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A study on the effect of human running cadence based on the bouncing ball model

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Abstract: Running is a very popular sport on professional and especially on hobby level. Professional athletes improve their body motion carefully, while hobby runners usually do not focus on the energy efficient and injury preventing running form. The running form is characterized by some fundamental parameters, like step size, stride frequency and strike pattern besides many other kinematic parameters. Much information can be found on the internet and in magazines about the correct running form, although these information are not based on scientific investigation in most of the cases. The main reason is that the running has a quite complex dynamics with many parameters leading to highly complex mechanical models. Thus it is hard to accomplish quantitative investigations that provides useful and practical conclusion. Although the running form characteristics can be investigated by pure mechanical calculations if a proper model exists. We propose a simple bouncing ball model to prove that runners should choose relatively high stride frequency. Cadence should be always kept around 180 steps/min according to the experience, while running speed should be modified by variing the stride length. We show that higher stride frequency implies lower risk of injury and energy efficiency. The model based estimations are supported by a large sample measurement data.

1. Introduction

Many works like [2, 23] contribute to the thorough understanding of bipedal locomotion, human walking and running. Several approaches have been developed which try to realize the healthy, injury preventing, energy efficient and natural way of running in practice [16–18]. Many papers study the effect of foot strike pattern and footwear experimentally [1, 7, 10, 13, 15, 19, 25].

A lot of complex high degree of freedom (DoF) mechanical models exist, which are suitable for motion capturing, dynamic and kinematic analysis of the human body and running motion carefully. However these investigations are hard to use for prediction regarding the effect of a parameter modification. A simplified dynamical model can be more predictive than a very complex model with large number of parameters. Starting from the most complex models, e.g. [21] towards the simplest ones, we can mention some low DoF segmental models [15, 26, 27] and some spring legged models [20, 24], besides many other examples.

Incrasing the comlexity of the model, the number of parameters can grow exponentially. The most fundamental parameters, with which the running form can be characterised, are the running speed, step size, stride frequency and strike pattern besides many other kinematic parameters. Many articles study the effect of cadence c, which is condiered to be one of the most important parameters, when running form is analysed [4, 5, 9, 14, 22]. This work also focuses on *stride frequency* which is also common to call *cadence*.

At a certain speed, an infinitely many variations of stride length and stride frequency can be chosen. The experiments explained in [4] showed that the optimal cadence, when the oxigen uptake (the indicator of physical loading of the body) is minimal, and the freely chosen convenient cadence are not the same for everyone.

In this paper we show by means of a simple dynamic model that cadence has a direct effect on energy efficiency and impact intesity. The estimations are based on the bouncing ball model. The bouncing ball model itself is validated by measurements.

2. The bouncing ball model

In order to achieve the minimally complex model, the mass of the body can be shrinked into one point mass when it is in flight phase. The model is valid if external forces, like aerodynamic forces are neglected. The parabolic path of the CoM during flight phase is depicted in figure 1. The motion of the centre of gravity (CoG) in flying phase is described by a parabolic curve:

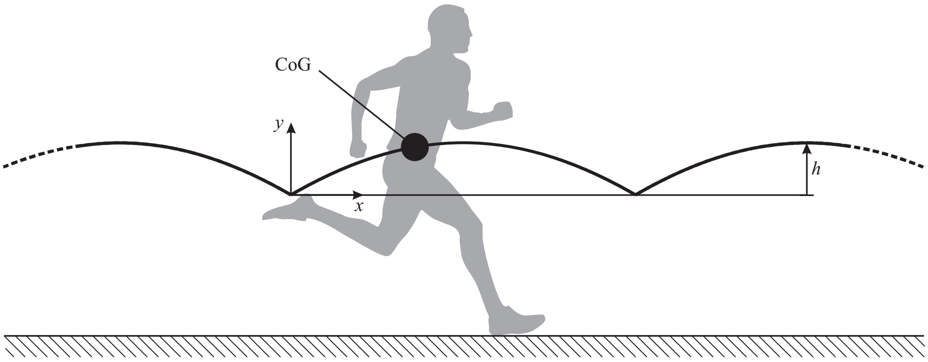

Figure 1. Idealized path of a runner's CoG

$$x(t) = x_0 + \dot{x}_0 t, \tag{1}$$

$$y(t) = y_0 + \dot{y}_0 t - \frac{g}{2} t^2, \tag{2}$$

where x_0 and y_0 are the initial position coordinates and $\dot{x}_0$ and $\dot{y}_0$ are the initial velocity components represented in a Cartesian system, depicted in figure 1.

