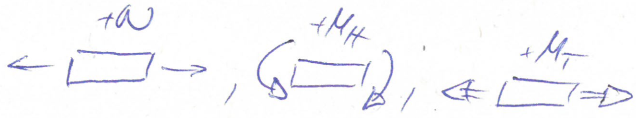
Betti-tétel Nagy betűs → valódi terhelésből, kisbetűs → "vizsgáló" terhelés

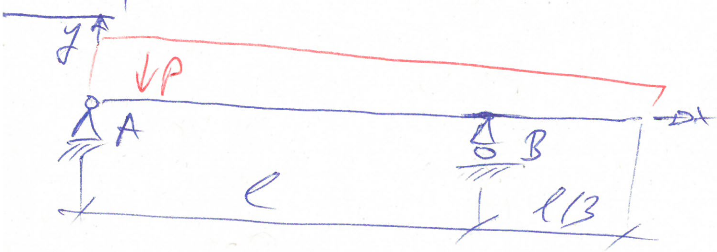
$$\int_{(l)} \frac{N}{AE} n + \frac{M_H}{I_z E} m_h + \frac{M_T}{I_p G} m_t \, ds = \dots$$

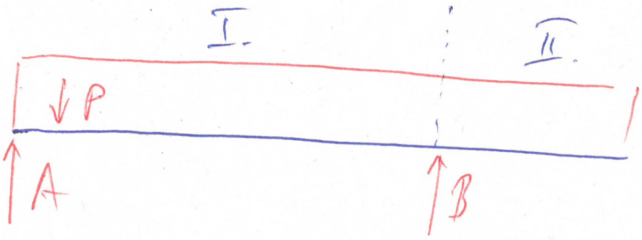


vizsgáló ~~...~~ terhelés → adott irányú erő → adott irányú elmozdulás
→ adott irányú nyomaték → — " — szögelfordulás

$F = 1N$, $M = 1Nm$

10.1,





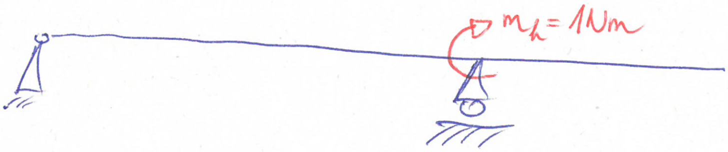
$l = 3m$, $p = 6\,kN/m$, $A = 8kN$, $B = 16kN$, $IE = 200\,kNm^2$

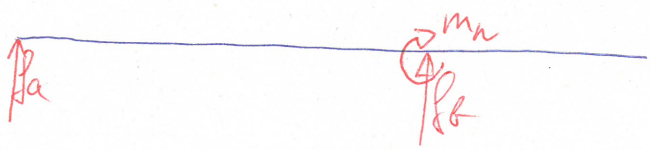
$\varphi_B = ?$

húzás nincs, csavarás nincs $N \equiv 0$, $M_T \equiv 0$

$$M_H^1 = -A \cdot x + p\frac{x^2}{2} \quad x \in [0, l]$$

$$M_H^2 = -A \cdot x - B(x - l) + p\frac{x^2}{2} \quad x \in [l, \tfrac{4}{3}l]$$





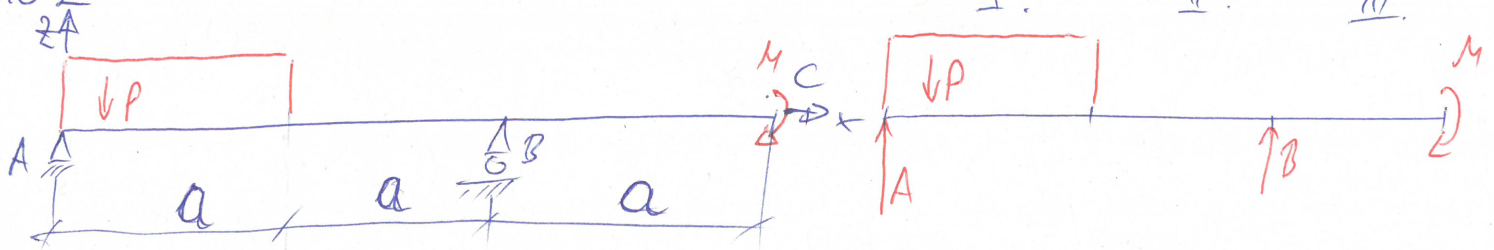
$$f_a + f_b = 0 \qquad f_a = -\frac{1}{l}$$

