Reduction of the effect of actuator saturation with periodic servo-constraints

László Bencsik, Ambrus Zelei, László L. Kovács

MTA-BME Research Group on Dynamics of Machines and Vehicles
Budapest, H-1111, Hungary
[bencsik, kovacs, zelei]@mm.bme.hu

ABSTRACT

The use of servo-constraints, or program definition in the task-space is quite common in trajectory tracking control of manipulators. The task definition is straight forward in case of fully actuated systems. Several algorithms providing the desired motion of the whole system can be found in the related literature. An originally fully actuated system can be handled as underactuated when some of the actuators reach their torque limits. In case of underactuated systems, the task is also given. Based on the task the controlled and uncontrolled directions can be separated. The uncontrolled motion, also referred as the internal dynamics of the system, has to be stable to ensure the stability of the whole system. Thus, the stability of the internal dynamics depend on the output represented by the servo-constraint. It means that in order to get a realizable task, the behaviour of the internal dynamics should be considered, and in many cases the original task has to be slightly modified. This paper presents a novel approach, when the servo-constraints are used for handling actuator saturation and not simply modified, but also switched periodically in time. It will be shown that the application of periodic servo-constraints is decreasing the trajectory tracking error in case of actuator saturation.

1 INTRODUCTION

Working with the control of underactuated mechanical systems can be inspired by several practical and theoretical problems. Beyond the classical underactuated problems, like cranes, flexible structures and aerial vehicles, the actuator saturation also can be mentioned as a characteristic underactuation because the saturated actuator cannot produce the required force or torque so it can be considered as a temporary underactuation [1].

Because of the wide variety of underactuated systems there is no general recipe for the control of them, but some metrics can be found - like the relative degree or the flatness of the system - to categorize them and find an appropriate control algorithm based on that categories. The mechanical modelling of underactuated systems as special type of complex multibody systems are well developed [2, 3]. Most of these rigid body dynamics based approaches use constraint equations in the mathematical model. On one hand, geometric or kinematic constraints give the relation of the dependent descriptor coordinates. Additional geometric constraints defined by the user are responsible for the task definition and therefore they are called servo-constraints. While the geometric or kinematic constraints are naturally satisfied, in some cases the servo-constrains are cannot be fulfilled by different reasons. Several publications [3, 4] deal with the modification of the original servo-constraints in order to get a realizable task. An other possible approach is the periodic variation of the servo-constraint. For underactuated robots the usefulness of the method was proven in [5].

In case of industrial robotic applications, when the trajectory design was careful and operating conditions are well defined in most cases the actuator saturation can be avoided, but always there will be a trade-off between selecting saturation preventing operational conditions and productivity. The most used practical technique to avoid the unwanted effect of actuator saturation is the so-called anti-windup scheme [6], when the original controller is subjected to a compensator which takes into account the difference between the saturated and ideal control inputs. In [1], the saturation of actuator(s) are handled as a temporary reduction of the number of available independent control inputs. In that work during the saturation the number of prescribed servo-constraints is reduced in order to get a realizable task. In contrast, present paper introduces the further generalization of the periodic variation of servo-constraints [5] in case of actuator saturation, so the servo-constraints are systematically switched during actuator saturation to redistribute the load on the actuators. When the actuators are not saturated, a general computed torque control scheme is applied to realize the desired motion.

2 PROBLEM FORMULATION

Most of the controlled mechanical systems possess complex multibody structure of which the mathematical modeling is convenient by using non-minimum set of descriptor coordinates. In such case, the general form of the equation of motion is written as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} = \mathbf{Q} + \mathbf{H}\mathbf{u}\,, \tag{1}$$

