COMPARISON OF THE MIXED-MODE BENDING AND THE PRESTRESSED END-NOTCHED FLEXURE SYSTEMS

András Szekrényes
Budapest University of Technology and Economics, Department of Applied Mechanics, Budapest 1111, Muegyetem rkp 5, Building MM

Abstract: *The mixed-mode bending system can be efficiently used for delamination testing of composites under mixed-mode I/II condition. The only drawback of the system is that a complex loading fixture is involved. Alternatively, the prestressed end-notched flexure system is suitable to replace the mixed-mode bending apparatus, because it is also able to test the material under any mode ratios. The present work performs a comparison of the two methods based on experiments. It is shown that the two methods lead to very similar results and the fracture envelopes obtained by the two different specimen types give almost the same fracture criteria.*

Keywords: *mixed-mode bending, prestressed end-notched flexure, composite, delamination*

1. INTRODUCTION

Composite materials - in general - exhibit relatively low interlaminar strength. Therefore, the measurement of the interlaminar fracture toughness is an important issue in the linear elastic fracture mechanics. Considering the mixed-mode I/II delamination there are large number of fracture systems, such as the mixed-mode bending (MMB) [1], the double-cantilever beam loaded with uneven bending moments (DCB-UBM) [2], the DCB specimen loaded by forces and moments [3] and – among others - the compact tension shear specimen (CTS) [4]. All of them involve complicated and expensive fixtures; however the mode ratio (G_{I}/G_{II}) is variable within wide ranges. On the other hand a study comparing the results obtained by two or more different setups for the same material has not yet been published.

Alternatively, the prestressed end-notched flexure (PENF) specimen – developed in 2006 [5] - can be used to replace the MMB setup, since it is able provide any mode ratio at crack initiation. The PENF is the simplest mixed-mode I/II setup ever developed, since it requires only a three point bending setup and few steel rollers with different diameters.

In the present work the MMB and PENF systems are compared to each other. Fracture experiments are performed on unidirectional glass/polyester specimens and the fracture envelope is determined by both the MMB and PENF systems. The experimental data is reduced by an improved beam theory (IBT) scheme.

2. MIXED-MODE BENDING TEST

The MMB specimen can be treated as the combination of the DCB and ENF specimens [1]. A 3-dimensional model of the setup is shown in Fig. 1. Considering the specimen as a slender beam the mode-I and mode-II energy release rates (ERR) of the MMB specimen are [6]:

$$G_I^{MMB} = \frac{P^2 a^2 (3c-L)^2}{16 b^2 h^3 E_{11} L^2}[12 + f_{W2} + f_T + f_{SV}], \qquad (1)$$

$$G_{II}^{MMB} = \frac{P^2 a^2 (c+L)^2}{16 b^2 h^3 E_{11} L^2}[9 + f_{SH2}], \qquad (2)$$

were the factors are [6]:

$$f_{W2} = 10.14\left(\frac{h}{a}\right)\left(\frac{E_{11}}{E_{33}}\right)^{\frac{1}{4}} + 8.58\left(\frac{h}{a}\right)^2\left(\frac{E_{11}}{E_{33}}\right)^{\frac{1}{2}}, \tag{3}$$

$$f_T = \frac{1}{k}\left(\frac{h}{a}\right)^2\left(\frac{E_{11}}{G_{13}}\right), \tag{4}$$

$$f_{SV} = \frac{12}{\pi}\left(\frac{h}{a}\right)\left(\frac{E_{11}}{G_{13}}\right)^{\frac{1}{2}}, \tag{5}$$

$$f_{SH2} = 1.96\left(\frac{h}{a}\right)\left(\frac{E_{11}}{G_{13}}\right)^{\frac{1}{2}} + 0.43\left(\frac{h}{a}\right)^2\left(\frac{E_{11}}{G_{13}}\right), \tag{6}$$

where P is the applied load, a=55 mm is the crack length c is the lever length, L=75 mm is the half span length, b=20 mm is the width, h=3.1 mm is the half thickness, E_{11}=33 GPa is the flexural modulus, E_{33}=7.2 GPa is the through-thickness modulus, G_{13}=3 GPa is the shear modulus of the material. The mode ratio (G_I/G_{II}) in each case may be obtained by combining Eqs. (1) and (2).

