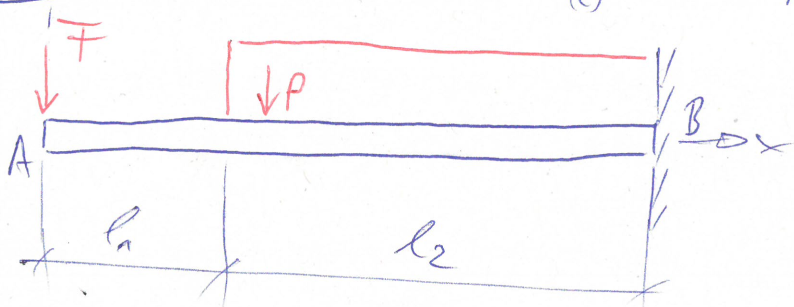
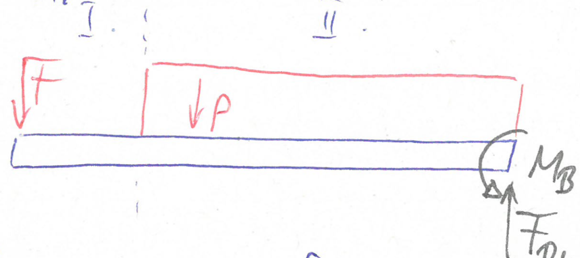
M.1.

$$V_i=\frac{1}{IE}\int_{(l)} M_H \frac{\partial M_H}{\partial F_i}\,dx+\frac{1}{AE}\int_{(l)} N\frac{\partial N}{\partial F_i}\,dx+\frac{1}{I_pG}\int_{(l)} M_t\frac{\partial M_t}{\partial F_i}\,dx$$





$F=2000\,N$, $p=500\,N/m$

$l_1=3\,m$, $l_2=8\,m$, $IE=10^8\,Nm^2$

$v_A=?$, $\varphi_A=?$

$$M_{H1}=F\cdot x \qquad x\in[0,l_1]$$

$$\frac{\partial M_{H1}}{\partial F}=x$$

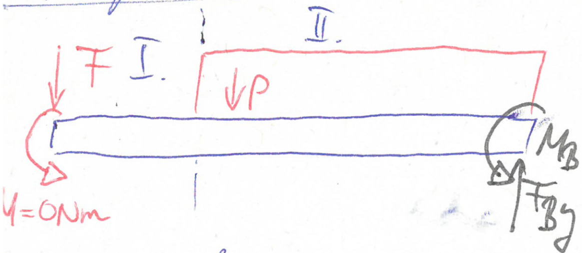
$$M_{H2}=F\cdot x+p\frac{(x-l_1)^2}{2} \qquad x\in[l_1,l_1+l_2]$$

$$\frac{\partial M_{H2}}{\partial F}=x$$

$$v_A=\frac{1}{IE}\int_{(l)} M_H\frac{\partial M_H}{\partial F}\,dx=\frac{1}{IE}\left\{\int_0^{l_1} M_{H1}\cdot\frac{\partial M_{H1}}{\partial F}\,dx+\int_{l_1}^{l_1+l_2} M_{H2}\frac{\partial M_{H2}}{\partial F}\,dx\right\}=\frac{1}{IE}\left(\frac{8F(l_1+l_2)^3+l_2^3(4l_1+3l_2)p}{24}\right)$$

$v_A=0.01271\,m \downarrow$

φ_A meghatározása



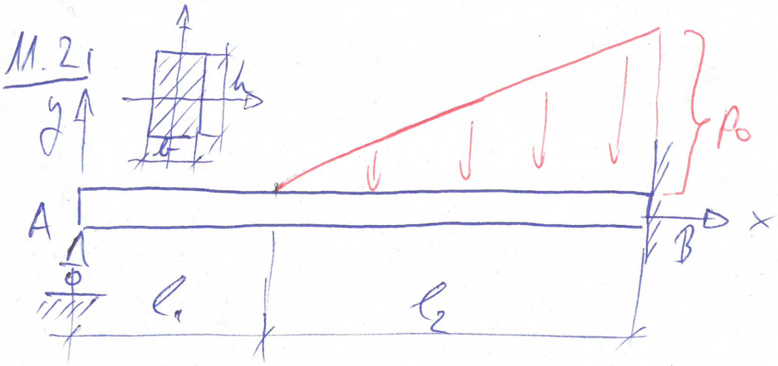
$$M_{H1}=M+F\cdot x\ ;\ x\in[0,l_1]\ ;\ \frac{\partial M_{H1}}{\partial M}=1$$