$$\boldsymbol{\phi}_g = \mathbf{0}\,, \tag{2}$$

where $\mathbf{q} \in \mathbb{R}^k$ are dependent coordinates, $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{k\times k}$ is the mass matrix, $\mathbf{\Phi}_{\mathbf{q}}(\mathbf{q},t) \in \mathbb{R}^{m\times k}$ is the Jacobian of geometric constraints $\boldsymbol{\phi}_g(\mathbf{q},t) \in \mathbb{R}^m$ and $\boldsymbol{\lambda} \in \mathbb{R}^m$ is the vector of the Lagrangian multipliers which is related to the magnitude of the constraint forces and troques. Matrix $\mathbf{H}(\mathbf{q}) \in \mathbb{R}^{k\times l}$ is the control input matrix and $\mathbf{u} \in \mathbb{R}^l$ contains the actuating forces and torques. In addition, $\mathbf{Q}(\mathbf{q},\dot{\mathbf{q}},t) \in \mathbb{R}^k$ denotes the remaining generalized forces.

The equation of motion (1) with the geometric constraints (2) forms a differential algebraic equation (DAE) with differentiation index 3. Using the method of Lagrange multipliers [7] the constraints are considered on the level of acceleration:

$$\ddot{\boldsymbol{\phi}}_g = \mathbf{\Phi}_{\mathbf{q}}\ddot{\mathbf{q}} + \dot{\mathbf{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}} + \dot{\boldsymbol{\phi}}_{\mathbf{g},\mathbf{t}}, \tag{3}$$

where

$$\mathbf{\Phi}_{\mathbf{q}} = \frac{\partial \boldsymbol{\phi}_g}{\partial \mathbf{q}}, \qquad \boldsymbol{\phi}_{g,t} = \frac{\partial \boldsymbol{\phi}_g}{\partial t}.$$

With the above index reduction technique the original DAE problem can be solved as an ordinary differential equation (ODE).

2.1 Task definition by means of servo-constraints

In fully actuated case the l number of independent actuators is equal to the $n = k - m$ degrees of freedom. In case of actuator saturation the l number of independent actuators becomes less than n. In such case the system is called underactuated.

The required motion is specified by the so-called servo-constraints [11, 2] $\boldsymbol{\phi}_s(\mathbf{q},t) \in \mathbb{R}^l$. The additional constraint equations

$$\boldsymbol{\phi}_s = \mathbf{0}\,, \tag{4}$$

have a mathematical form similar to the geometric constraints. With the loss of generality we suppose that the relative degree for all outputs are $r = 2$ [13]. If this condition is not satisfied by the servo-constraints, further generalization is possible to be carried out. The physical meaning of $r = 2$ is that the input forces or torques have a direct effect on the system in the controlled directions. Mathematically it means that the differentiation index is still 3 when the system (1) and (2) are subjected to the servo-constraint equation (4). Thus using the method of Lagrange multipliers the servo-constraints also can be considered on the level of acceleration as

$$\ddot{\boldsymbol{\phi}}_s = \mathbf{G}_{\mathbf{q}}\ddot{\mathbf{q}} + \dot{\mathbf{G}}_{\mathbf{q}}\dot{\mathbf{q}} + \dot{\mathbf{c}}, \tag{5}$$

where

$$\mathbf{G_q} = \frac{\partial \boldsymbol{\phi}_s}{\partial \mathbf{q}}, \qquad \mathbf{c} = \frac{\partial \boldsymbol{\phi}_s}{\partial t},$$

similarly to the geometric ones. In case of the servo-constraints the control input **u** plays a similar role like the Lagrangian multipliers of the geometric constraints.

2.2 Computed torque control method

During the control design of mechanical systems it is often expedient to calculate the inverse dynamics of a system with respect to the desired task. The solution of the inverse dynamics can be seen as a feedforward control action that realizes the desired motion without considering any disturbances and modelling errors. This approach is frequently referred in the literature as computed torque control method. In case of underactuated systems the inverse dynamics is not well defined. Some degrees-of-freedoms cannot directly be controlled, and the corresponding generalized coordinates depend on the system dynamics only. Still, the input forces can be computed from the following equation constructed by using equations (1), (3) and (5) [4].

