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Stability of underactuated multibody systems subjected to periodic servo constraints

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Abstract: Motion control of underactuated multibody systems involves a lot of mathematical problems. This is mainly due to the fact that in case of underactuated systems the number of independent inputs are less than the degrees-of-freedom. Besides, the modelling of multibody systems is chalenging in itself. Flexible manipulators, cranes and robots with passive joints can be mentioned as characteristic examples. Methods available in the literature are often provided to solve specific problems of specific systems. The general application of these method may lead to unstable dynamic behaviour. The present work assumes a general multibody description and proposes the use of periodic servo-constraints in order to enhance the dynamic properties of the system.

1. Introduction

In the motion simulation of multibody systems it is a standard procedure to use the non-minimum set of coordinates to describe the system [2]. Therefore between the dependent coordinates geometric constraints should be considered which leads to differential algebraic equations (DAE). Using this idea the control task is also can be formulated as an additional set of constraints that are called servo-constraint [4]. While the geometric or kinematic constraints are naturally satisfied, in some cases the servo-constrains cannot be fulfilled by different reasons. Several publications [6, 7] deal with the modification of the original servo-constraints in order to get a realizable task. In those cases the original task is simply modified using a linear combination of the newly selected set of servo-costraints considering the internal dynamics of the system. An other possible approach is the periodic variation of the servo-constraint. For underactuated robots the usefulness of the method was comfirmed in [5]. The aim of this paper is to investigate the dynamic properties, the applicability of the periodic-servo constraint based control and to find the stable control parameters for the periodic servo-constraints. In the numerical studies a service-robotic application is used.

2. Problem Formulation

The equation of motion of an underactuated system can be derived using the non-minimum set of descriptor coordinates resulting the Lagrange equation of the first kind in the form:

$$\mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathsf{T}}\boldsymbol{\lambda} = \mathbf{Q} + \mathbf{H}\mathbf{u} \tag{1}$$

$$\boldsymbol{\phi}_g = \mathbf{0}\,, \tag{2}$$

where $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n\times n}$ is the mass matrix, $\boldsymbol{\Phi}_{\mathbf{q}}(\mathbf{q}) \in \mathbb{R}^{m\times n}$ is the Jacobian of geometric constraints $\boldsymbol{\phi}_g(\mathbf{q}, t) \in \mathbb{R}^m$ and $\boldsymbol{\lambda} \in \mathbb{R}^m$ is the vector of the Lagrangian multipliers. Matrix $\mathbf{H}(\mathbf{q}) \in \mathbb{R}^{n\times r}$ is the control input matrix and $\mathbf{u} \in \mathbb{R}^r$ contains the independent control inputs. In addition, $\mathbf{Q}(\mathbf{q}, \dot{\mathbf{q}}, t) \in \mathbb{R}^n$ denotes the remaining generalized forces. In similar form with geometric constraints (2) the desired motion also can be formulated as an additional constraint:

$$\boldsymbol{\phi}_S(\mathbf{q}, t) = \mathbf{0} \tag{3}$$

as a function of generalized coordinates and time, which called as servo constraint [4]. Using the method of Lagrangie multipliers [2], the geometric constraints (2) should be considered on the level of acceleration in order to compute the acceleration $\ddot{\mathbf{q}}$, and the Lagrange multiplier $\boldsymbol{\lambda}$. As it is presented in [6] [9] additionally the servo constraints are also considered on the level of acceleration, while the control input $\mathbf{u}$ should satisfy the constraints which can be calculated as:

$$\begin{bmatrix} \mathbf{M} & \boldsymbol{\Phi}_{\mathbf{q}}^T & -\mathbf{H} \\ \boldsymbol{\Phi}_{\mathbf{q}} & \mathbf{0} & \mathbf{0} \\ \mathbf{G}_{\mathbf{q}} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{Q} \\ -\dot{\boldsymbol{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}} \\ -\dot{\mathbf{G}}_{\mathbf{q}}\dot{\mathbf{q}} - \dot{\mathbf{c}} - K_D\dot{\boldsymbol{\phi}}_s - K_P\boldsymbol{\phi}_s \end{bmatrix}. \tag{4}$$

In equation (4) the servo constraints are stabilized with a simple linear regulator where K_P and K_D are the proportional and derivative gains respectively. It is quite similiar to the Baumgarte stabilization technique which is used in the solution of DAE equations of motion.

