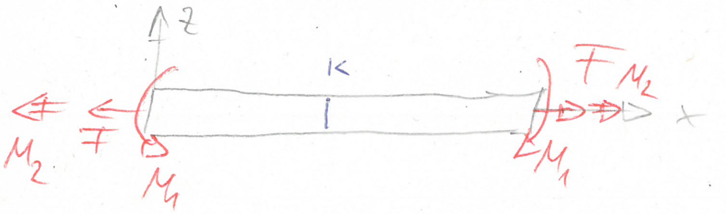
9.1.1 $l = 1\,m$, $d = 10\,mm$, $F = 1\,kN$ húzó, M_1 hajlító, M_2 csavaró

M_1/F, M_2/F ? azonos alakváltozási energia, teljes alakváltozási E,

$\underline{\sigma} = ?$, $\sigma_g = ?$, $\underline{\sigma}_d = ?$ veszélyes pontban

$\varepsilon_{dx} = ?$ $E = 200\,GPa$, $\nu = 0.3$



$$U_N = \frac{1}{2}\int_{(l)} \frac{F^2}{AE}\,dx,\quad A = \frac{d^2\pi}{4};\quad U_H = \frac{1}{2}\int_{(l)} \frac{M_1^2}{I_y E}\,dx,\quad I_y = \frac{d^4\pi}{64};\quad U_T = \frac{1}{2}\int_{(l)} \frac{M_2^2}{I_p G};\quad I_p = \frac{d^4\pi}{32}$$

$$U_N = \frac{1}{2}\frac{F^2 l}{AE};\quad U_H = \frac{1}{2}\frac{M_1^2 l}{I_y E};\quad G = \frac{E}{2(1+\nu)},\quad U_T = \frac{1}{2}\frac{M_2^2 l}{I_p \frac{E}{2(1+\nu)}}$$

$$U_N = U_H = U_T \rightarrow U_N = U_H \rightarrow \frac{I_y}{A} = \frac{M_1^2}{F^2} \rightarrow \underline{\frac{M_1}{F} = \frac{d}{4}} \rightarrow M_1 = 2.5\,Nm$$

$$U_N = U_T \rightarrow \frac{I_p}{A\cdot 2(1+\nu)} = \frac{M_2^2}{F^2} \rightarrow \underline{\frac{M_2}{F} = \frac{d}{4\sqrt{(1+\nu)}}} \rightarrow M_2 = 2.19\,Nm$$

$$U = U_N + U_H + U_T = 3U_N = \underline{0.095\,J}$$

Igénybevételek konstansok → tetszőleges keresztmetszetet tekinthetünk

húzás - hajlítás → σ_x; csavarás → τ_{xt}

$$\sigma_x = \frac{F}{A} + \frac{M_1}{I_y} z \qquad \sigma_x^P = \frac{F}{A} + \frac{M_1}{I_y}\frac{d}{2} = 38.19\,MPa$$

$$\tau_{xt} = \frac{M_2}{I_p}\cdot r \qquad \tau_{xt}^P = \frac{M_2}{I_p}\frac{d}{2} = 11.17\,MPa$$

$$\underline{\sigma}_{(x,r,t)} = \begin{bmatrix} \sigma_x^P & 0 & \tau_{xt}^P \\ 0 & 0 & 0 \\ \tau_{xt}^P & 0 & 0 \end{bmatrix}$$

$$\underline{\underline{\sigma}}_g = \frac{1}{3}\sigma_I \underline{\underline{E}} \quad , \quad \sigma_I = \sigma_x + \sigma_r + \sigma_t = 38.20\,\text{MPa} \quad , \quad \underline{\underline{\sigma}}_g = 12.73\,\text{MPa} \cdot \underline{\underline{E}}$$

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}_g + \underline{\underline{\sigma}}_d \rightarrow \underline{\underline{\sigma}}_d = \underline{\underline{\sigma}} - \frac{1}{3}\sigma_I \underline{\underline{E}} = \begin{bmatrix} 25.46 & 0 & 11.17 \\ 0 & -12.73 & 0 \\ 11.17 & 0 & -12.73 \end{bmatrix} [\text{MPa}]$$