$$\begin{bmatrix} \mathbf{M} & \boldsymbol{\Phi}_\mathbf{q}^T & -\mathbf{H} \\ \boldsymbol{\Phi}_\mathbf{q} & \mathbf{0} & \mathbf{0} \\ \mathbf{G_q} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{Q} \\ -\dot{\boldsymbol{\Phi}}_\mathbf{q}\dot{\mathbf{q}} - \dot{\boldsymbol{\phi}}_{g,t} \\ -\dot{\mathbf{G}}_\mathbf{q}\dot{\mathbf{q}} - \dot{\mathbf{c}} - K_D\dot{\boldsymbol{\phi}}_s - K_P\boldsymbol{\phi}_s \end{bmatrix}, \qquad (6)$$

were the control gains K_P and K_D are the proportional and derivative gains have similar role like the parameters of the Baumgarte stabilization technique [12] which is quite common for stabilizing geometric constraints in multibody simulations. We use (6) in the case of the original fully actuated system and also in the underactuated case, when some of the actuators temporarily saturate.

2.3 The periodic servo-constraints

As it is presented in the previous section the task of the manipulator can be formulated as additional constraints called servo-constraints. Many techniques can be found in the literature to solve the resulting DAE problem. In reference [4] and in [2] the origianal task thus the servo-constraints are slightly modified in order to stabilize the internal dynamics or get a feasible problem. This modification makes the stable control possible. Obviuously, this modification has drawbacks and it results larger, but still acceptable, tracking errors. Reference [5] introduces a different approach when the servo-constraints are switched periodically in time. In one period the servo-constraints are responsible for realizing the desired motion, while in the subsequent, typically shorter period a different set of servo-constraints are formalized to stabilize the internal dynamics.

2.4 The issue of actuator saturation and frame algorithm

We focus on fully actuated manipulators with n degrees of freedom and $l = n$ actuators performing a strictly $l = n$ dimensional task. If an actuator reaches its torque limit than it will not be able to provide the required force. As a consequence, if m number of actuators saturate than the maximum dimension of the independently feasible task is reduced to $l = n - m$.

The proposed method of periodic servo-constraints is divided into two main parts. In the case, when the actuators are not saturated, a simple computed torque control scheme is applied. During saturation new set(s) of servo-constraints are introduced and they are switched in time. The periodic servo-constraints requires a switching pattern introduced in the section 3. In order to determine the new set(s) of servo-constraints the reduced control input matrix $\hat{\mathbf{H}}$ should be derived. For the partitioning of the servo-constraints the relative degree analysis [13] gives us a hand. The relative degree r_{ij} should be analysed between the i^{th} saturated actuators ($i = 1, 2...m$) and the j^{th} servo-constraints ($j = 1, 2...n$). If $r_{ij} > 2$, then the i^{th} saturated actuator has not effect on j^{th} constraint on the acceleration level. These servo-constraints are involved in every new sets in their original form. If $r_{ij} = 2$, then the saturated actuator has direct effect on the j^{th} servo-constraint. In

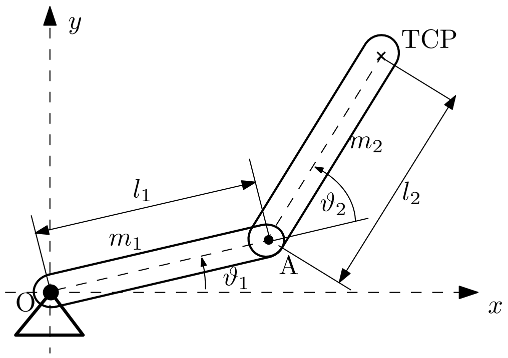

Figure 1. Two-link manipulator model

this case we have to find an independent actuator among the $g = 1, 2...n-m$ unsaturated actuators which has also affect on the j^{th} servo-constraint, so that $r_{gj} = 2$. Hereupon the servo-constraints corresponding to g^{th} actuator are switched in order to achieve that $l = n-m$ number of servo-constraints are used simultaneously. The proper selection of the switching pattern guarantee the minimization of the servo-constraints' violations. Section 3 will introduce the simplest example on which the idea of periodic servo-constraint can be used to reduce the effect of actuator saturation.