$$m_h^a := 0 = f_b \cdot l - m_h = 0 \rightarrow f_b = \frac{1}{l} \Bigg\} \quad m_h^1 = \frac{1}{l}x, \quad m_h^2 \equiv 0$$

$$\varphi_B = \int_{(l)} \frac{M_H}{IE} m_h \, dx = \int_0^l \frac{M_H^1}{IE} m_h^1 \, dx + \int_l^{\frac{4}{3}l} \frac{M_H^2}{IE} \underbrace{m_h^2}_{0} \, dx = \int_0^l \frac{1}{lIE}\left(-Ax^2 + p\frac{x^3}{2}\right) dx =$$

$$= \frac{1}{lIE}\left[-A\frac{x^3}{3} + p\frac{x^4}{8}\right]_{x=0}^{x=l} = \frac{1}{IEl}\left(-A\frac{l^3}{3} + p\frac{l^4}{8}\right) = -0.01875\ rad = -1.0743$$

10.2



$a = 0.5\,m$, $p = 8\,kN/m$, $M = 2\,kNm$, $A\ 1kN$, $B = 3\,kN$, $I_yE = 50\,kNm^2$

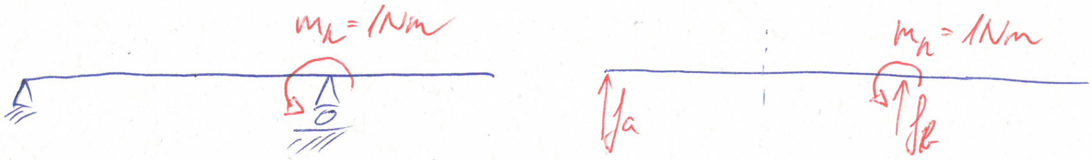
$\varphi_B = ?$; $w_C = ?$

$$M_H^1 = -Ax + p\frac{x^2}{2} \quad x \in [0, a]$$

$$M_H^2 = -Ax + pa\left(x - \frac{a}{2}\right) \quad x \in [a, 2a]$$

$$M_H^3 = -Ax + pa\left(x - \frac{a}{2}\right) - B(x - 2a) \quad x \in [2a, 3a]$$

φ_B meghatározása



$$f_a + f_B = 0 \rightarrow f_a = \frac{1}{2a}$$

$$m_h^a := 0 = f_B \cdot 2a + m_h = 0 \rightarrow f_B = -\frac{1}{2a}$$

$$m_h^1 = -\frac{1}{2a}x \ , \quad m_h^2 = -\frac{1}{2a}x \ ; \quad m_h^3 \equiv 0$$

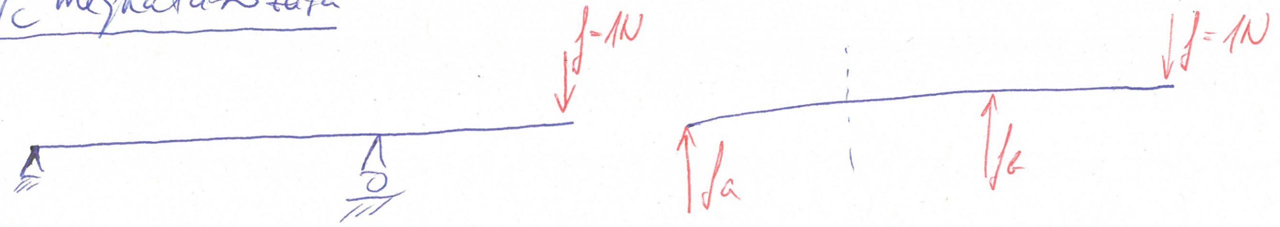
$$\varphi_B = \int_{(e)} \frac{M_H}{I_yE} m_h \, dx = -\frac{1}{2a} \cdot \frac{1}{I_yE}\left(\int_0^a -Ax^2 + p\frac{x^3}{2}\,dx + \int_a^{2a} -Ax^2 + pa\left(x^2 - \frac{a}{2}x\right)dx + 0\right) =$$

