# Servo Constraint Based Computed Torque Control of ACROBOTER

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**Abstract** This paper presents the computed torque control of the cable suspended service robot platform ACROBOTER. The dynamics of the non-conventional robotic structure is modeled by using a redundant set of descriptor coordinates that are coupled by geometric constraints. The dynamical model is given in the form of differential algebraic equations. The control task of the under–actuated robot is defined by additional servo constraint equations. The proposed controller is based on projecting the equations of motion into the nullspace of the input transmission matrix and utilizing the servo constraints to synthesize a PD compensator.

## **1** Introduction

Service robots operating in indoor environments have to share space with humans and have to overcome various obstacles on the floor. Therefore, from the viewpoint of mobility, ceiling based robots has many advantages over floor based concepts [1, 4]. The ACROBOTER service robot platform is a new ceiling based robot, which is designed to perform pick and place tasks and to carry other service robots with lower mobility in the whole inner space of a room [5]. The first prototype of the platform is shown in Fig. 1. The Climbing Unit (CU) is a planar robotic arm that swaps between anchor points in the plane of the suspended ceiling. The windable suspending cable holds the Swinging Unit (SU) to which the carried objects can be connected. The system has a pendulum like structure, but compared to [4], the positioning of the payload is controlled also by windable orienting cables and ducted fan actuators. The concept combines the planar stepping motion of the CU and the thrusted-hoisted pendulum-like motion of the SU resulting a redundant and underactuated multi-body system.

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Fig. 1 The ACROBOTER service robot platform

## 2 The planar ACROBOTER model

In the planar model of ACROBOTER (see Fig. 2) the CU is a linear drive. The swinging unit is modeled as a rod, while the cable connector is a point mass. These elements are connected by ideal windable cables. The thrust force provided by the ducted fan actuator acting upon the SU at point T, and the centre of gravity of the SU is denoted by CM.

Suppose that the CU follows its desired trajectory perfectly and/or its position can be fed back to the position controller of the SU. Then the 5 DoFs subsystem formed by the SU and the cable connector can easily be described by the 6 Cartesian coordinates of points P<sub>2</sub>, P<sub>3</sub> and P<sub>4</sub>. The use of the descriptor (natural) coordinates  $\mathbf{q} = [x_2 \ y_2 \ x_3 \ y_3 \ x_4 \ y_4]^T$  and the corresponding geometric constraint equation  $\mathbf{\phi} = (x_3 - x_4)^2 + (y_3 - y_4)^2 - l = 0$  yield the equation of motion in the form

$$\mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\lambda} = \mathbf{Q}_{g} + \mathbf{H}\mathbf{u} \quad \text{with} \quad \boldsymbol{\phi} = \mathbf{0}, \tag{1}$$



Fig. 2 The planar model of ACROBTOER

2

where  $\mathbf{M} \in \mathbb{R}^{n \times n}$  is the mass matrix and  $\mathbf{\Phi}_{\mathbf{q}} = \partial \mathbf{\phi} / \partial \mathbf{q} \in \mathbb{R}^{m \times n}$  is the constraint Jacobian associated with the geometric constraints. In addition,  $\mathbf{Q}_g \in \mathbb{R}^n$  and  $\mathbf{H} \in \mathbb{R}^{n \times l}$  are the generalized gravity force and the input transmission matrix, respectively. The control input  $\mathbf{u} \in \mathbb{R}^l$  here is given as  $\mathbf{u} = [F_1 F_2 F_3 F_T]^T$ , where  $F_1$  is the cable force applied through the main suspending cable, the forces  $F_2$  and  $F_3$  are provided by the orienting cables and  $F_T$  is the thrust force. Due to space limitations, for the derivation of the mass matrix and the generalized forces the reader is referred to [2].