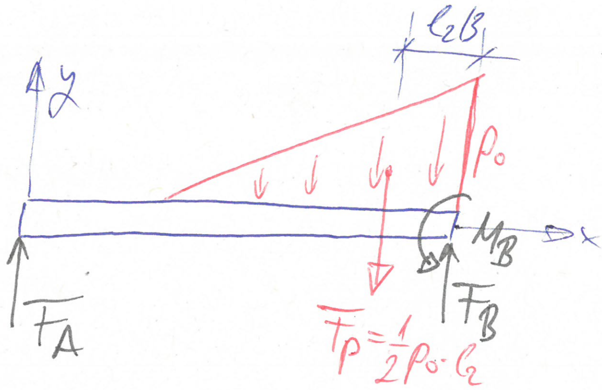
$$M_{H2}=M+F\cdot x+p\frac{(x-l_1)^2}{2}\ ;\ x\in[l_1,l_1+l_2]\ ;\ \frac{\partial M_{H2}}{\partial M}=1$$

$$\varphi_A=\frac{1}{IE}\left\{\int_0^{l_1} M_{H1}\cdot\frac{\partial M_{H1}}{\partial M}\,dx+\int_{l_1}^{l_1+l_2} M_{H2}\frac{\partial M_{H2}}{\partial M}\,dx\right\}=\frac{1}{IE}\left\{\int_0^{l_1}(0+F\cdot x)\cdot 1\,dx+\int_{l_1}^{l_1+l_2}\left(0+F\cdot x+p\frac{(x-l_1)^2}{2}\right)\cdot 1\,dx\right\}=$$

$$\varphi_A=\frac{3F(l_1+l_2)^2+l_2^3p}{6IE}=1.637\cdot 10^{-3}\,rad=0.0938^\circ$$

11.2.





$l_1 = 4m$, $l_2 = 6m$, $p_0 = 5000\,N/m$, $h = 0.14m$, $b = 0.1m$, $E = 200\,GPa$

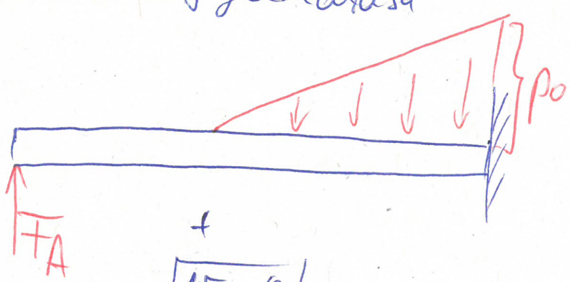
$F_A = ?$

$y: \; F_A + F_B - p_0 \cdot l_2 \cdot \frac{1}{2} = 0$

$x: \; 0 = 0$

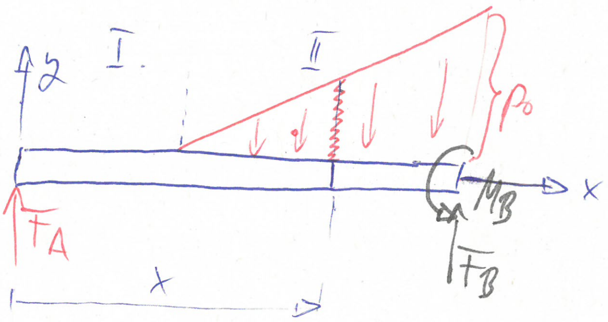
$z: \; M_A^B := 0 = F_A(l_1 + l_2) - F_p \cdot l_2/3 - M_B = 0$

feladat átfogalmazása



$+$

$\boxed{v_A = 0}$



$M_{H1} = -F_A \cdot x \; ; \; x \in [0, l_1] \; ; \; \frac{\partial M_{H1}}{\partial F_A} = -x$