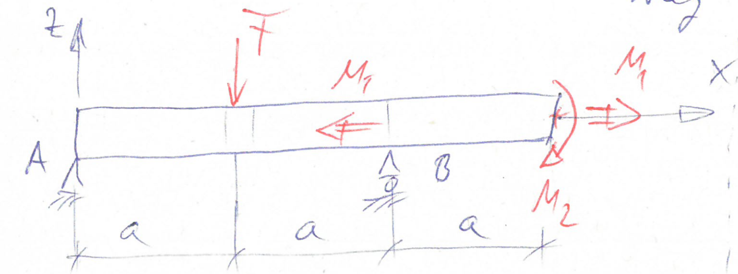
$$\underline{\underline{\varepsilon}} = \frac{1+\nu}{E}\left(\underline{\underline{\sigma}} - \frac{\nu}{1+\nu}\underline{\underline{E}}\,\sigma_I\right) \rightarrow \varepsilon_I \rightarrow \underline{\underline{\varepsilon}}_d = \underline{\underline{\varepsilon}} - \frac{1}{3}\varepsilon_I \underline{\underline{E}}$$

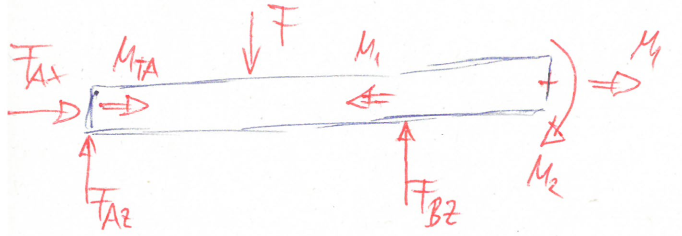
$$\varepsilon_I = \frac{1-2\nu}{E}\sigma_I = 76.39 \cdot 10^{-6}$$

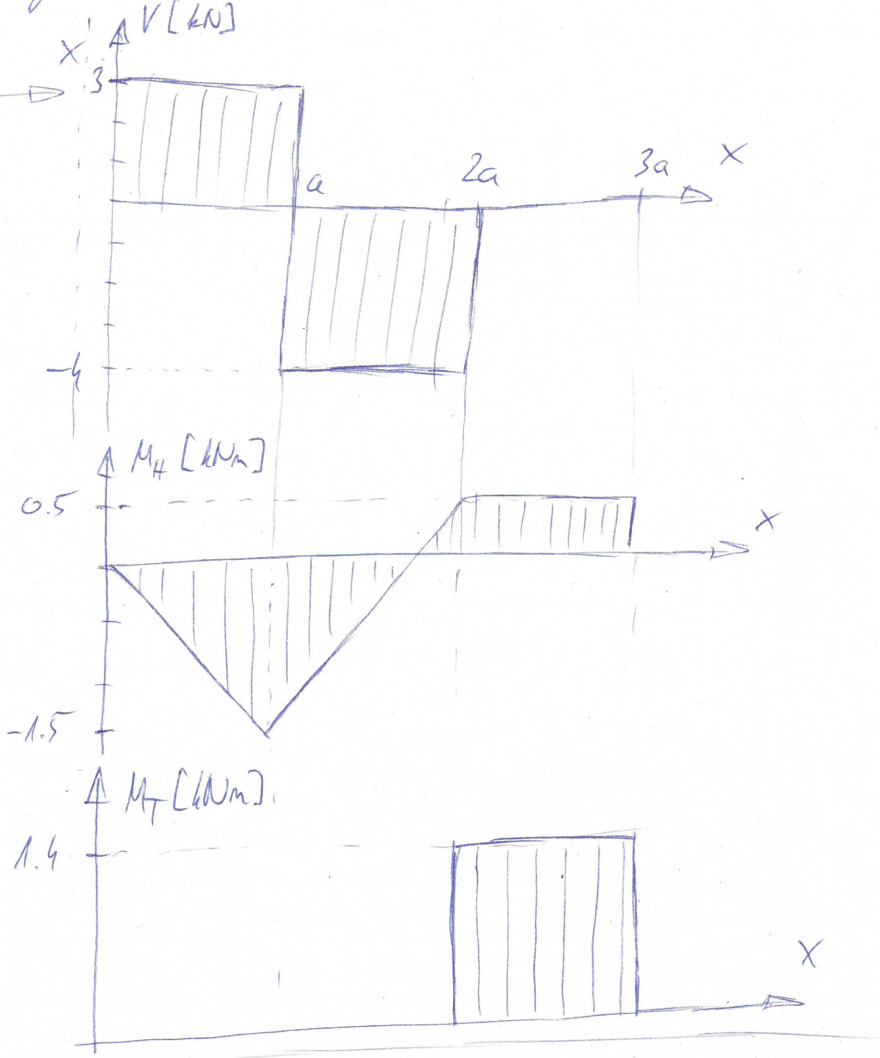
$$\underline{\underline{\varepsilon}}_d = \frac{1+\nu}{E}\left(\underline{\underline{\sigma}}_d - \frac{\nu}{1+\nu}\underbrace{\underline{\underline{E}}_d \sigma_I}_{=\underline{\underline{0}}}\right) = \begin{bmatrix} 165.5 & 0 & 72.58 \\ 0 & -82.76 & 0 \\ 72.51 & 0 & -82.76 \end{bmatrix} \cdot 10^{-6}$$