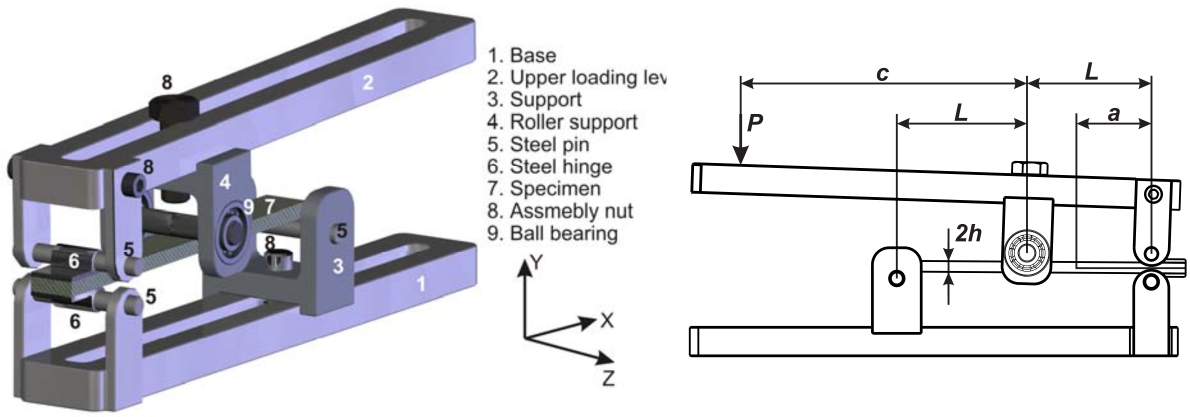


Fig. 1. *The mixed-mode bending method for delamination testing.*

3. PRESTRESSED END NOTCHED FLEXURE TEST

It has been shown that the PENF specimen is also the superposition of the DCB and ENF specimens [5], as it is shown in Fig. 2. The mode-I part of the ERR is provided by the prestressed state of the specimen, while the mode-II ERR is from the three-point bending of the coupon. The mode-I and mode-II ERRs are given by [5]:

$$G_I = \frac{h^3 E_{11}\delta_{DCB}^2}{64a^4}\frac{[12 + f_{W2} + f_T + f_{SV}]}{[1 + (f_{W1} + f_T + f_{SV}/2)/4]^2}, \tag{7}$$

$$G_{II} = \frac{P_{ENF}^2 a^2}{16b^2 h^3 E_{11}}[9 + f_{SH2}], \tag{8}$$

$$f_{W1} = 5.07\left(\frac{h}{a}\right)\left(\frac{E_{11}}{E_{33}}\right)^{\frac{1}{4}} + 8.58\left(\frac{h}{a}\right)^2\left(\frac{E_{11}}{E_{33}}\right)^{\frac{1}{2}} + 2.08\left(\frac{h}{a}\right)^3\left(\frac{E_{11}}{E_{33}}\right)^{\frac{3}{4}}, \tag{10}$$

where δ_{DCB} is the crack opening displacement of the DCB specimen, which is eventually equal to the diameter of the prsetresser roller.