$$= -\frac{1}{2a}\frac{1}{I_yE}\left(\left[-A\frac{x^3}{3} + p\frac{x^4}{8}\right]_{x=0}^{x=a} + \left[-A\frac{x^3}{3} + pa\left(\frac{x^3}{3} - \frac{a}{2}\frac{x^2}{2}\right)\right]_{x=a}^{x=2a}\right) =$$

$$= -\frac{1}{2a}\frac{1}{I_yE}\left(-A\frac{a^3}{3} + p\frac{a^4}{8} + \left(-A\frac{7}{3}a^3 + pa\underbrace{\left(\underbrace{\left(\frac{8}{3} - 1\right)}_{\frac{5}{3}}a^3 - \underbrace{\left(\frac{1}{3} - \frac{1}{4}\right)}_{\frac{1}{12}}a^3\right)}_{19/12\,a^3}\right)\right) = \ldots =$$

$$= \underline{-0.010417\,rad} = \underline{-0.5968^\circ}$$

w_C meghatározása



$f_a + f_b - 1 = 0 \rightarrow f_a = -\frac{1}{2}$

$m_h^a = 0 = f_b \cdot 2a - 3a \cdot 1 = 0 \rightarrow f_b = \frac{3}{2}$

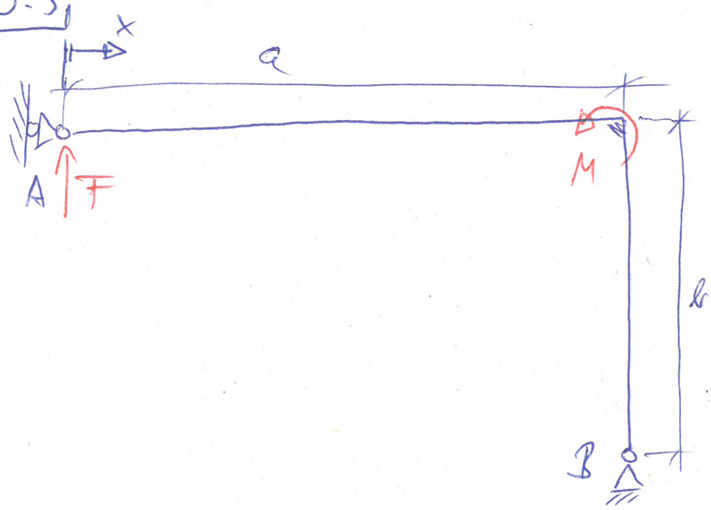
$m_h^1 = \frac{1}{2}x \;,\; m_h^2 = \frac{1}{2}x \;,\; m_h^3 = \frac{1}{2}x - f_b(x-2a) = 3a - x$

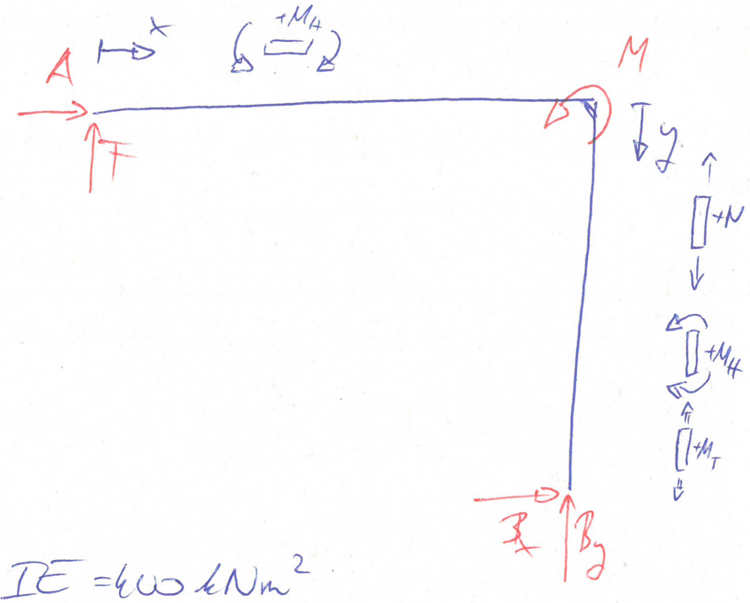
$$w_C = \int_{(l)} \frac{M_H}{I_y E} m_h \, dx = \frac{1}{I_y E}\left\{ \frac{1}{2}\int_0^a -Ax^2 + p\frac{x^3}{2}\,dx + \frac{1}{2}\int_a^{2a} -Ax^2 + pa\left(x^2 - \frac{a}{2}x\right)dx + \int_{2a}^{3a} \dots \, dx \right\} =$$

$$\frac{1}{I_y E}\frac{a^3}{18}\left(85ap - 120A - 8B\right) = \underline{0.0102\,m}$$