The time period T_f of the flight phase can be expressed based on time derivative of equation (2). The vertical velocity is zero, when the CoG is on the top of the parabola at $t = T_f/2$, so that we can write:

$$0 = \dot{y}_0 - g\frac{T_f}{2}. \tag{3}$$

Besides, we can apply the principle of conservation of mechanical energy in vertical direction, from which it is easy to determine the initial vertical velocity magnitude as the function of the height h of the parapolic path.

$$\dot{y}_0 = \sqrt{2gh}. \tag{4}$$

Combining equations (3) and (4) we obtain the height h of the parabola as a function of the flying phase time period:

$$h = \frac{1}{8}gT_f^2. \tag{5}$$

Equation (5) clearly shows that the height of the parabolic path is a function of the time period of each step hence it is a parameter of cadence.

The total time period T[s] of one step is in direct relation with cadence c which possesses [steps/min] unit:

$$T = \frac{60}{C}, \tag{6}$$

and the time period T_f of the flying phase and cadence has the relation:

$$T_f = \frac{60}{C}r_f, \tag{7}$$

where $r_f = T_f/T$ is the ratio of the flying phase time duration. Its typical value is in the range $r_f = 0.4...0.7$. In this work we assumed $r_f = 0.6$ airborn phase ratio. The path in the flying phase is depicted in figure 2 in case of different cadence values.

3. Measurements

In order to prove the validity of the bouncing ball model we accomplished an experiment, when the motion of 41 runners was videocaptured. Every investigated person was hobby runner in the age from 15 to 50 and from both sex. The measured people were told to run a distance of 5km with a convenient speed on an open air running track. Their motion was recorded on a 3m long distance by a high resolution videocamera after the first 4 km. The speed of the camera was 50 frames/s and the resolution was set to 1920 × 1080 pixels.

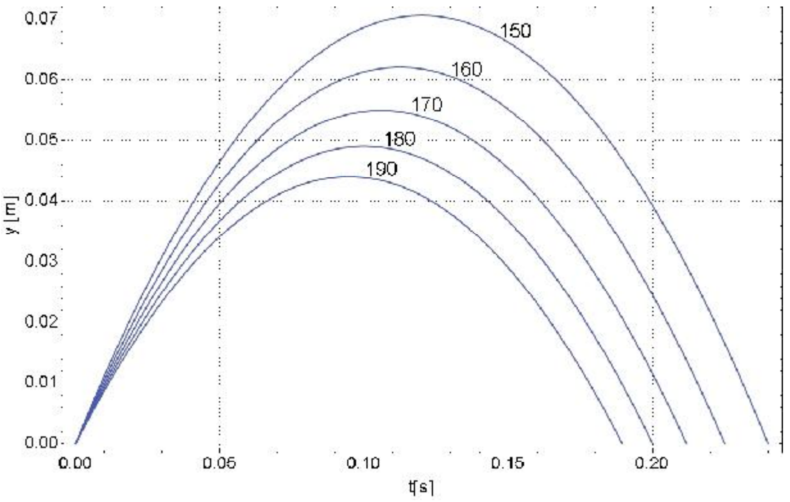

Figure 2. Parabolic path of CoG during flying phase in different cadence values (150...190)

The foot landing position and the vertical elevation A of the head was registered based on the video frames. The vertical displacement A of the head gives an acceptable estimation of the vertical displacement of the CoG of the body, however reference [8] provides a comparison of methodologies and the results of a large scale data experiment which aims to measure the vertical displacement of runners. Besides, the characteristic time durations, like stride period T, and flight phase duration T_f was determined based on the frame indices. The velocity of each person was determined based on the time duration that was needed to complete a 2.5m predefined distance. The parameters listed in table 1 were determined in case of each runner: horizontal speed v_x, stride length s, cadence c and vertical displacement A.