$M_{H2} = -F_A \cdot x + \frac{1}{2} \frac{p_0}{l_2}(x - l_1)(x - l_1)\frac{1}{3}(x - l_1) = -F_A \cdot x + \frac{1}{6}\frac{p_0}{l_2}(x - l_1)^3 ; \; x \in [l_1, l_1 + l_2]$

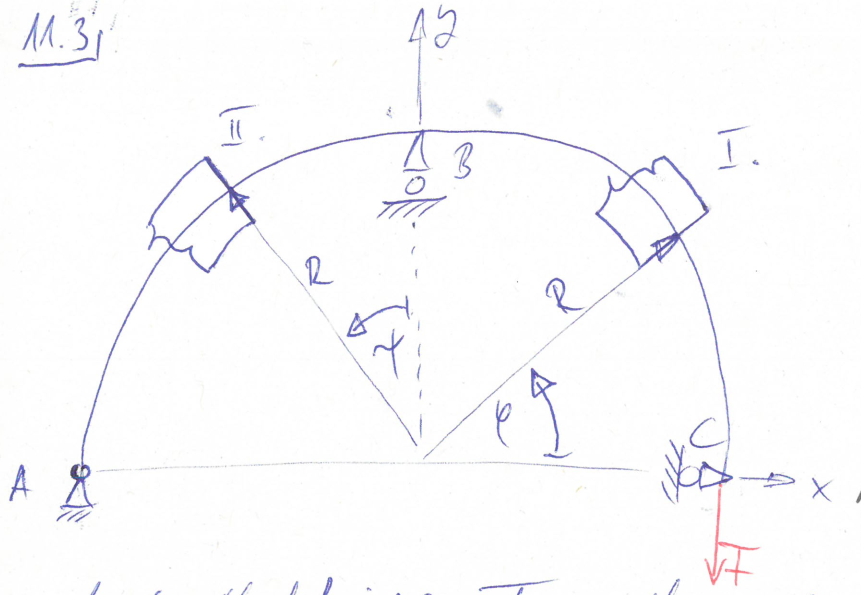
$\frac{\partial M_{H2}}{\partial F_A} = -x$

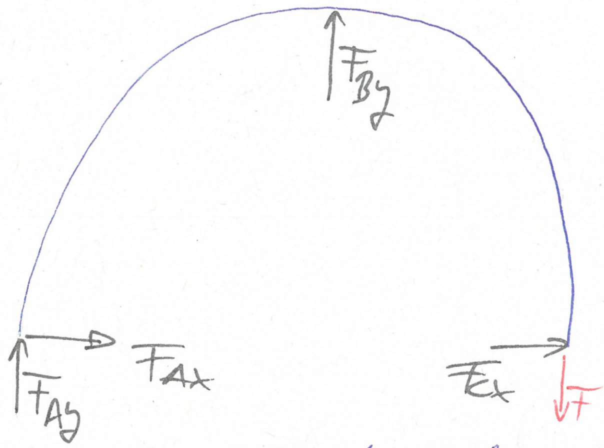
$v_A = \frac{1}{IE}\int_{(l)} M_H \frac{\partial M_H}{\partial F_A} dx = \frac{1}{IE}\left\{\int_0^{l_1} M_{H1}\frac{\partial M_{H1}}{\partial F_A}dx + \int_{l_1}^{l_1+l_2} M_{H2}\frac{\partial M_{H2}}{\partial F_A}dx\right\} = 0$

$I = \frac{b h^3}{12}$

$F_A = \frac{(5 l_1 l_2^3 + 4 l_2^4) p_0}{40 (l_1 + l_2)^3} = 1188N$

11.3.





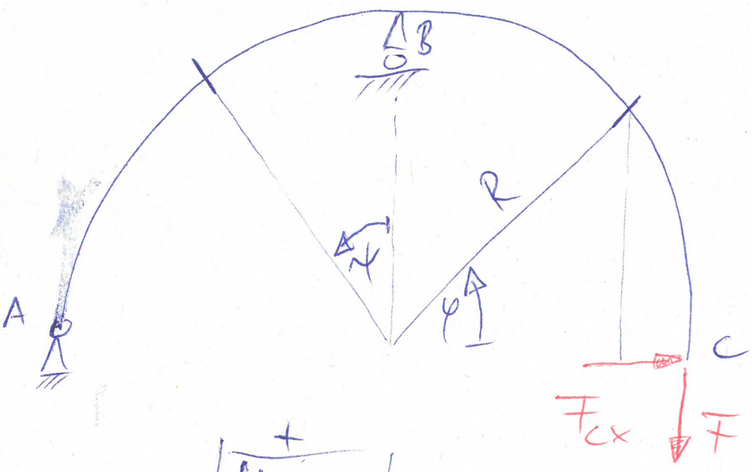
reakcióerők kifejezése F segítségével, v_C elmozdulásához szükséges deriválás

