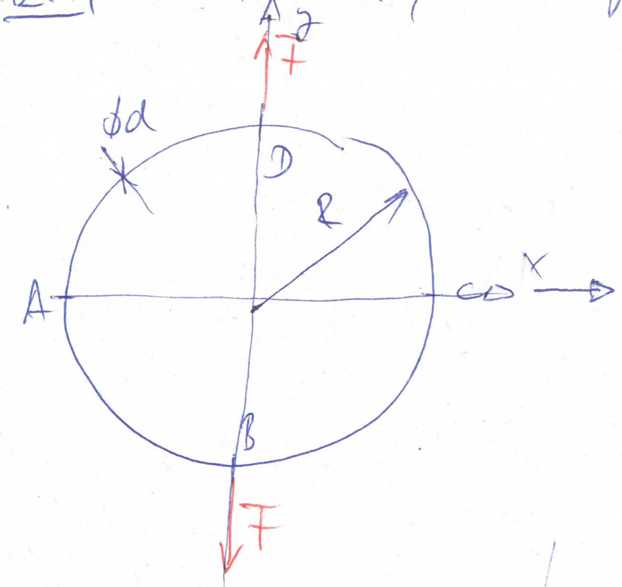
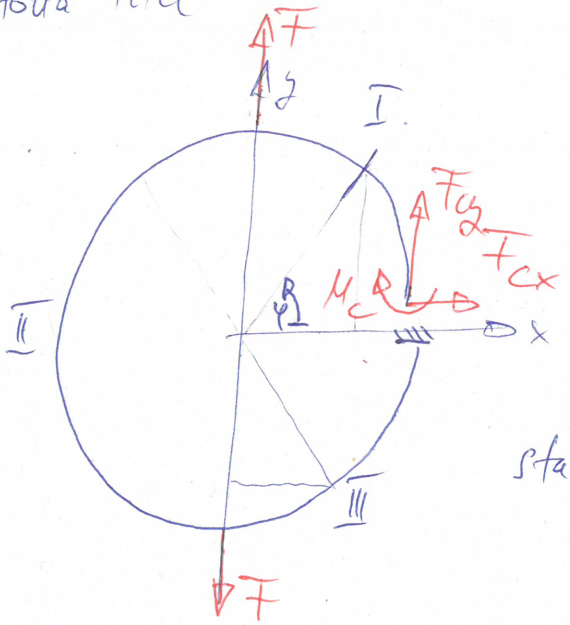
12.1.1 zárt keret, → nyitottás tétel



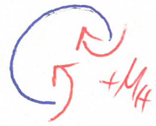


$u_C=0$

$v_C=0$

$\varphi_C=0$

statikai egyenesthetőség



R, F, d ismert

hajlítónyomaték maximuma? A, B, C, D?

$$M_{H1}=M_C-F_{Cx}R\sin\varphi-F_{Cy}(1-\cos\varphi)R\quad \varphi\in[0,\pi/2]$$

$$M_{H2}=M_C-F_{Cx}R\sin\varphi-F_{Cy}R(1-\cos\varphi)-FR\underbrace{\sin\left(\varphi-\frac{\pi}{2}\right)}_{-\cos\varphi}$$

$$\varphi\in[\pi/2,3\pi/2]$$

$$M_{H3}=M_C-F_{Cx}R\sin\varphi-F_{Cy}R(1-\cos\varphi)+FR\cos\varphi-FR\underbrace{\sin\left(\varphi-\frac{3\pi}{2}\right)}_{\cos(\varphi)}\quad \varphi\in\left[\frac{3\pi}{2},2\pi\right]$$

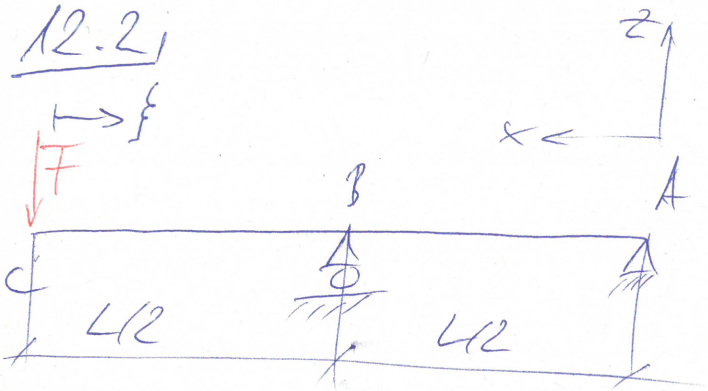
$$u_C=\frac{1}{I_zE}\int_{(l)}M_H\frac{\partial M_H}{\partial F_{Cx}}ds=\frac{1}{I_zE}\left\{\int_0^{\pi/2}M_{H1}\frac{\partial M_{H1}}{\partial F_{Cx}}R\,d\varphi+\int_{\pi/2}^{3\pi/2}M_{H2}\frac{\partial M_{H2}}{\partial F_{Cx}}R\,d\varphi+\int_{3\pi/2}^{2\pi}M_{H3}\frac{\partial M_{H3}}{\partial F_{Cx}}R\,d\varphi\right\}=0$$