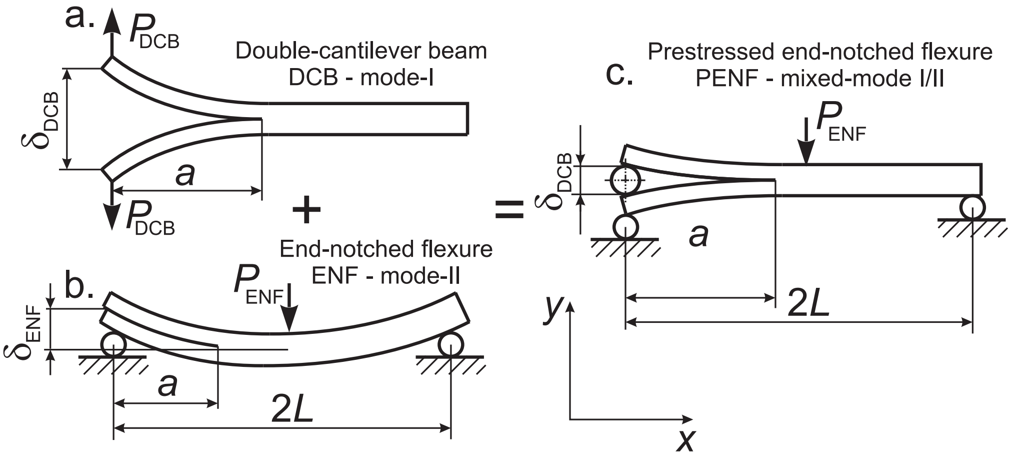
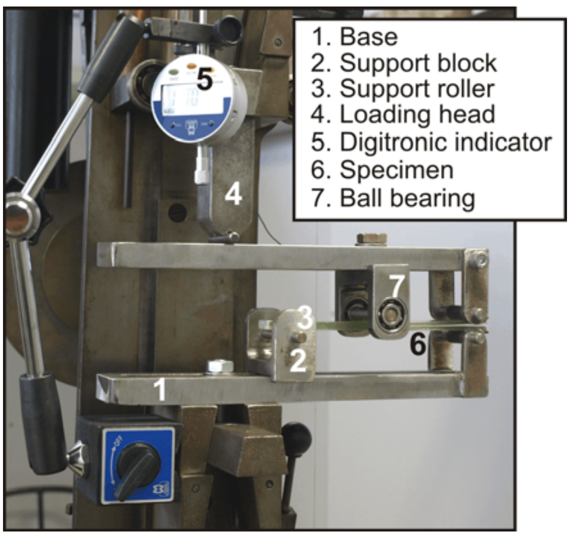


Fig.2. *The concept of the prestressed end-notched flexure system.*

4. EXPERIMENTS

First, the pure mode-I and pure mode-II critical ERR were determined performing DCB and ENF tests. The test setups are schematically shown by Fig. 2a-b. In both cases the specimens were loaded up to fracture initiation and the critical loads were measured. The DCB test requires steel hinges bonded to the specimen surfaces, an epoxy-based glue was used. In the ENF test a simple (non-standard) three-point bending setup was applied. For both tests four specimens were used. The critical ERRs were calculated by using an IBT scheme [5]. The results are: G_{IC}=260.9 J/m^2 and G_{IIC}=770.8 J/m^2.

For the MMB test the setup shown in Fig.3a was applied. In fact the same steel hinges were used as those of the DCB test. The specimens were tested at six different mode ratios providing six additional points in the G_I-G_{II} plane. Each specimen was loaded up to fracture initiation and the critical force was recorded. The deflection of the specimen was measured by a digitronic indicator. The mode-I and mode-II ERRs were calculated using Eqs. (1)-(6).



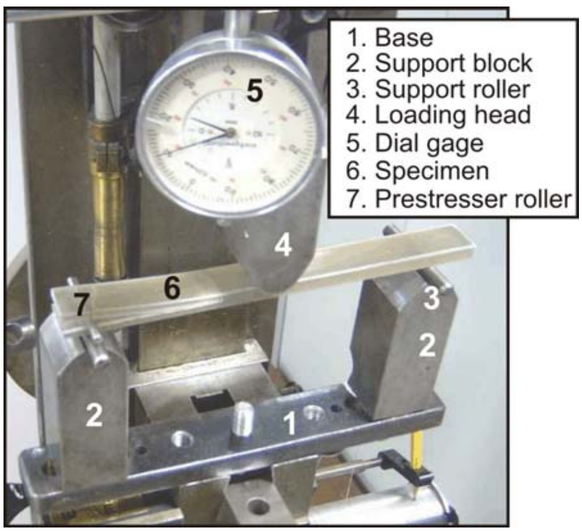


Fig.3. *The experimental equipment for the mixed-mode bending (a) and prestressed end-notched flexure tests (b).*