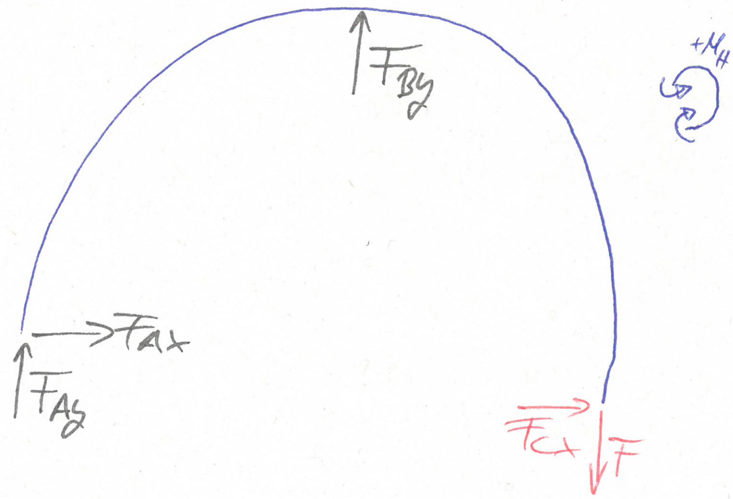
x: $F_{Ax}+F_{Cx}=0$

y: $F_{Ay}+F_{By}-F=0$ $\quad \boxed{F_{Ay}=-F}$

z: $M_H^A=0=F_{By}\cdot R-F\cdot 2R \rightarrow \boxed{F_{By}=2F}$

feladat átfogalmazása





$$\boxed{\overset{+}{u_C}=0}$$

$$M_{H1}=F\cdot R(1-\cos\varphi)-F_{Cx}\cdot R\sin\varphi\ ;\ \varphi\in[0,\pi/2]\ ;\ \frac{\partial M_{H1}}{\partial F_{Cx}}=-R\sin\varphi$$

$$M_{H2}=+FR(1+\sin\psi)-F_{Cx}R\cos\psi-\underbrace{F_{By}}_{2F}R\sin\psi\ ;\ \psi\in[0,\pi/2]\ ;\ \frac{\partial M_{H2}}{\partial F_{Cx}}=-R\cos\psi$$

$$u_C=\frac{1}{IE}\int_{(l)}M_H\frac{\partial M_H}{\partial F_{Cx}}ds=\frac{1}{IE}\left\{\int_0^{\pi/2}M_{H1}\frac{\partial M_{H1}}{\partial F_{Cx}}R\,d\varphi+\int_0^{\pi/2}M_{H2}\frac{\partial M_{H2}}{\partial F_{Cx}}R\,d\psi\right\}=0\rightarrow F_{Cx}=\frac{2F}{\pi}$$

Ha M_H-n kívül N-et is figyelembevesszük

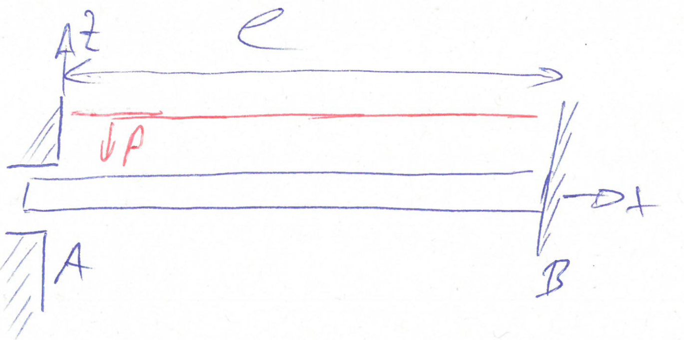
$$F_{cx} = \frac{2F}{\pi} \cdot \frac{AR^2 - I}{AR^2 + I} \quad \text{pl } A = \frac{d^2\pi}{4} \quad I = \frac{d^4\pi}{64} \quad d \ll R \Rightarrow I \ll AR^2$$