3 SIMULATION CASE STUDY

In order to illustrate the applicability of the presented approach a simulation study was accomplished on a two-link manipulator shown in Fig. 1. The corresponding members of the equation of motion (1) of the manipulator was derived from the Euler-Lagrange formula using the minimum set of generalized coordinates $\mathbf{q} = [\vartheta_1, \vartheta_2]^{\mathrm{T}}$ (see Fig 1). The mass matrix is

$$\mathbf{M} = \begin{bmatrix} J_0 + J_{CM2} + \frac{1}{4}l_2^2 m_2 + l_1 l_2 \cos\vartheta_2 & J_{CM2} + \frac{1}{2}l_1 l_2 m_2 \cos\vartheta_2 \\ J_{CM2} + \frac{1}{2}l_1 l_2 m_2 \cos\vartheta_2 & J_{CM2} + \frac{1}{4}m_2 l_2^2 \end{bmatrix}. \tag{7}$$

The coefficient matrix of the actuator forces $\mathbf{u} = [\tau_1, \tau_2]^{\mathrm{T}}$ can be derived from the virtual power of the actuators

$$\mathbf{H} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{8}$$

while the remaining forces are

$$\mathbf{Q} = \begin{bmatrix} l_1 l_2 m_2 \sin\vartheta_2(\dot\vartheta_1 - \frac{1}{2}\dot\vartheta_2)\dot\vartheta_2 \\ \frac{1}{2}l_1 l_2 m_2 \sin\vartheta_2 \dot\vartheta_1^2 \end{bmatrix}. \tag{9}$$

The links are supposed as homogeneous rigid bodies, and the physical parameters can be found in Table 1. The manipulator was placed perpendicular to the gravity field, like a typical SCARA robot application, thus the effect of gravity was not present.

The tool center point (TCP) of the manipulator was commanded to follow the desired trajectory. Using the idea of servo-constraints the task was written as

$$\phi_{\mathbf{s}} = \begin{bmatrix} \phi_{s1} \\ \phi_{s2} \end{bmatrix} = \begin{bmatrix} l_1\cos(\vartheta_1) + l_2\cos(\vartheta_1+\vartheta_2) - x_D \\ l_1\sin(\vartheta_1) + l_2\sin(\vartheta_1+\vartheta_2) - y_D \end{bmatrix}, \tag{10}$$

where x_D, y_D describe the desired TCP position on horizontal plane. In order to understand the operation of the proposed controller only the shoulder actuator can saturate. During saturation the input matrix is reducing as $\hat{\mathbf{H}} = [0 \quad 1]^{\mathrm{T}}$ in the inverse dynamics calculation, and the saturated actuator force is applied as a constant force appearing in $\mathbf{Q}$.

Table 1. MECHANICAL PARAMETERS

Description	Parameter	Value
Mass of the first arm	m_1	0.2 [kg]
Mass of the second arm	m_2	0.2 [kg]
Length of the first arm	l_1	0.4 [m]
Length of the second arm	l_2	0.4 [m]
Inertia of the first arm	J_O	0.00107 [kgm^2]
Inertia of the second arm	J_{CM2}	0.0027 [kgm^2]

3.1 The tuning of the pattern of the periodic servo-constraints

The key question of the method is the switching pattern of the servo-constraints. This switching rule can be derived in several ways. The following method fucuses on the previously introduced 2 degrees of freedom example. During the actuator saturation the unsaturated actuator has to produce different accelerations in different directions, while the effect and necessity of the actuator on the different servo-constraint directions is not equal. Thus, based on the ratio of necessity we can determine a pattern for the switched servo-constraints. In order to approximate this ratio the massmatrix $\mathbf{M}$ and $\hat{\mathbf{H}}$ should be transformed into the space of servo-constraints. The time differentiation of the servo-constraint equations gives the following formulae on the velocity level:

$$\dot{\boldsymbol{\phi}}_s = \mathbf{G_q}\dot{\mathbf{q}} + \mathbf{c}, \tag{11}$$

where $\mathbf{G_q}$ playes the role of a Jacobian matrix, thus the massmatrix in the space of the servo-constraints is

$$\mathbf{W} = \mathbf{G_q^{-T}}\mathbf{M}\mathbf{G_q^{-1}}. \tag{12}$$

The virtual power of the available actuators can be written as,

$$\delta P = \delta\dot{\boldsymbol{\phi}}_s^{\mathrm{T}}\mathbf{G_q^{-T}}\hat{\mathbf{H}}\hat{\mathbf{u}}, \tag{13}$$

from where control input matrix in the space of the servo-constraints is

$$\mathbf{B} = \mathbf{G_q^{-T}}\hat{\mathbf{H}}. \tag{14}$$

If the TCP is controlled by servo-constraints, the above mentioned transformation has a same result as the operational space control [10]. A possible approximation of the effect and necessity of the remaining actuators can be estimated by dividing the desired inertial force $\mathbf{w}$ by each columns of the operational space control input matrix $\mathbf{B}$

$$z_i = w_i/B_{1,i}\,; \quad i = 1, 2 \ldots n\,. \tag{15}$$

The elements in $\mathbf{z}$ will show the influence of the remaining independent actuators on the servo-constraints. The desired inertial force $\mathbf{w}$ is calculated by the product of the effective (operational space) mass matrix $\mathbf{W}$ and desired servo-constraint accelerations $\dot{\mathbf{c}}$ as:

$$\mathbf{w} = \mathbf{W}\dot{\mathbf{c}}. \tag{16}$$

The remaining centrifugal and Coriolis forces are neglected in this approximation. The periodic pattern is constructed along the trajectory based on ratio $\mathbf{z}$.

In case of the presented example only one actuator can saturate, so $m = 1$ and the remaining actuator has effect on the both servo-constraints. Thus two servo-constraint are switched, while the matrix $\mathbf{B}$ has only one column. The periodic pattern can be constructed before the simulation based on the desired quantities.

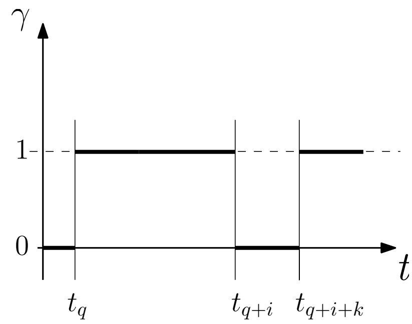

Figure 2. Switching pattern of servo-constraints

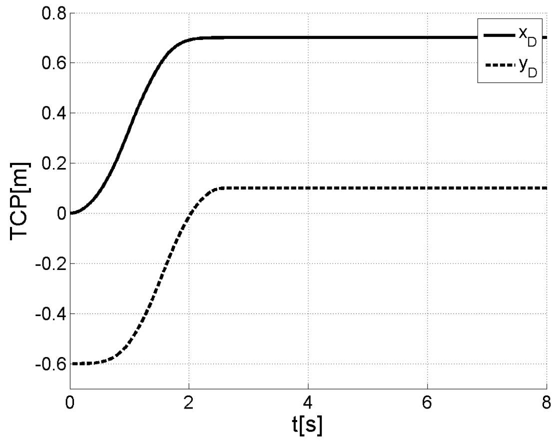

Figure 3. The desired value of the coordinates in time

In the present example $\mathbf{z} \in \mathbb{R}^2$. Based on that we split the time periods into two parts when for i time steps the first servo-constraints is valid and for k time steps the second servo-constraint is valid. The ratio of i and k are directly proportional with the ratio of the elements of $\mathbf{z}$:

$$i = int(|z_1|\frac{p}{|z_1|+|z_2|}), \tag{17}$$

where $p = i + k$ is size of the time period. We calculate this ratio for each time period along the desired trajectory, thus i and k can be different in every period. Function γ realizes the periodic switching pattern, see Fig. 2. The periodic servo constraint is $\hat{\phi}_s = \gamma\phi_{s1} + (1-\gamma)\,\phi_{s2}$. In the case of the two-link arm x position of TCP was controlled for i time steps and y position was controlled for k time steps.