~~függőleges mid maximalis x irányú elmozdulása ≈ 0.08mm~~

10-3,





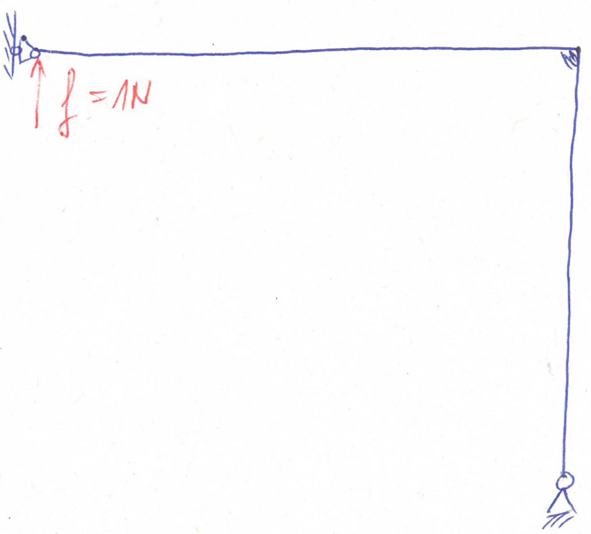
$a = 2m, \; b = 1m, \; M = 4.5\,kNm, \; F = 2\,kN, \; IE = 400\,kNm^2$

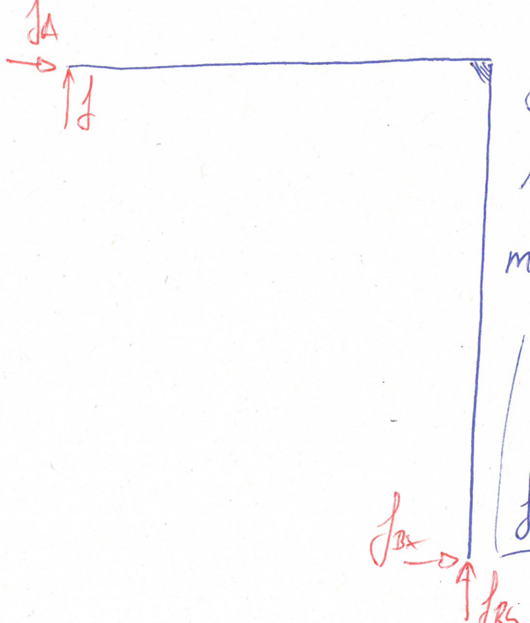
$A = 0.5\,kN, \; B_x = -0.5\,kN, \; B_y = 2\,kN$

$v_A = ?$

$M_{H1} = -Fx$

$M_{H2} = -F \cdot a + M - A \cdot y$





$f_A + f_{Bx} = 0$

$1 + f_{By} = 0 \rightarrow \boxed{f_{By} = -1}$

$m_h^A = 0 = f_{Bx} \cdot b + f_{By} \cdot a = 0$

$$\boxed{\begin{aligned} f_{Bx} &= \frac{a}{b} \\ f_A &= -\frac{a}{b} \end{aligned}}$$