5.2, méretezés HMH, kör km $a = 0.5\ m$, $F = 7\ kN$, $M_1 = 1.4\ kNm$, $M_2 = 0.5\ kNm$

$\sigma_{meg} = 140\ MPa$







$x: F_{Ax} = 0$

$z: F_{Az} - F + F_{Bz} = 0$

$y: M_A = -F \cdot a + F_{Bz} \cdot 2a - M_2 = 0$

$x\ M_T = M_{TA} - M_1 + M_1 = 0$

$F_{Ax} = 0 \quad N(x) = 0$

$F_{Az} = \left(\frac{F \cdot a - M_2}{2a}\right) = 3000 N$

$F_{Bz} = \ldots = 4000 N$

$M_{TA} = 0$

$x = a$, $x = 2a$ lehetséges veszélyes km.

→ $x=a$ vizsgálata → méretezés, $x=2a$ ellenőrzés

$x=a$ - ban egytengelyű feszültségállapot

$$\sigma_x^a = \frac{M_H(a)}{\underbrace{\frac{d^4\pi}{64}}_{I_y}} \cdot z \rightarrow \sigma_x^{a\,MAX} = \frac{M_H(a)}{\frac{d^4\pi}{64}} \cdot \frac{d}{2}$$

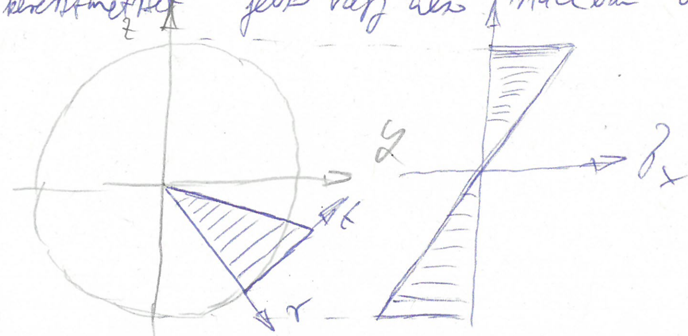
$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_x^{a\,MAX} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \underline{\underline{\sigma}}_d = \underline{\underline{\sigma}} - \frac{1}{3}\sigma_x^{a\,MAX} \cdot \underline{\underline{E}} = \begin{bmatrix} 2/3\,\sigma_x^{a\,MAX} & 0 & 0 \\ 0 & -1/3\,\sigma_x^{a\,MAX} & 0 \\ 0 & 0 & -1/3\,\sigma_x^{a\,MAX} \end{bmatrix}$$

$$\sigma_{egy}^{HMH} = \sqrt{\frac{3}{2}\underline{\underline{\sigma}}_d : \underline{\underline{\sigma}}_d} = \sigma_x^{a\,MAX}\sqrt{1} \le \sigma_{meg}$$

$d \ge 47.8\ mm \rightarrow \underline{\underline{d' = 48\ mm}}$

$x \in [2a, 3a]$ ellenőrzés

kör keresztmetszet felső vagy alsó szálában veszélyes pont



$$\sigma_x^{2a} = \frac{M_H(2a)}{I_y} \cdot z \rightarrow \sigma_x^{2a\,MAX} = \frac{M_H(2a)}{I_y}\frac{d'}{2}$$