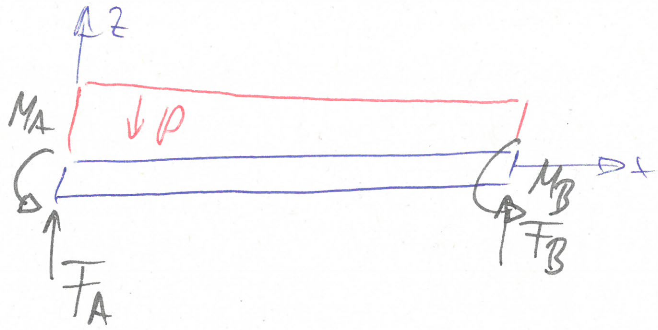
v_C meghatározásához deriváltak

$$\frac{\partial M_{H1}}{\partial F} = R(1-\cos\varphi) \; ; \; \frac{\partial M_{H2}}{\partial F} = R(1+\sin\varphi) - 2R\sin\varphi$$

$$\underset{\sim}{v_C} = \frac{1}{IE}\left\{\int_0^{\pi/2} M_{H1} \cdot \frac{\partial M_{H1}}{\partial F} R\,d\varphi + \int_0^{\pi/2} M_{H2} \frac{\partial M_{H2}}{\partial F} R\,d\varphi\right\} = \underline{\frac{FR^3}{2IE} \, \frac{3\pi^2 - 8\pi - 4}{\pi}}$$

11.4





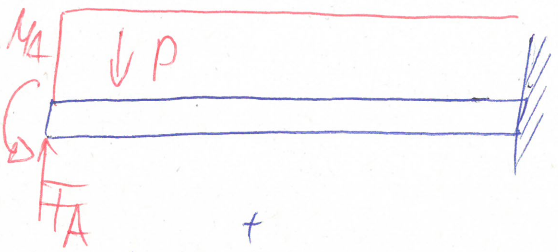
reakcióerők meghatározása

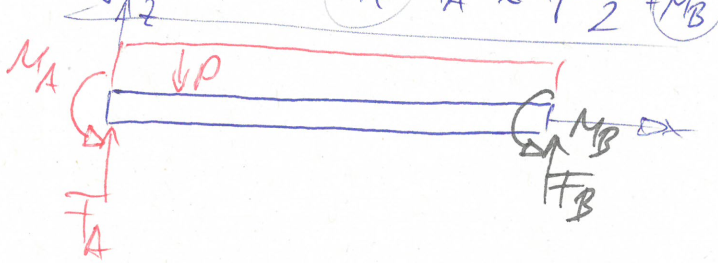
IE adott

x: $0=0$ ✓

z: $F_A+F_B-p\cdot l=0$

y $M_H^B := 0 = M_A - F_A\cdot l + p\frac{l^2}{2} + M_B$





+

$w_A=0$ és $\varphi_A=0$

$$M_H = M_A - F_A\cdot x + p\frac{x^2}{2};\quad x\in[0,l];\quad \frac{\partial M_H}{\partial F_A}=-x;\quad \frac{\partial M_H}{\partial M_A}=1$$

$$w_A=\frac{1}{IE}\int_{(l)} M_H\cdot\frac{\partial M_H}{\partial F_A}dx=\frac{1}{IE}\int_0^l\left(M_A-F_A\cdot x+p\frac{x^2}{2}\right)\cdot(-x)\,dx=0$$

$$\varphi_A=\frac{1}{IE}\int_{(l)} M_H\frac{\partial M_H}{\partial M_A}dx=\frac{1}{IE}\int_0^l\left(M_A-F_A\cdot x+p\frac{x^2}{2}\right)\cdot 1\,dx=0$$

$$F_A=\frac{pl}{2},\quad M_A=\frac{pl^2}{12}$$

$$F_B=\frac{pl}{2},\quad M_B=-\frac{pl^2}{12}$$