3.2 Numerical results

In the numerical simulations the manipulator had to follow an arc of a circle shown in Fig. 3. The whole desired path in the plane, the initial configuration (marked with 0) and the end configuration (marked with 1) are presented on Fig. 4. As it is mentioned, the actuator in the first joint can saturate at the value of $|\tau_{1max}| = 0.6$[Nm] and the torque limit of the second actuator is not considered. The proportional and the derivative gains in (6) was set to $K_P = 40$ $K_d = 20$ respectively in every simulation scenarios.

In the first simulation scenario the effect of actuator saturation is not handled by periodic switching of the servo-constraints. The control torques was calculated based on equation (6) and Fig. 5 clearly shows that the first actuator is saturated. Fig. 6 presents that during the saturation the servo-constraint violation was significantly increased.

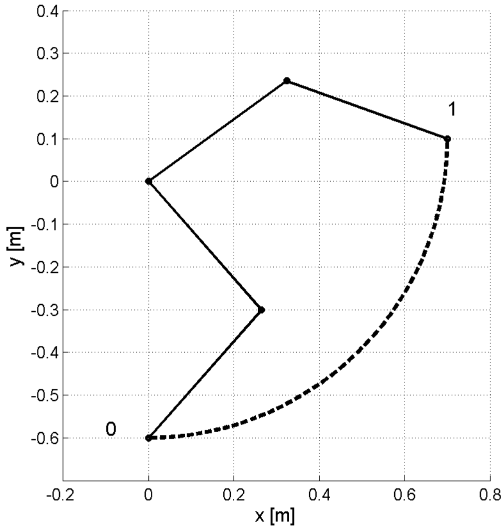

Figure 4. The desired path and the initial and end configuration

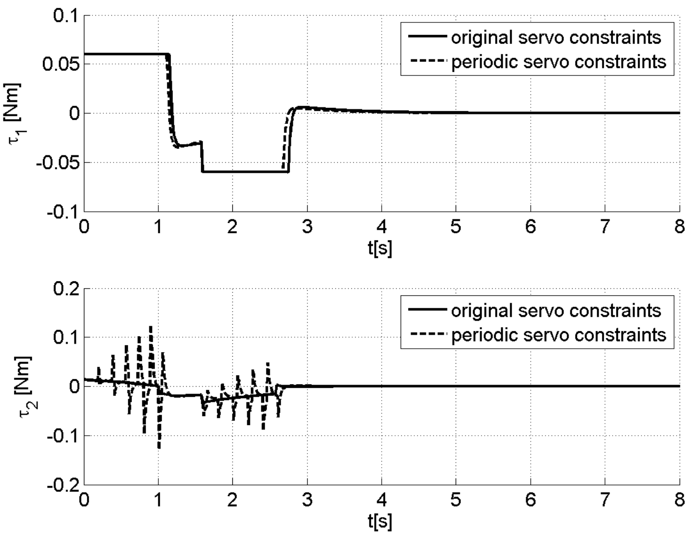

Figure 5. Simulation results - actuator torques

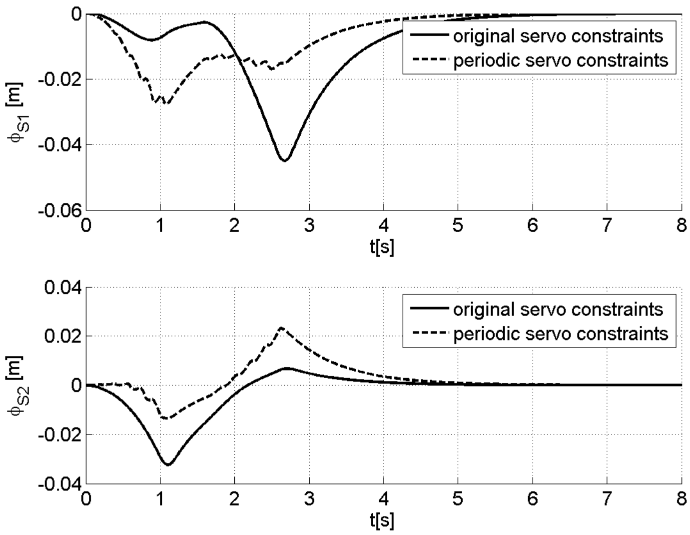

Figure 6. Simulation results - servo-constraints

In the second simulation scenario the proposed control algorithm were implemented on test example. In the initialization phase the whole path was divided into time periods. Each time period built up by p samplings and function γ was constructed based on equation (17). The number of samplings is $p = 20$. During the actuator saturation the periodic servo-constraints generate a control force as it presented in Fig. 5. The corresponding violations of the servo-constraints are visible in Fig. 6. Simulation results clearly show that the violation of the servo-constraints are significantly smaller. Besides, the system gets out from the saturation a little bit faster, as Fig. 5 shows. As a marginal drawback, the computed torques has a periodic-like oscillation, which can be provided by an actuator with average dynamical properties.