The experimental equipment for the PENF test is demonstrated in Fig. 3b. The span length was *2L*=150 mm, the crack length of interest was *a*=55 mm. The reason for the latter was that the critical crack opening measured from the DCB test is about 4.5 mm (if a=55 mm) and the crack tip is far enough (20 mm) from the point of load application. The stiffness, the compliance and the mode-II ERR of the PENF specimen are identical to those of the ENF specimen. We applied six steel rollers to control the mode-I part of the ERR including the following diameters: d_0=1, 1.5, 2, 2.4, 3 and 4 mm. It was assumed that the crack opening displacements (δ_{DCB}) in Eq. (7) are equal to these values. Similarly to the DCB and ENF tests, we applied four coupons at each steel roller. The load-deflection data was measured by using the scale of the testing machine and a dial gage (see Fig. 3a). The mode-I, mode-II ERRs and the mode ratio was calculated using Eqs. (7)-(8).

5. RESULTS AND DISCUSSION

Tables 1 and 2 list the results of the experiments reduced by the IBT scheme. It is shown that both method is suitable to cover the necessary mode ratios. The values in the tables are used to construct interlaminar fracture envelopes in the G_I-G_{II} plane.

	c [mm]	0(ENF)	28.5	35	42	50	70	170	(DCB)
Impoved	G_I/G_{II}	0	0.015	0.11	0.24	0.52	1.26	4.59	∞
beam theory	G_I [J/m^2]	0	8.62	35.1	50.9	78.9	110.5	153.8	260.9
(IBT)	G_{II} [J/m^2]	770.8	574.8	323.5	208.9	150.4	87.5	33.5	0
	$G_{I/II}$ [J/m^2]	770.8	583.4	358.6	259.8	229.3	197.9	187.3	260.9

Table 1. *The mode-I and mode-II critical ERRs calculated by using the IBT method, MMB test.*

	δ_{DCB} [mm]	0(ENF)	1	1.5	2	2.4	3	4	4.51(DCB)
Impoved	G_I/G_{II}	0	0.02	0.09	0.22	0.43	1.05	6.09	∞
beam theory		-	±7e-4	±4e-3	±4e-4	±0.02	±0.04	±0.4	-
(IBT)	G_I [J/m^2]	0	12.8	28.9	51.3	73.9	115.4	205.2	260.9
	G_{II} [J/m^2]	770.8	553.9	334.1	238.2	173.5	109.6	33.7	0
		±18.8	±16.6	±17.3	±4	±8.2	±3.6	±2.3	-
	$G_{I/II}$ [J/m^2]	770.8	566.7	363	289.5	247.4	225	238.9	260.9

Table 2. *The mode-I and mode-II critical ERRs calculated by using the IBT method, PENF test.*

In accordance with the power criterion [5] the following relation may be established between the mode-I and mode-II ERRs:

$$\left(\frac{G_I}{G_{IC}}\right)^{p_1} + \left(\frac{G_{II}}{G_{IIC}}\right)^{p_2} = 1, \tag{11}$$

On the other hand Williams' criterion [7] recommends the following expression:

$$\left(\frac{G_I}{G_{IC}} - 1\right)\left(\frac{G_{II}}{G_{IIC}} - 1\right) - I_i\left(\frac{G_I}{G_{IC}}\right)\left(\frac{G_{II}}{G_{IIC}}\right) = 0, \tag{12}$$

where I_i is the interaction parameter between the mode-I and mode-II ERRs. In Eqs. (11)-(12) G_{IC} is the critical ERR rate under pure mode-I (calculated from the data of the DCB specimen), G_{IIC} is the mode-II critical ERR (calculated from the data of the ENF specimen). The results of the MMB and PENF tests (listed in Tables 1 and 2) were used to provide six additional points in the G_I-G_{II} plane. The power parameters (p_1, p_2) in Eq. (11) and the interaction parameter (I_i) in Eq. (12) were determined by a curve-fit technique.

The fracture envelopes are plotted in Figs. 4 and 5 calculated by the MMB and $PENF_{I/II}$ tests, respectively. It is interesting that the interaction parameter I_i was not the same in both figures; although the differences between the power parameters are also small ones. Figs 4 and 5 show that there are negligible differences between the fracture envelopes determined by two different methods. Moreover, the difference between the power and Williams' criteria is negligible, both describes the same failure locus. However, the application of Williams' method is simpler.

