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Discovery and Online Interactive Representation of the Dimensionless Parameter-Space of the Spring-Loaded Inverted Pendulum Model of Legged Locomotion Using Surface Interpolation

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Abstract: The spring-loaded inverted pendulum (SLIP) is a widely used model of legged locomotion. However, a complete map of the dimensionless parameter regions of stable periodic solutions and the basin of attraction cannot be found in the literature. In this work, the minimum set of independent physical parameters was found using the Buckingham π theorem. The three-dimensional space of two dimensionless physical parameters and the dimensionless total mechanical energy of the conservative system was discovered by means of numerical continuation. The fundament of the stability analysis of the piecewise-smooth system was provided by the numerical calculation of the fundamental solution matrices and the monodromy matrix. The energy conserving and non-energy conserving perturbations were addressed in the stability analysis. An effective iteration procedure based on the Nelder-Mead method is presented which tunes the model parameters in order to imitate the motion characteristics of specific animals and locomotion types such as running, trotting and galloping. The results will be available online in the form of an interactive platform.

Keywords: legged locomotion, spring-loaded inverted pendulum, dimensional analysis, numerical continuation, piecewise-smooth dynamical systems

1. Introduction

The SLIP model [1-4], which combines the flight (F) and the ground (G) phases, interprets the CoM trajectory of a real pedal system. The model consists of a mass m at dimensionless position ξ, η and a spring k with natural length r_0. The physical parameters are the dimensionless stiffness $\gamma = k\, r_0/(mg)$ and the pre-touchdown leg angle β. The mechanical energy is the third independent parameter. We aim to create a database of pre-computed isosurfaces of eigenvalues and basin of attraction.

2. Results, Discussion and Conclusion

Figure 1. shows the behaviour of the SLIP model, which is already intricate for a particular physical parameter set. It is even more challenging to obtain a global picture, such as the assessment of the stable region in Fig 2. Here, the curves represent $|\mu|=1$ for the non-spurious eigenvalue. Isosurfaces defined by $|\mu|=c$ are also generated. Based on the continuation results, we develop an interactive online platform which helps researchers, teachers and students to understand the SLIP model and to choose parameters. The platform provides informative plots regarding the parameter zones, where the stable/unstable solutions exist. Surface interpolation techniques make the real-time rendering and therefore the travelling to different segments of the parameter space possible.

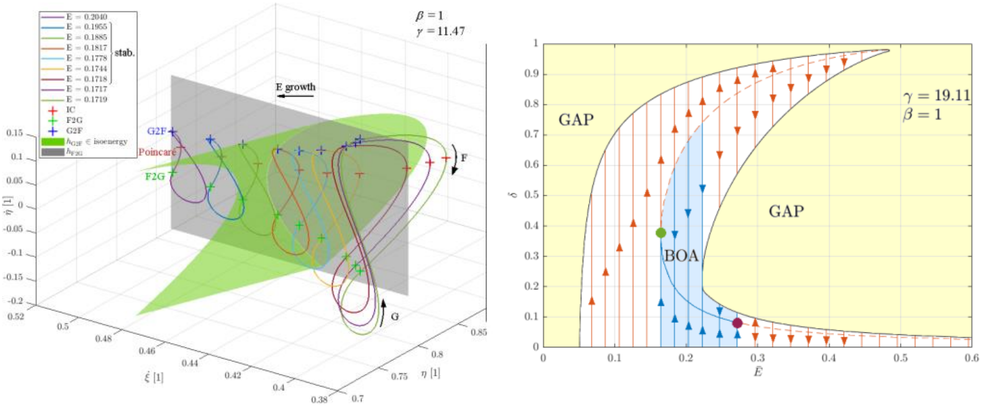

Fig. 1. Stable/unstable periodic solutions on different mechanical energy levels, event- and isoenergy surfaces in the relevant subspace of the state variables (left panel). Stable and unstable periodic solutions shown by blue solid and red dashed curves, respectively; basin of attraction and the gap region, where the Poincaré-return map is not defined, indicated by light blue and pale yellow shading, respectively (right panel).

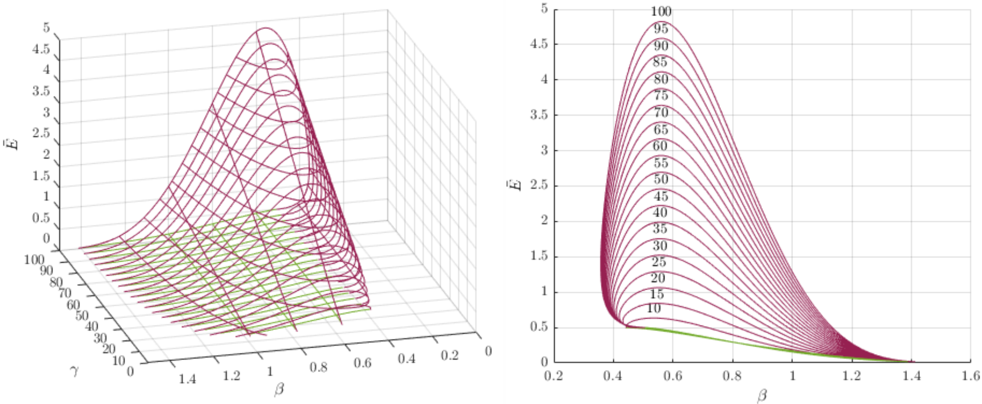

Fig. 2. Enveloping curves of the stable region in the three-dimensional parameter space (left panel). Boundary curves of the stable region for a variety of numerical values of γ (right panel).

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