4. Validation of the model and discussion of the results

In order to validate the model, the measured data was plotted in figure 3 in which the theoretical height h of the parabolic path as a function of cadence c is shown by solid line. The measured values are depicted by dots, while the a curve fitted on the measurement data is plotted by a dashed curve. Qualitatively good coherence can be observed between the measured data and the theoretical curve, however the quantitave data have 20% error in average. The average of the measured displacement is larger than the theoretically predicted value. The possible reason is the further vertical displacement in reality when the leg is grounded. For the investigation of this phenomena many researches are available, e.g. [20,24].

The relation between the parabola height h and the vertical velocity component $v_y^- = \dot{y}_0$ right before the impact is given by (4). It is shown by [12] and [6] that the impact forces

Table 1. Measured data of 41 people: running speed v_x, stride length s, cadence c, vertical displacement A

no.	v_x km/h	s m	c 1/min	A mm
1	16.1	1.45	176	65
2	13.6	1.3	167	80
3	9.4	0.9	171	52
4	9.2	0.9	176	76
5	9.8	0.975	167	74
6	9.4	0.875	188	70
7	9.8	0.96	167	66
8	9.4	0.91	171	74
9	11.0	1.125	158	93
10	8.8	0.975	150	88
11	13.6	1.3	171	91
12	10.6	1.11	162	96
13	7.5	0.725	171	63
14	9.6	0.901	171	40
15	10.2	0.975	176	59
16	9.7	0.99	162	86
17	6.8	0.725	150	56
18	6.8	0.725	150	44
19	13.8	1.375	167	91
20	9.8	0.88	171	85
21	9.8	1.08	150	92
22	12.3	1.27	167	76
23	8.0	0.8	171	52
24	8.8	0.97	150	76
25	8.3	0.845	162	73
26	8.2	0.945	146	104
27	10.6	0.95	182	46
28	9.4	1.05	146	85
29	9.5	1.0	158	65
30	7.3	0.725	167	38
31	10.3	0.925	182	55
32	9.8	1.075	150	107
33	7.5	0.835	150	70
34	9.6	0.995	178	64
35	11.5	1.105	171	69
36	9.7	0.99	162	85
37	8.0	0.79	171	66
38	10.6	0.995	176	44
39	11.4	1.08	176	68
40	12.9	1.23	171	87
41	7.6	0.855	150	70

correlates with the kinetic energy content which is absorbed due to the foot impact. It is called *constrained motion space kinetic energy* (CMSKE) in the literature. CMSKE is directly proportional to the impulse of the contact reaction force and also to the peak reaction force [11, 12]. The related effective mass concept for foot impact is introduced in [3] for a one DoF model. The cited studies showed that foot strike intensity can be characterised by the CMSKE which depends on the pre-impact configuration and velocity and the effective mass matrix. All in all, the lower the vertical velocity is, the smaller the impact intensity is.

Since, the vertical direction motion is constrained by the ground in the bouncing ball model, CMSKE is calculated from the vertical velocity component only:

$$E_c = \frac{1}{2} m \dot{y}_0^2. \tag{8}$$

We substitute equation (4) into (8) which provides the relation between the pre-impact velocity and the height of the parabolic path. We obtain the following *linear relation* between

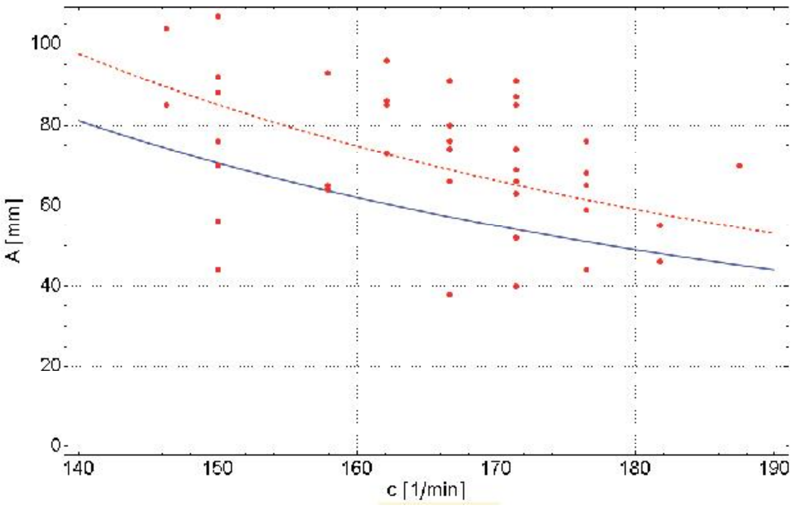

Figure 3. Height h of the parabolic path as the function of cadence is plotted by solid line. The measurement data for the vertical displacement A is depicted by dots. The dashed line shows the curve fitted to the measurement data.