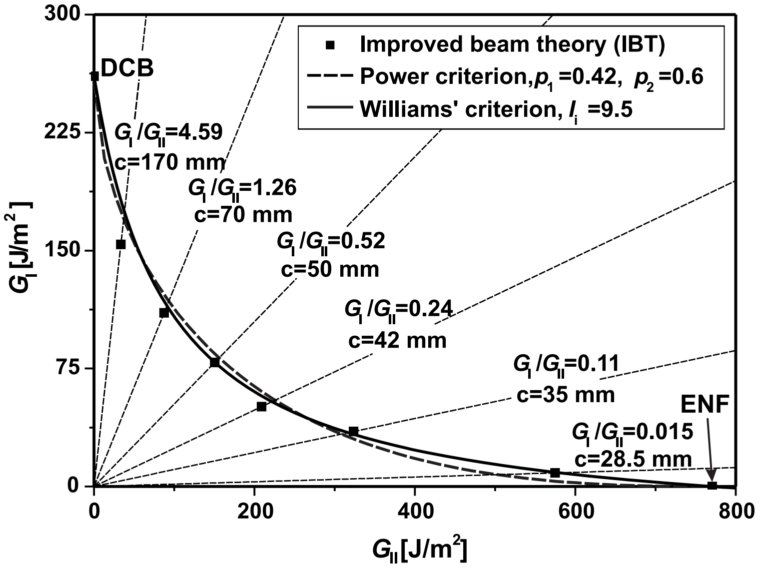


Fig. 4. *Interlaminar fracture envelope in the G_I-G_{II} plane determined by the MMB test.*

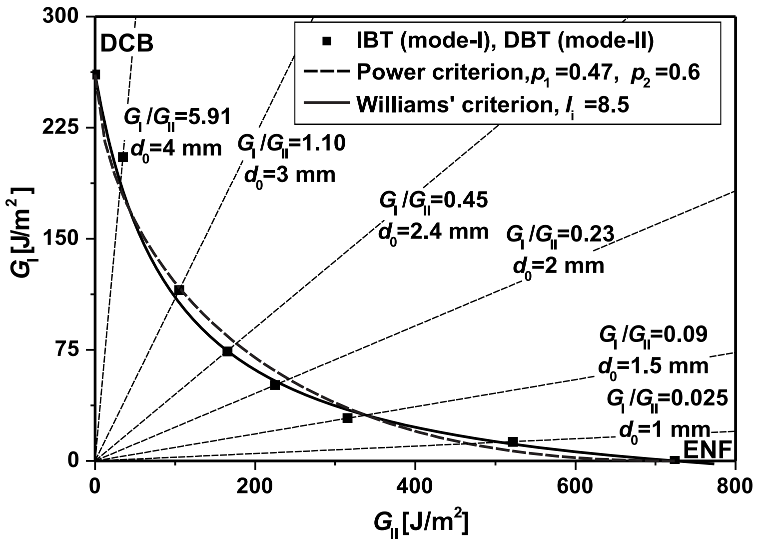


Fig. 5. *Interlaminar fracture envelope in the G_I-G_{II} plane determined by the PENF test.*

6. CONCLUSIONS

The literature of composite fracture mechanics proposes many excellent mixed-mode I/II tests, however a significant incompleteness is that experimental results are available only for different materials obtained from different tests. Therefore to address this void two different configurations – namely the mixed-mode bending and the prestressed end-notched flexure - were used to determine the interlaminar fracture envelopes of a unidirectional glass/polyester composite material.

The two methods apply the same superposition scheme and specimen geometry. The results indicate that the critical energy release rates under mixed-mode I/II loading lead to essentially the same interlaminar fracture envelopes. Furthermore, it follows that the prestressed end-notched flexure system is a useful alternative of the mixed-mode bending apparatus if the aim is to determine the fracture envelope of the material. Finally the comparison of different mixed-mode I/II configurations is an important issue, because it my help us to choose the optimal solution for a given material.

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