Numerical Stability Analysis of the Conservative SLIP Model with a Hill-Type Muscle

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EXTENDED ABSTRACT

1 Introduction

The human balancing process is an interesting research topic which requires multidisciplinary knowledge. Various models are used to analyze the hopping, running movements and the underlying causes of its stability in case of humans, animals and biomimetic robots. Several types of muscle models can be used to consider the non-linearities of the muscle system.

2 Mechanical model

We study the spring-loaded inverted pendulum (SLIP) model [1]. However instead of the spring, we attached a simplified Hill-type muscle to the point-mass, which consists of the active element only. Note that since the muscle can only exert pulling force, we need to invert the muscle model such as in the work of Häufle et al. [2], where they analyzed a similar model for a vertical hopping. However, we consider running movement assuming stability arises from this kind of movement. Let us denote the state variables of the system as $\mathbf{x}$:

$$\mathbf{x} = [x, y, \dot{x}, \dot{y}]^{\mathrm{T}}, \tag{1}$$

where x, y are the Cartesian-coordinates of the point-mass. The intrinsic properties of the muscle model can be considered in different complexities. At this point, we consider conservative autonomous systems only. Hence we neglect the force-velocity relation of the active element of the muscle so that we assume the relation is constant, moreover we assume constant maximal muscle activation such as in [2]. Three alternative force-length relation is studied: constant, linear and non-linear Hill-type.

3 Methods

The system is non-smooth, and it is capable to move on a periodic trajectory, which consists of a flight and a stance phase. The flight phase equation of motion can be solved in closed form, but the stance phase equation of motion cannot be due to the geometric nonlinearities. First let us choose a Poincaré-section, for convenience let it be the apex point of the flight phase, when $\dot{y}$ is zero. The x variable is a quasi-cyclic coordinate since it has no effect on the motion in the flight phase. However, it appears in the equations regarding the stance phase, but only its relative position matters from the ground attachment point. In case of periodic motion with time period T the solution coincides the Poincaré-section at the same point. Therefore, if we treat the behavior of the system as a black box between t_0 and $t_0 + T$, where $\mathbf{x}(t_0)$ is on the Poincaré section, only two variables affect the stability of the system, y and $\dot{x}$. Note that at this point we only consider autonomous conservative systems, therefore if we fix the mechanical energy of the system, we can express one of these variables as the function of the other. Therefore, the stability of the system can be analyzed via a 1D return map, as shown in Figure 1.

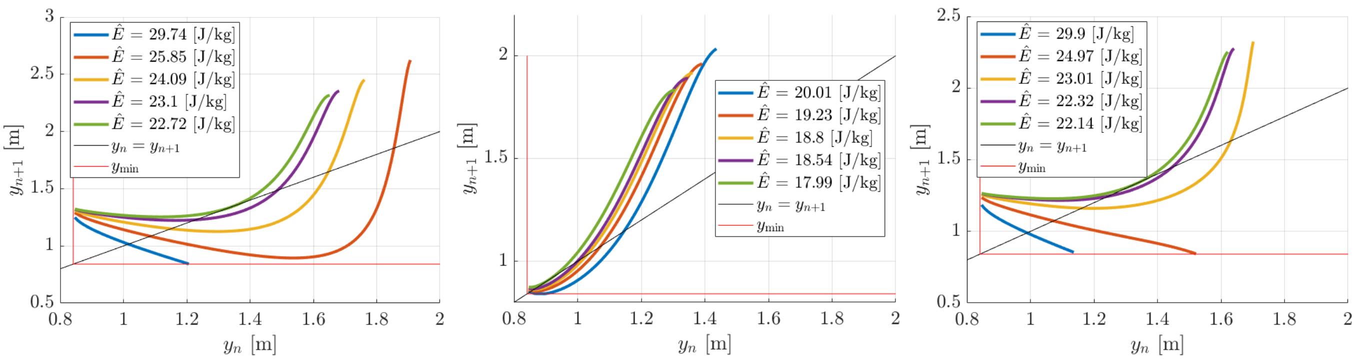

Figure 1: 1D return maps of the system from left to right the force-length relation is: constant, linear, non-linear Hill-type

According to this method, we can conclude that with all the three force-length relations orbital asymptotic stability can be achieved within a mechanical energy range. Moreover, the basin of attraction and bifurcations can also be determined from these maps. In every mechanical energy level where stable hopping is possible, the system has an unstable periodic trajectory too. The stable hopping is associated with lower jumping heights but faster running velocities. As the mechanical energy level is increasing, these two fix points grow apart, and since the upper limit of the basin of attraction (BoA) is the jumping height of the corresponding unstable motion, the BoA is increasing. However, the BoA size can shrink abruptly, because high energy curves intersect the limiting minimal jumping height, which results in a fall-over since the system is not able to take off at the end of the stance phase. Another interesting phenomenon can be observed; there is a lower energy limit for periodic orbits connected with a saddle-node bifurcation.