$$v_C=\frac{1}{I_zE}\int_{(l)}M_H\frac{\partial M_H}{\partial F_{Cy}}ds=\frac{1}{I_zE}\{\dots\}=0;\quad \varphi_C=\frac{1}{I_zE}\int_{(l)}M_H\frac{\partial M_H}{\partial M_C}ds=\frac{1}{I_zE}\{\dots\}=0$$

$$F_{Cx}=0;\ F_{Cy}=-\frac{F}{2};\ M_C=-FR\frac{\pi-2}{2\pi}=M_A\quad M_D=FR\cdot\frac{1}{\pi}=M_B$$

$$|M_D|>|M_C|\quad M_{HMAX}=\frac{FR}{\pi}$$

12.2



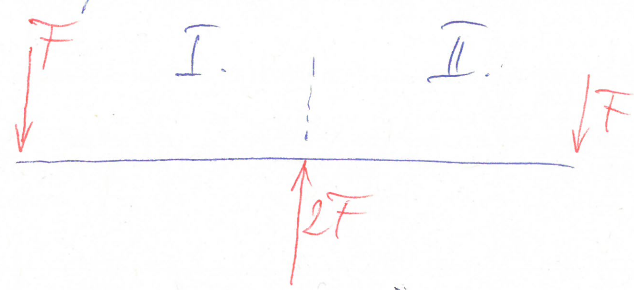
tartó

maximális elmozdulás AB között?

w_C =? rugalmassal DE-vel

L, F adott, IE = állandó

Rápillantással



$M_{H1} = F\cdot\xi \quad ; \xi\in[0, L/2]$

$M_{H2} = F\cdot\xi - 2F(\xi - L/2) = F\cdot L - F\xi$

$\xi\in[L/2, L]$

Rugalmassal DE-e

$-\frac{M_{H1}}{IE} = w_1''(\xi); \xi\in[0, L/2]$

$-\frac{M_{H2}}{IE} = + w_2''(\xi); \xi\in[L/2, L]$

+ Kapcsolódási és Peremfeltételek

(1) $w_1(L/2) = w_2(L/2)$

(2) $w_1'(L/2) = w_2'(L/2)$

+

(3) $w_1(L/2) = 0$ vagy $w_2(L/2) = 0$

(4) $w_2(L) = 0$

$w_1'(\xi) = \int -\frac{F}{IE}\xi\, d\xi = -\frac{F}{IE}\left(\frac{\xi^2}{2} + C_{11}\right)$

$w_1(\xi) = \int w_1'(\xi)\, d\xi = -\frac{F}{IE}\left(\frac{\xi^3}{6} + C_{11}\xi + C_{12}\right)$

$w_2'(\xi) = -\frac{F}{IE}\int(-\xi + L)\, d\xi = -\frac{F}{IE}\left(-\frac{\xi^2}{2} + L\xi + C_{21}\right)$

$w_2(\xi) = \int w_2'(\xi)\, d\xi = -\frac{F}{IE}\left(-\frac{\xi^3}{6} + L\frac{\xi^2}{2} + C_{21}\xi + C_{22}\right)$

$C_{11}, C_{12}, C_{21}, C_{22}$ meghatározása

1) $-\cancel{\frac{F}{IE}}\left(\frac{L^3}{48} + C_{11}\frac{L}{2} + C_{12}\right) = -\cancel{\frac{F}{IE}}\left(-\frac{L^3}{48} + L\frac{L^2}{8} + C_{12}\frac{L}{2} + C_{22}\right)$

(2) $-\frac{F}{IE}\left(\frac{L^2}{8}+C_{11}\right) = -\frac{F}{IE}\left(-\frac{L^2}{8}+\frac{L^2}{4}+C_{21}\right)$