#### **3** Computed torque control

The equation of motion (1) can be projected into the nullspace of the actuator forces by applying the transformation  $\mathbf{V} = \text{Null}(\mathbf{H}^{T})^{T}$ . The resulting n - l equations describe the uncontrolled dynamics of system. In reference [3] these equations are solved sequentially for some uncontrolled coordinates by considering a PD controller. The results are used to calculate the reference signals for the uncontrolled coordinates. Instead, this paper proposes to incorporate the servo constraints  $\mathbf{\phi}_s = \mathbf{g}(\mathbf{q}) + \mathbf{p}(t)$  into the projected system of equations, where  $\mathbf{g}(\mathbf{q})$  defines the task and  $\mathbf{p}(t)$  is the desired performance. Then by calculating the servo constraints at the acceleration level as  $\mathbf{G}_{\mathbf{q}}\ddot{\mathbf{q}} = -\dot{\mathbf{G}}_{\mathbf{q}}\dot{\mathbf{q}} - \dot{\mathbf{c}}$  with  $\mathbf{G}_{\mathbf{q}} = \partial \mathbf{g}(\mathbf{q})/\partial \mathbf{q}$  and  $\mathbf{c} = \partial \mathbf{p}(t)/\partial t$ , the internal dynamics of the closed loop system is described by

$$\begin{bmatrix} \mathbf{V}\mathbf{M} \ \mathbf{V}\mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}} \\ \mathbf{\Phi}_{\mathbf{q}} & \mathbf{0} \\ \mathbf{G}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{V}\mathbf{Q}_{g} \\ -\dot{\mathbf{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}} \\ -\dot{\mathbf{G}}_{\mathbf{q}}\dot{\mathbf{q}} - \dot{\mathbf{c}} - \mathbf{K}_{P}\phi_{s} - \mathbf{K}_{D}\dot{\phi}_{s} \end{bmatrix}, \quad (2)$$

where  $\mathbf{K}_P$  and  $\mathbf{K}_D$  are the proportional and differential gain matrices, respectively. In addition, the transformation  $\mathbf{H}^T$  projects eq. (1) into the image of  $\mathbf{H}$  forming *l* independent equations for the control inputs in the form

$$\mathbf{u} = \mathbf{H}^{\dagger} \left( \mathbf{M} \ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}} \boldsymbol{\lambda} - \mathbf{Q}_{g} \right) \quad , \quad \mathbf{H}^{\dagger} = (\mathbf{H}^{\mathrm{T}} \mathbf{H})^{-1} \mathbf{H}^{\mathrm{T}} . \tag{3}$$

To verify the applicability of the method, let the task is to move the center of mass of the SU along a straight line with a trapezoidal velocity profile characterized by the maximum velocity  $v_{max} = 0.25$  m/s and acceleration  $a_{max} = 0.5$  m/s<sup>2</sup>. The desired motion is given by vector  $\mathbf{p}(t) = -[h_{CC}^d x_{CM}^d y_{CM}^d 0]^T$  which defines the elevation of the cable connector above the SU, the position of the point CM and the desired zero tilting of the SU. In terms of coordinates the corresponding task can be given as

$$\mathbf{g}(\mathbf{q}) = \begin{bmatrix} y_2 - \frac{y_3 + y_4}{2} & (1 - \gamma)x_2 + \gamma \frac{x_3 + x_4}{2} & \frac{y_3 + y_4}{2} & y_3 - y_4 \end{bmatrix}^1 , \quad (4)$$

where the factor  $\gamma = 0.8$  is used to couple the dynamics of the cable connector and the SU. The simulation results presented in Fig. 3 clearly show that the proposed controller stabilizes the possible oscillations of the cable connector. To demonstrate



this, a horizontal force impulse (10N for 20ms) is applied from t = 1.5s to the cable connector as an external disturbance.

## **4** Conclusions

In this paper the servo constraint based computed torque control of ACROBOTER was discussed. The proposed formulation makes it possible the simple and fast computation of the control inputs. The presented simulation show satisfactory results even in the case of experimentally tuned control gain matrices. Future work include the stability analysis of the system and the elaboration of methods for the relaxation of the servo constraints in order to avoid the saturation of the actuators forces.

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