$m_{h1} = -1 \cdot x$

$m_{h2} = -1 \cdot a - f_A \cdot y = -a + \frac{a}{b} y$

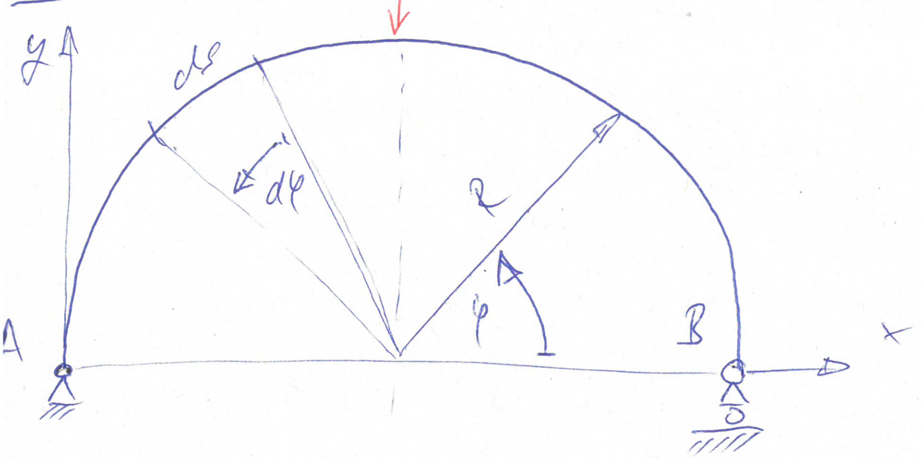
$$v_A = \int_{(l)} \frac{M_H}{IE} m_h \, ds = \int_0^a \frac{M_{H1}}{IE} m_{h1} \, dx + \int_0^b \frac{M_{H2}}{IE} m_{h2} \, dy = \frac{1}{IE}\left\{\frac{Fa^3}{3} + \int_0^b \underline{Fa^2} - \underline{Ma} + Aay - F\frac{a^2}{b}y + M\frac{a}{b}y - A\frac{a}{b}y^2 \, dy\right\} = \frac{1}{IE}\left\{\frac{Fa^3}{3} + \left[Fa^2 y - May + Aa\frac{y^2}{2} - F\frac{a^2}{b}y^2/2 + M\frac{a}{b}\frac{y^2}{2} - A\frac{a}{b}\frac{y^3}{3}\right]_{y=0}^{y=b}\right\}$$

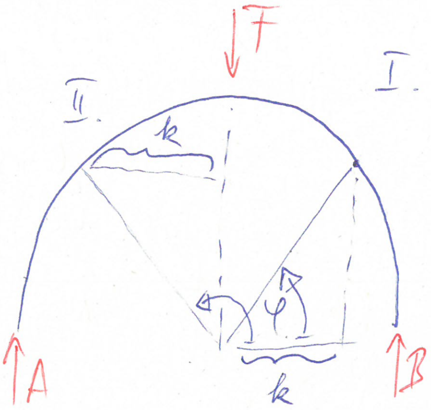
$$\frac{1}{IE}\left\{\frac{Fa^3}{3}+Fa^2b-Mab+Aa\frac{b^2}{2}-Fa^2\cdot\frac{b}{2}+Ma\frac{b}{2}-Aa\frac{b^2}{3}\right\}=$$

$$=\frac{1}{IE}\left\{\frac{Fa^3}{3}+Fa^2\frac{b}{2}-M\frac{ab}{2}+Aa\frac{b^2}{6}\right\}=0.0125\,m=\underline{\underline{12.5\,mm}}.$$

függőleges és maximális x irányú elmozdulása $\approx 0.08\,mm$

10.41





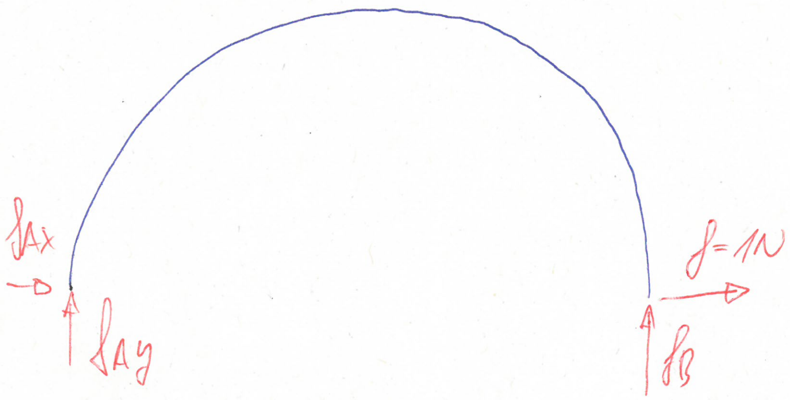
$A = B = F/2$,

F, R, IE adott $u_B = ?,\ \varphi_B = ?$

$$M_{H1} = -R(1-\cos\varphi)\frac{F}{2} \quad \varphi \in [0, \pi/2]$$

$$M_{H2} = -R(1-\cos\varphi)\frac{F}{2} + R\underbrace{\sin\left(\varphi-\frac{\pi}{2}\right)}_{-\cos\varphi}F = -\frac{RF}{2}(1+\cos\varphi) \quad \varphi \in [\pi/2, \pi]$$