3. Periodic servo-constraints

Using the above explained *task based* formalism, the controlled- and the internal dynamics of the system can be separated. The internal dynamics it often referred ad passive dynamics which should be stable to ensure the stability of the system. However, the stability of the internal dynamics depend on the controlled task. If the stability of the internal dynamics can not be guaranteed, the original task (servo-constraint) should be modified slightly as it is presented in [7] and [8]. This modification makes the control stable with an acceptable violation of the original task. In reference [5] a different approach is introduced, when the

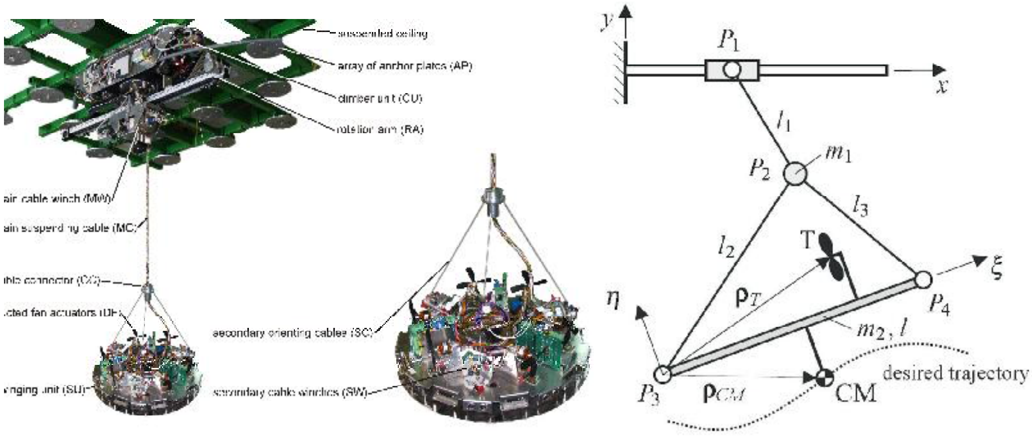

Figure 1. The prototype and the planar model of the ACROBOTER platform

servo-constraint is not simply modified but also periodically changed in time. In one period the original servo-constraint is considered for realizing the desired motion, while in the subsequent period (typically shorter) a modified servo-constraint is applied to stabilize the unstable internal dynamics. Thus slightly modified servo constraints are applied to stabilize the otherwise unstable internal dynamics. This modification makes the stable control possible, and result in larger, but still acceptable, tracking errors. The goal of this paper is to investigate dynamic properties and the advantages of the periodic servo-constraint based control algorithm.

4. Stability and dynamics

The dynamics of the presented control approach will be investigated in case of service robot wich will be briefly introduced in the following.

4.1. Service robotic example

The introduced method will be presented via the example of the motion control of the ACROBOTER service robot [1] (see Fig. 1), which is a suspended pendulum like underactuated manipulator. The mechanical structure of ACROBOTER can be divided into two parts; the climber unit (CU) carries the swinging unit (SU), which hangs on the main cable (MC) and three orienting secondary cables (SC) as shown in Fig. 1. The length of the cables are adjusted by servo motors, and the positioning of the SU is assisted by ducted fan actuators. Despite of the large number of actuators the system is underactuated. In order to understand the behaviour of the control, the planar model of the system is investigated. The

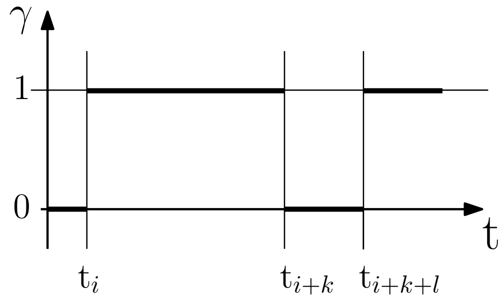

Figure 2. Servo constraint switching scheme

planar model is shown on the right hand side of Fig. 1. While the model has five DoF and the number of independent actuators is four only thus it is still underactuated. To describe the geometry of the model depicted in Fig. 1 the most way convenient is to use Cartesian coordinates $\mathbf{q} = [x_2\ y_2\ x_3\ y_3\ x_4\ y_4]^\intercal$, where the last four elements are the so-called natural coordinates that belong to the planar rigid body that represents the SU. Thus, according to [2], the mass matrix of the planar ACROBOTER model can be assembled as a constant block diagonal matrix $\mathbf{M} = \mathrm{diag}(\mathbf{M}_{CC}\ \mathbf{M}_{SU})$. During the control tests the planar ACROBOTER has to follow a linear path with horizontal orientation, when the CC is above the SU with h^d_{CC} height. As it was mentioned the passive motion (lateral motion of the CC) should be stabilized by periodically changing servo constraints. Using the servo-constraint based formalism the modified task can be defined as:

$$\boldsymbol{\phi}_s = \begin{bmatrix} y_{CC} - \frac{y_3+y_4}{2} - h^d_{CC} \\ (1-\gamma)x_2 + \gamma\frac{x_3+x_4}{2} - x^d_{SU} \\ \frac{y_3+y_4}{2} - y^d_{SU} \\ y_3 - y_4 \end{bmatrix} . \quad (5)$$

In equation (5) the function γ is responsible for the switching of servo constraints as it is shown in Fig. 2. For l time steps $\gamma = 0$ the passive part of the motion is considered in the servo-constraint set, while for k time steps $\gamma = 1$ and the original task is realized in the control scheme.

4.2. Stability investigation

In order to chose an effective switching pattern linear stability analysis was carried out. Considering the discrete behaviour of the digitally controlled system a picewise solution of the equation of motion should constructed for eigenvalue analysis. This solution is known analytically if the system is linear, thus we have to linearize the system around the investigated configurations. During the stability investigation it is assumed that the control forces calculated at the n^{th} time instant are based on the $(n-1)^{\text{th}}$ measured values which are held by a zero-order-hold (ZOH) until the end of the $(n+1)^{\text{th}}$ sampling instant. The stability investigation is based on equation (2) where the input force $\mathbf{u}$ is calculated via the solution of equation (4).

The equation of motion can be linearized around an arbitrary configuration and after that the equation of the controlled system can be written in the general first order form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{x}(t_{n-1}), \quad t \in [t_n, t_{n+1}]. \tag{6}$$

By using the state variables at the end of the n^{th} sampling interval the solution can be calculated as:

$$\mathbf{x}(t_{n+1}) = \mathbf{A}_d\mathbf{x}(t_n) + \mathbf{B}_d(t_{n-1}), \tag{7}$$

where $\mathbf{A}_d$ and $\mathbf{B}_d$ can be calcualated utilizing the following property [3]

$$e^{\mathbf{W}\Delta t} = \begin{bmatrix} \mathbf{A_d} & \mathbf{B_d} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \tag{8}$$

where Δt is the sampling time of the digital controller, and the matrix W can be constructed as:

$$\mathbf{W} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{9}$$

Based on equation (7) the mapping $\mathbf{z}_{n+1} = \mathbf{H}\mathbf{z}_n$ can be composed where $\mathbf{z}_n = [\mathbf{x}_{n-1}, \mathbf{x}_n]^\mathsf{T}$. The control law is switched in time therefore the state-equation is also changed during the control. The discrete mapping for the whole pattern (see: Fig. 2) can be constructed as:

$$\mathbf{z}_{n+k+l} = \prod_{j=1}^{k+l} \mathbf{H}_{n+k+l-j}\, \mathbf{z}_n. \tag{10}$$

From the computed eigenvalues $\boldsymbol{\rho}$ of the mapping (10) the avaraged eigenvalue for one time step can be computed as:

$$\bar{\boldsymbol{\rho}} = \sqrt[k+l]{\boldsymbol{\rho}}. \tag{11}$$

In the stability analysis the internal dynamics is stabilized for $l = 1$ time step only. The stability charts in Fig. 3 show the stable domains of operation with different switching periods ($k = 1 \ldots 9$) in the plane of the control parameters K_P and K_D. It can be concluded that the area of the stable domains is the biggest, when $k = 5$. The fastest decay can be achieved when $k = 1$, which means that in every second time step the original constraint is repeated with the modified one.