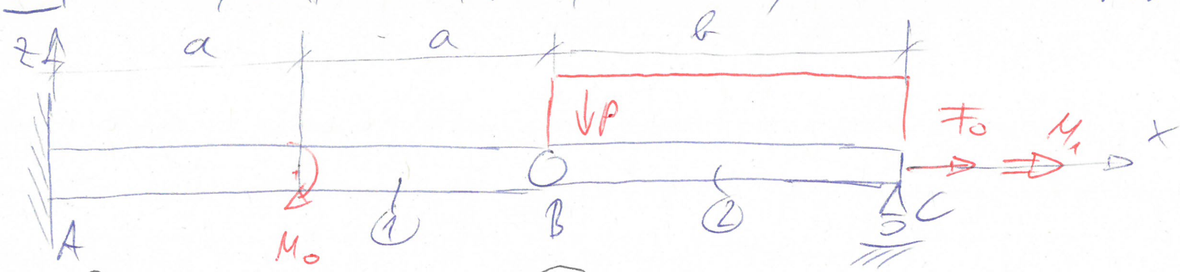
$$\tau_{xt}^{2a\,MAX} = \frac{M_T(2a)}{I_p}\frac{d'}{2}$$

$$\underset{=}{\sigma}^{2a}_{(x,r,t)} = \begin{bmatrix} \sigma_x^{2a\,MAX} & 0 & \tau_{xt}^{2a\,MAX} \\ 0 & 0 & 0 \\ \tau_{xt}^{2a\,MAX} & 0 & 0 \end{bmatrix} \Rightarrow \underset{=}{\sigma}_d^{2a} = \underset{=}{\sigma}^{2a} - \frac{1}{3}\sigma_I^{2a}\,\underset{=}{E}$$

$$\sigma_I^{2a} = \sigma_x^{2a\,MAX}$$

$$\sigma_{egg}^{HMH} = \sqrt{\frac{3}{2}\,\underset{=}{\sigma}_d^{2a} : \underset{=}{\sigma}_d^{2a}} = 120.8\ MPa < \sigma_{meg} \checkmark$$