(3) $-\frac{F}{IE}\left(\frac{L^3}{48}+C_{11}\frac{L}{2}+C_{12}\right)=0$

(4) $-\frac{F}{IE}\left(-\frac{L^3}{6}+\frac{L^3}{2}+C_{21}\cdot L+C_{22}\right)=0$

$C_{11}=-\frac{5L^2}{24};\ C_{12}=L^3/12;\ C_{21}=-\frac{11L^2}{24};\ C_{22}=+L^3/8$

w_{max} meghatározása $\equiv$ szélsőérték keresés

$w_{2,max} \to w_2'(\xi_0)=0$

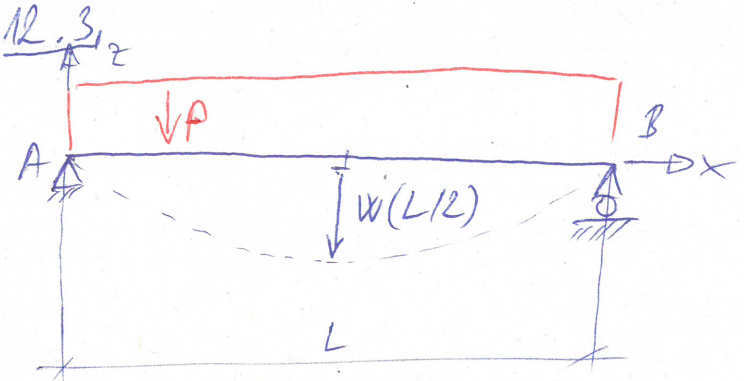
$-\frac{F}{IE}\left(-\frac{\xi^2}{2}+L\xi+\left(-\frac{11}{24}L^2\right)\right)=0 \qquad \xi_{1,2}=\frac{-L\pm\sqrt{L^2-\frac{11}{12}L^2}}{-2\cdot\frac{1}{2}}=L\left(1\pm\frac{1}{\sqrt{12}}\right)$

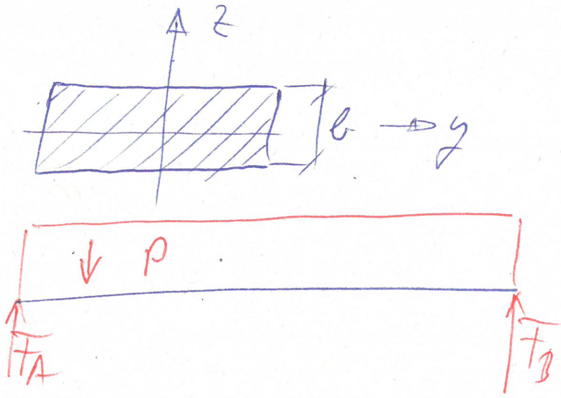
$\xi_1=L\left(1+\frac{1}{\sqrt{12}}\right)$ rúdon kívül található

$\xi_2=\xi_0=L\left(1-\frac{1}{\sqrt{12}}\right) \qquad w_2(\xi_0)=w_{max}=\frac{FL^3}{IE}\frac{1}{36\sqrt{12}}$

$w_C=w_1(0)=-\frac{F}{IE}\cdot\frac{L^3}{12}$

12.3.





$\sigma_{MAX} = 60\,MPa$, $w(L/2) = -\frac{L}{600}$, $E = 200\,GPa$, $b/L = ?$

RP. $F_A = F_B = \frac{pL}{2}$

$M_H = -\frac{pL}{2}x + p\frac{x^2}{2}$ $\qquad M_{HMAX} = M_H\big|_{x=L/2} = -\frac{pL}{2}\cdot\frac{L}{2} + p\frac{L^2}{8} = -\frac{pL^2}{8}$

$$\sigma_{MAX} = \frac{M_{HMAX}}{I_y}\cdot\left(-\frac{b}{2}\right) = \boxed{\frac{pL^2}{2\cdot 8 I_y}\cdot b = 60\,MPa} \quad (*)$$

$$w''(x) = -\frac{M_H}{I_y E}$$

$$w'(x) = -\int \frac{p}{2I_yE}\left(-Lx + x^2\right)dx = -\frac{p}{2I_yE}\left(-L\frac{x^2}{2} + \frac{x^3}{3} + C_1\right)$$

$$w(x) = \int w'(x)\,dx = -\frac{p}{2I_yE}\left(-L\frac{x^3}{6} + \frac{x^4}{12} + C_1x + C_2\right)$$