CMSKE (E_c) and parabola height:

$$E_c = mgh. \tag{9}$$

Equation (9) shows that all of the potential energy of level h is absorbed by the constraint that arises when the foot touches the ground at the end of the flight phase. The main message is that that the impact intensity is in linear relation with the vertical displacement of the body. The vertical displacement values A on figure 3 are directly proportional with the impact intensity, that can be characterised by the impulse I_F of the vertical component of the contact force:

$$A = \gamma I_{Fy}, \tag{10}$$

where γ is a scale factor. The the impulse of the contact force is obtained by integrating it on the time duration of the impact:

$$I_{Fy} = \int F_y. \tag{11}$$

Figure 3 shows that infinitely high stride frequency and zero stride length should be chosen theoretically in order to reach minimal energy cost and impact intensity. It is obvious, that it is not feasible in reality. The optimal stride frequency is limited by the muscular activity that depends on the stretch-shortening cycle of the muscles, which is not included by the model. Nevertheless, the model predicts correctly, that higher cadence should be kept in order to achieve better energy efficiency and lower risk of impact induced injury.

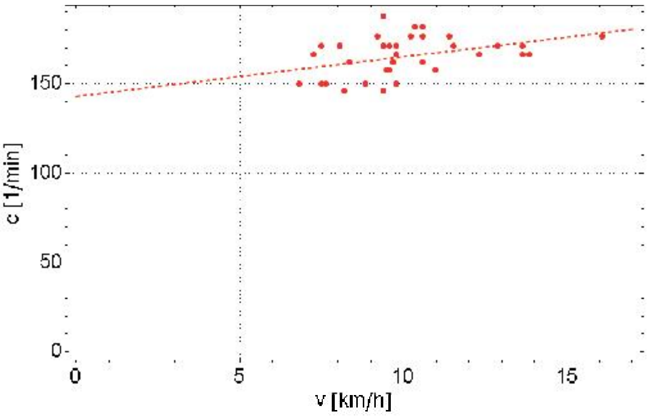

Figure 4. Running speed (v_x) versus cadence (c): measured data and the best fit line

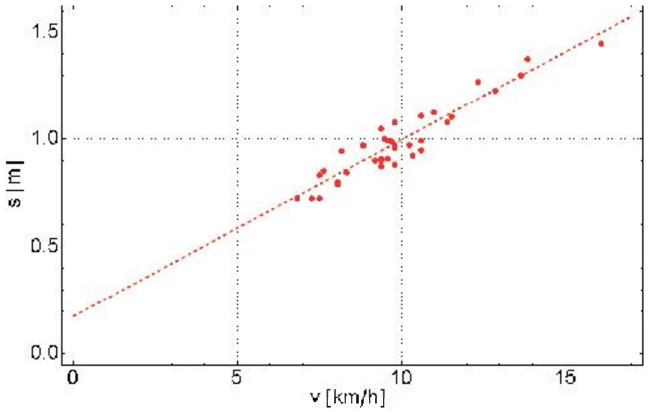

Figure 5. Running speed (v_x) stride length (s): measured data and the best fit line

As a secondary result shown by figures 4 and 5 the measurements confirmed that people tend to chose a larger stride length and they do not change the cadence, when they are running in different speed. So in case of the examined people, the running speed is set by changing the stride length and not the cadence. The dashed line shows a line fitted on the measured values.

5. Conclusions

The bouncing ball model was proposed and large scale experimental data was collected in order to validate the model. It is experimentally confirmed that the bouncing ball model is the minimally complex dynamic model when cadence and its effect on ground impact intensity and energy efficiency of running are studied. According to the model, the vertical displacement is directly proportional to the impact intensity, characterized by the impulse of the ground-foot contact force. We showed by means of a very simple dynamical model that stride frequency has a direct effect on energy efficiency of human running and impact intesity when foot collides with the ground. The model verified that higher cadence is preferable, when energy efficiency and injury preventing running form is developed.

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