u_B meghatározása



$f_{Ax} + 1 = 0 \quad \boxed{f_{Ax} = -1}$

$f_{Ay} + f_B = 0 \quad \boxed{f_{Ay} = 0}$

$m_h^A := 0 = f_B \cdot 2R \rightarrow \boxed{f_B = 0}$

$m_{h1} = -R\sin\varphi \quad \varphi \in [0, \pi/2]$

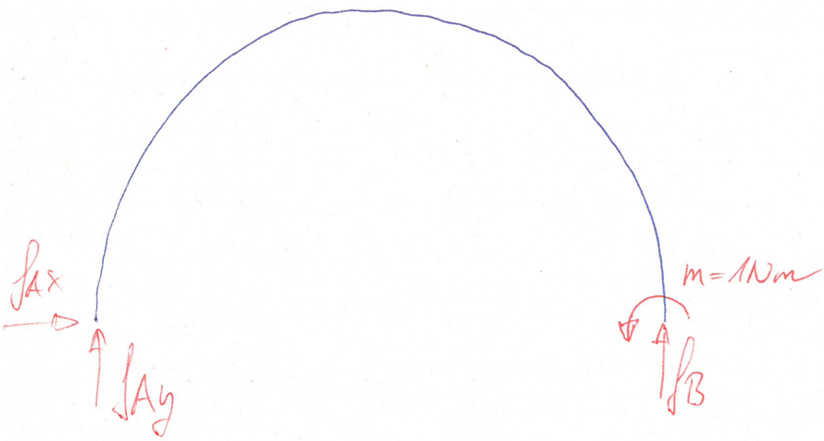
$m_{h2} = -R\sin\varphi \quad \varphi \in [\pi/2, \pi]$

$$\int_{(l)} \frac{M_H \cdot m_h}{IE}\, ds = \frac{1}{IE}\int_0^{\pi} M_H \cdot m_h \overbrace{R\, d\varphi}^{ds} = \frac{1}{IE}\left\{\int_0^{\pi/2} \frac{FR^3}{2}(\sin\varphi - \sin\varphi\cos\varphi)\, d\varphi + \right.$$

$$\left. + \frac{FR^3}{2}\int_{\pi/2}^{\pi} \sin\varphi + \sin\varphi\cos\varphi\, d\varphi\right\} = \frac{FR^3}{2IE}\left\{\left[-\cos\varphi - \frac{1}{2}\sin^2\varphi\right]_{\varphi=0}^{\varphi=\pi/2} + \left[-\cos\varphi + \frac{1}{2}\sin^2\varphi\right]_{\varphi=\pi/2}^{\varphi=\pi}\right\} =$$

$$\frac{FR^3}{2IE}\left\{\left(0-\frac{1}{2}\right) - (-1-0) + (1+0) - \left(-0+\frac{1}{2}\right)\right\} = \frac{FR^3}{2IE}$$

φ_B meghatározása



$\cancel{f_{Ay} + f}$

$f_{Ay} + f_B = 0 \qquad \boxed{f_{Ay} = \frac{1}{2R}}$

$\boxed{f_{Ax} = 0}$

$m_h^A = 0 = f_B \cdot 2R + 1 = 0 \to \boxed{f_B = -\frac{1}{2R}}$

$m_{h1} = -1 - f_B(1-\cos\varphi)\cdot R = -1 + \frac{1}{2}(1-\cos\varphi) \quad \varphi \in [0, \pi/2]$

$m_{h2} = -1 + \frac{1}{2}(1-\cos\varphi) \quad \cancel{\varphi \in [\pi/2, \pi]} \quad \varphi \in [\pi/2, \pi]$

$\varphi_B = \frac{1}{IE}\int_0^{\pi/2} M_{H1}\, m_{h1}\, R\, d\varphi + \frac{1}{IE}\int_{\pi/2}^{\pi} M_{H2}\, m_{h2}\, R\, d\varphi = \frac{R^2 F}{4IE}(\pi - 2)$