peremfeltételek

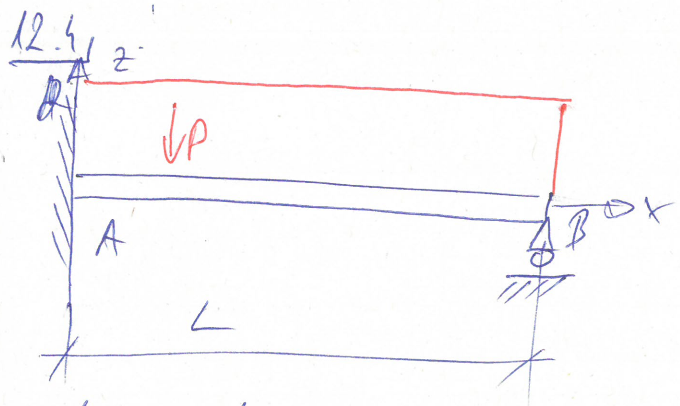
$w(0) = 0 \rightarrow \underline{C_2 = 0}$

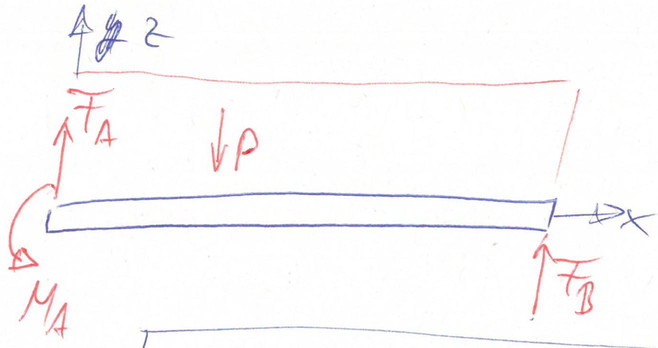
$$w(L) = 0 = -\cancel{\frac{p}{2I_yE}}\left(-\frac{L^4}{6} + \frac{L^4}{12} + C_1\cdot L\right) \rightarrow \underline{C_1 = \frac{L^3}{12}}$$

$$w\left(\frac{L}{2}\right) = -\frac{p}{2I_yE}\left(-\frac{L}{6}\cdot\frac{L^3}{8} + \frac{L^4}{12\cdot 16} + \frac{L^4}{24} + 0\right) = \boxed{-\frac{p}{2I_yE}\cdot\frac{5L^{\cancel{4}3}}{192} = -\frac{\cancel{L}1}{600}} \quad (**)$$

$(*)/(**) \rightarrow b/L = 3/80 = 0.0375$

12.4





$\boxed{+\ w(0)=0,\ w'(0)=0,\ w(L)=0}$

reakcióerők meghatározása rugalmassal DE segítségével

$$M_H = M_A - F_A\cdot x + p\frac{x^2}{2}$$

$$w''(x) = -\frac{M_H}{IE} \rightarrow \quad w'(x) = \int -\frac{1}{IE}\left(M_A - F_A\cdot x + p\frac{x^2}{2}\right)dx$$

$$w'(x) = -\frac{1}{IE}\left(M_A x - F_A\frac{x^2}{2} + p\frac{x^3}{6} + C_1\right)$$

$$w(x) = \int w'(x)dx = \int -\frac{1}{IE}\left(M_A x - F_A\frac{x^2}{2} + p\frac{x^3}{6} + C_1\right)dx$$

$$w(x) = -\frac{1}{IE}\left(M_A\frac{x^2}{2} - F_A\frac{x^3}{6} + p\frac{x^4}{24} + C_1 x + C_2\right)$$

C_1, C_2 meghatározása

3 pf. közül 2 választása

$w(0)=0,\ w'(0)=0 \rightarrow C_1=0,\ C_2=0$

M_A, F_A, F_B meghatározása aktív terhelés függvényeként

$$\left.\begin{aligned} z:&\quad F_A - pL + F_B = 0\\ y:&\quad M_H^A = M_A - p\frac{L^2}{2} + F_B L = 0\\ &w(L) = -\frac{1}{IE}\left(M_A\frac{L^2}{2} - F_A\frac{L^3}{6} + p\frac{L^4}{24}\right) = 0\end{aligned}\right\} \quad \begin{aligned} M_A &= pl^2/8\\ F_A &= \frac{5}{8}pl\\ F_B &= \frac{3}{8}pl\end{aligned}$$