable for tuning the mechanical model.

### 4. Cost function and parameter tuning

Our goal is to tune the parameters \( p \in \mathbb{R}^{14} \) in such way that the scalar cost function \( F(p) \), which is composed by the experimental data shown in Section 3 and the simulated data, is minimal. The cost function is formulated by using the experimental data collected in a vector: \( \mathbf{V}(t) = [\alpha_a(t), \tau_a(t), P_a(t), \alpha_k(t), \tau_k(t), P_k(t), \alpha_h(t), \tau_h(t), P_h(t)]^T \):

\[
F(p) = \int_0^{T_s} \left( \mathbf{V}(t) - \tilde{\mathbf{V}}(p, t) \right)^T \mathbf{W} \left( \mathbf{V}(t) - \tilde{\mathbf{V}}(p, t) \right) dt, \tag{3}
\]

where the time histories coming from the simulated motion with parameter set \( p \) are collected in \( \tilde{\mathbf{V}}(p, t) = [\tilde{\alpha}_a(p, t), \tilde{\tau}_a(p, t), \tilde{P}_a(p, t), \tilde{\alpha}_k(p, t), \tilde{\tau}_k(p, t), \tilde{P}_k(p, t), \tilde{\alpha}_h(p, t), \tilde{\tau}_h(p, t), \tilde{P}_h(p, t)]^T \), and the weight matrix \( \mathbf{W} \in \mathbb{R}^{9 \times 9} \) is diagonal with \( W = \text{diag}(w_1, \ldots, w_9) \). For first trial, \( \mathbf{W} \) is chosen as identity.

For demonstration purposes, we show a simulation result in the human-like range of locomotion speed (0.15 m/s) and hopping elevation (10 cm). Here, the parameters in \( p \) are adopted from [28]. Fig. 11 shows the stroboscopic view of the hopping motion on flat surface. The corresponding cost functions are collected in Fig. 12. These are compared with the experimental results shown in Section 3.

In case of the joint angles (upper three panels), the simulation results are fair. Even the experimentally obtained ankle angles have controversies, thus we couldn’t expect good correspondence of the measurements and the simulations. The simulated knee angle time history is quite close to the experiments. The trends of the hip angle curve are also good, however it could be enhanced quantitatively.

The joint torques in the middle row of Fig. 12 seem unnatural comparing to the experimental data. It is caused by the ground-to-flight and flight-to-ground switches of the controller and by the artificially introduced saturation of the torques. Consequently, not only the parameter tuning but the review of the control law equations (1) and (2). The abrupt switching should be avoided.

The most important difference of the mechanical power of the joint torques from the experimental data is that the controller intentionally ensures positive power in the entire ground phase. This induces again the review of the control law. However, the magnitude of the mechanical power is realistic.
Figure 11. Stroboscopic view of the hopping motion

Figure 12. Components of the cost function in case of a typical simulation result: $\alpha_a(t)$, $\tau_a(t)$, $P_a(t)$, $\alpha_k(t)$, $\tau_k(t)$, $P_k(t)$, $\alpha_h(t)$, $\tau_h(t)$, $P_h(t)$.

5. Conclusions
We have collected experimental data from the literature for the joint angles, joint torques and the joint power for human running. We have shown that most of these data can be used as a template motion pattern, and we formulated a cost function with which the model parameters are tuned. The experimental data were compared to the not yet optimized simulated motion and we have concluded that in spite of the fact that the joint angles are fair, the joint torques and power are not realistic because of the nature of the controller. The
parameter optimization will be done (in future work) after the review of the control law.

For future investigations, there are several ways to improve the data collection. Using larger amount of input data can lead to more supported result, which is much closer to the "average" human running. The data collection from graphs can be fully automatized. Creating new measurement data, the same conditions and methods can surely be applied for each individual, e.g., the movement of all participants is recorded with the same equipment, the data is evaluated with the same method, and the participants runs with the same speed. In future work, the measurement dataset will be categorized based on age, sex and training level. For these different datasets, different model parameters will be found.

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