The question arises; what happens when the system meets with a disturbance that causes it to change its mechanical energy level? For this kind of analysis, the 1D return map is not appropriate. We should find the monodromy matrix $\mathbf{C}$ of the system, which is constructed numerically via the help of the fundamental matrices $\mathbf{\Phi}$ of each phase and the saltation matrices $\mathbf{S}$ connecting these phases [3]. In our case, since the Poincaré-section is in the middle of the flight phase, the principal matrix is as follows:

$$\mathbf{C} = \mathbf{\Phi}_{\mathrm{F}}(t_{\mathrm{apex}})\mathbf{S}_{\mathrm{S2F}}(t_{\mathrm{S2F}})\mathbf{\Phi}_{\mathrm{S}}(t_{\mathrm{S2F}})\mathbf{S}_{\mathrm{F2S}}(t_{\mathrm{F2S}})\mathbf{\Phi}_{\mathrm{F}}(t_{\mathrm{F2S}}). \tag{2}$$

The construction of the matrices in (2) is detailed in the work of Piiroinen et al. [3]. The indexes S and F denote the stance and flight phases respectively. S2F denotes the phase change from stance to flight and vice versa. According to the Floquet theory, the eigenvalues of the principal matrix determine the stability of a solution. In our case, we have two eigenvalues with the value 1, one zero Floquet multiplier and one which depends on the initial conditions. Since the system is autonomous, one of the 1-valued eigenvalue is the trivial one associated with the trivial eigenvector. The zero multiplier is associated with the x direction, since the x coordinate is quasi-cyclic it is projected in every stance phase such a way that the attachment point to the ground is in the origin. The initial condition dependent multiplier is the same as one can determine from the 1D return maps. The remaining 1-valued multiplier suggests that the system can only achieve neutral stability for all force-length complexities in case of arbitrary disturbance. This finding is in correspondence with the fact that the system is conservative, therefore cannot change its mechanical energy level on its own. Therefore, a perturbated solution will converge to the periodic solution on a given iso-energy surface and since the initial conditions which result in periodic motions are continuous, some of the error between the original and the new trajectory will disappear, but if the mechanical energy have changed, a remaining difference can be observed; as shown for dimensionless generalized coordinates in Figure 2.

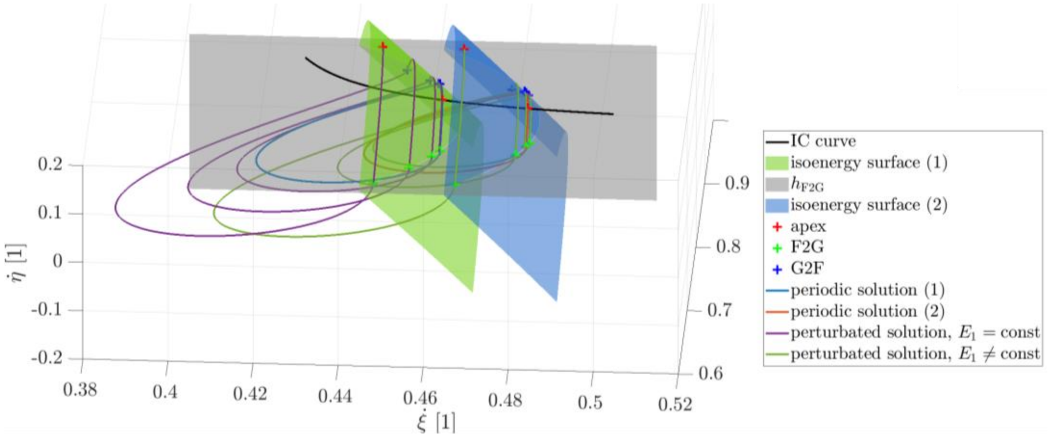

Figure 2: Relation between the iso-energy surfaces and trajectories, h is the switching function

4 Conclusion

For energy conserving perturbations, orbital asymptotic stability can be achieved by a conservative autonomous SLIP model with a simple Hill-type muscle using any of the three (constant, linear and non-linear Hill-type) intrinsic force-length relations. Therefore, running movement provide stability in contrast to vertical hopping, where orbital asymptotic stability cannot be achieved unless at least linear force-velocity relation was considered, and the system lost its conservatism [2].

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