In order to make a quantitative comparison, the norm of the servo-constraint violation was plotted on the same chart on Fig. 7. The maximum error was $|\phi_s| \approx 45[mm]$ when the actuator saturation is not handled. When the periodic servo-constraints were used, the maximum error of the servo-constraints was only $|\phi_s| \approx 30[mm]$. To get a comparable metric the Root-mean square (RMS) value of the norm of the servo-constraints was computed as:

$$\bar{\phi}_s = \sqrt{\frac{1}{n}\sum_{i=1}^{n} |\boldsymbol{\phi}_s|^2(t_i)}. \tag{18}$$

When the original servo-constraints was used the RMS value was $\bar{\phi}_s = 0.0174$ however for periodic servo-constraints the RMS value decreased to $\bar{\phi}_s = 0.0122$. Consequently the proposed control algorithm was able to reduce the effect of actuator saturation with approximately 30%.

4 CONCLUSIONS

In this paper the extension of periodically switched servo-constraint was proposed for the control of saturated systems. The proposed controller was tested on a two-link manipulator case study application. The results showed that with the application of the introduced method the violation of servo-constraints was much more acceptable during the actuator saturation. Thus we can conclude that with the application of periodically switched servo-constraints the precision of trajectory tracking can be enhanced during actuator saturation. An optimal swithcing pattern was generated for the two link case study example. However the optimalization method for general case is possible within a further research.

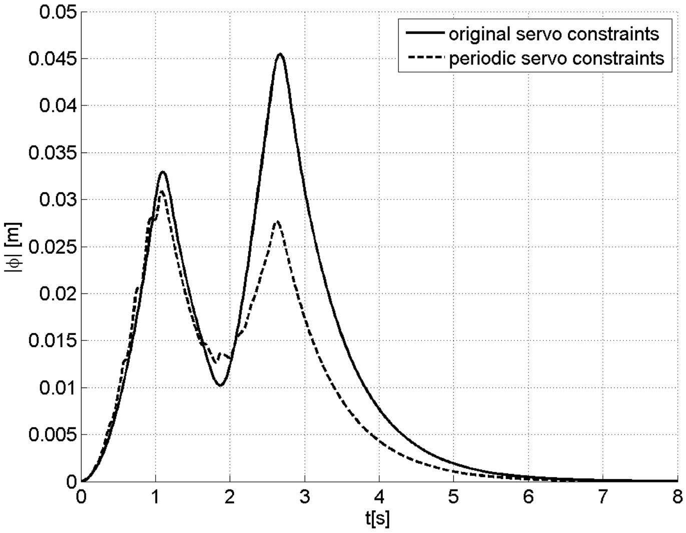

Figure 7. Norm of the servo-constraints

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