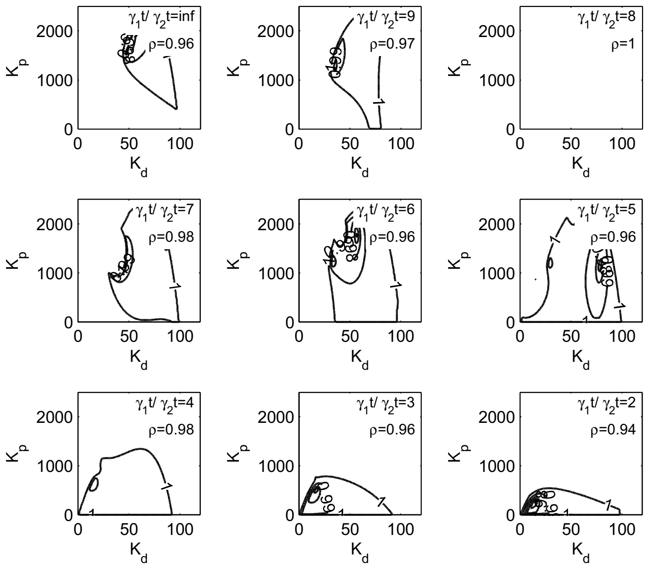

Figure 3. Stability charts in case of periodically switched servo constraints

5. Simulation case study

We confirmed the results shown by the stabilty diagrams by numerical simulations of the original non-linear system. In the first simulation scenario the control command is computed using the original servo constraints only. In this case the position of the SU is controlled only during the motion. In order to check the robustness of the control a horizontal perturbation was applied on the CC at $t = 1.5s$. The result of the trajectory tracking is shown on Fig. 4. The violation of the servo-constraints clearly show that neglecting of the internal dynamics can lead to unstable dynamics behaviour.

In the second simulation scenarion the periodic-servo constraints are used. It means that the horizontal displacement of the CC is used instead of the SU in the servo constraints at certain periodic time instances. The pattern was chosen based on the presented stability investigation (Sec. 4.2). The servo-constraints' violation is presented in Fig. 5. With the use of the same perturbation the results show that with the application of the periodic servo-constraints the investigated system can be controlled in stable way.

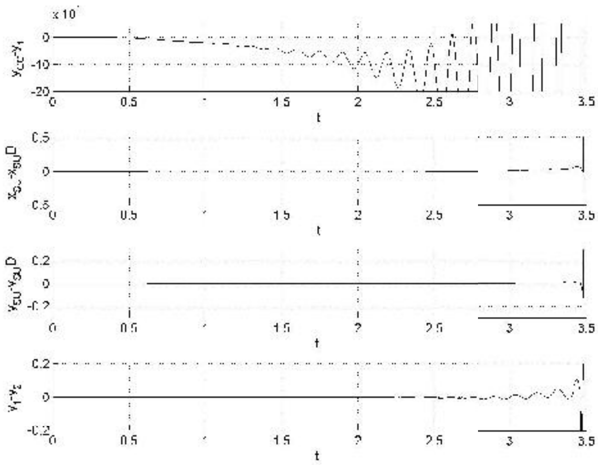

Figure 4. Simulation results with original servo constraints

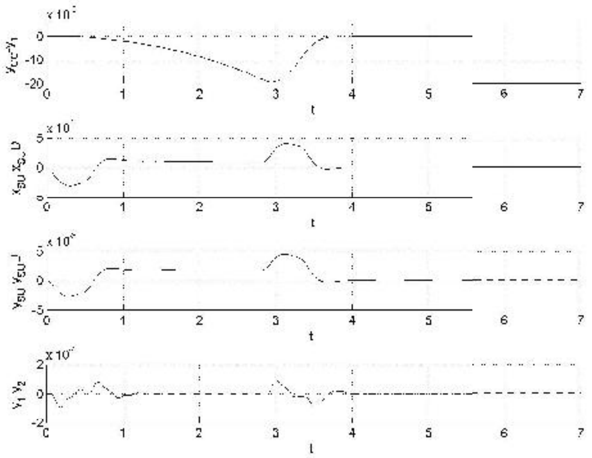

Figure 5. Simulation results with the periodic servo constraints

6. Conclusion

Present paper analysed the idea of periodic servo-constraints. Based on the stability analysis it can be concluded that the presented approach can effectively enhance the dynamical properties of the controlled system. The stability analysis was carried out in case of a service robot example. The result of that is applied in case of trajectory following problem which also shows that the application of periodic servo-constraints makes the original task feasible in stable way. For real-life application the optimal switching pattern requires a further research in order avoid the numerically expensive stability investigation.

Acknowledgments

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