5.3, Mohr, HMH méretezés, kör km, ellenőrzés N figyelembevételével



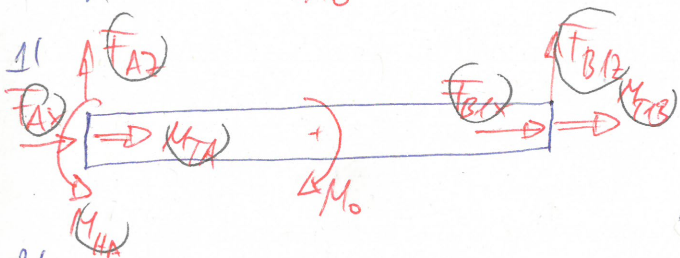
$a = 1m$, $b = 2m$

$F_0 = 2kN$, $p = 1.5kN/m$

$M_0 = 2kNm$, $M_1 = 2kNm$

$\sigma_{meg} = 100 MPa$

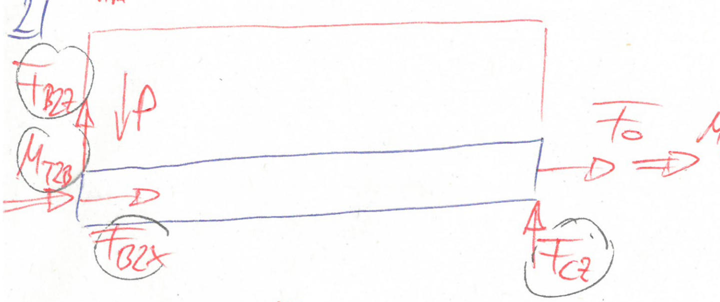
1/



$F_{AX} + F_{B1X} = 0$, $F_{AZ} + F_{B1Z} = 0$, $M_{TA} + M_{T1B} = 0$

$M_A := M_{HA} - M_0 + 2a F_{B1Z} = 0$

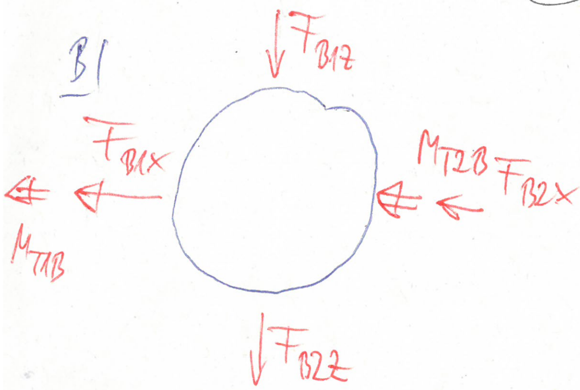
2/



$F_{B2X} + F_0 = 0$, $F_{B2Z} - p \cdot b + F_{CZ} = 0$, $M_1 + M_{T2B} = 0$

$M_B := -p \cdot b \cdot \frac{b}{2} + F_{CZ} \cdot b = 0$

B/



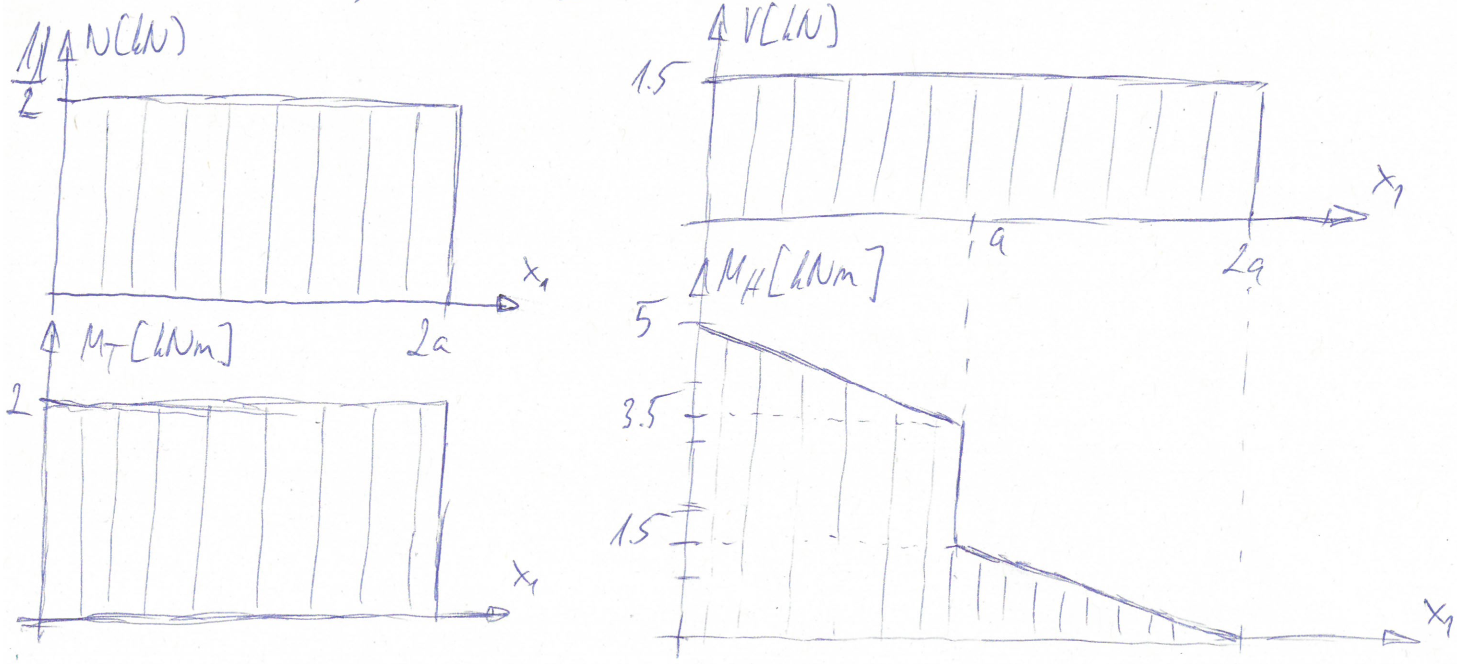
$-F_{B1X} - F_{B2X} = 0$, $-F_{B1Z} - F_{B2Z} = 0$, $-M_{T1B} - M_{T2B} = 0$

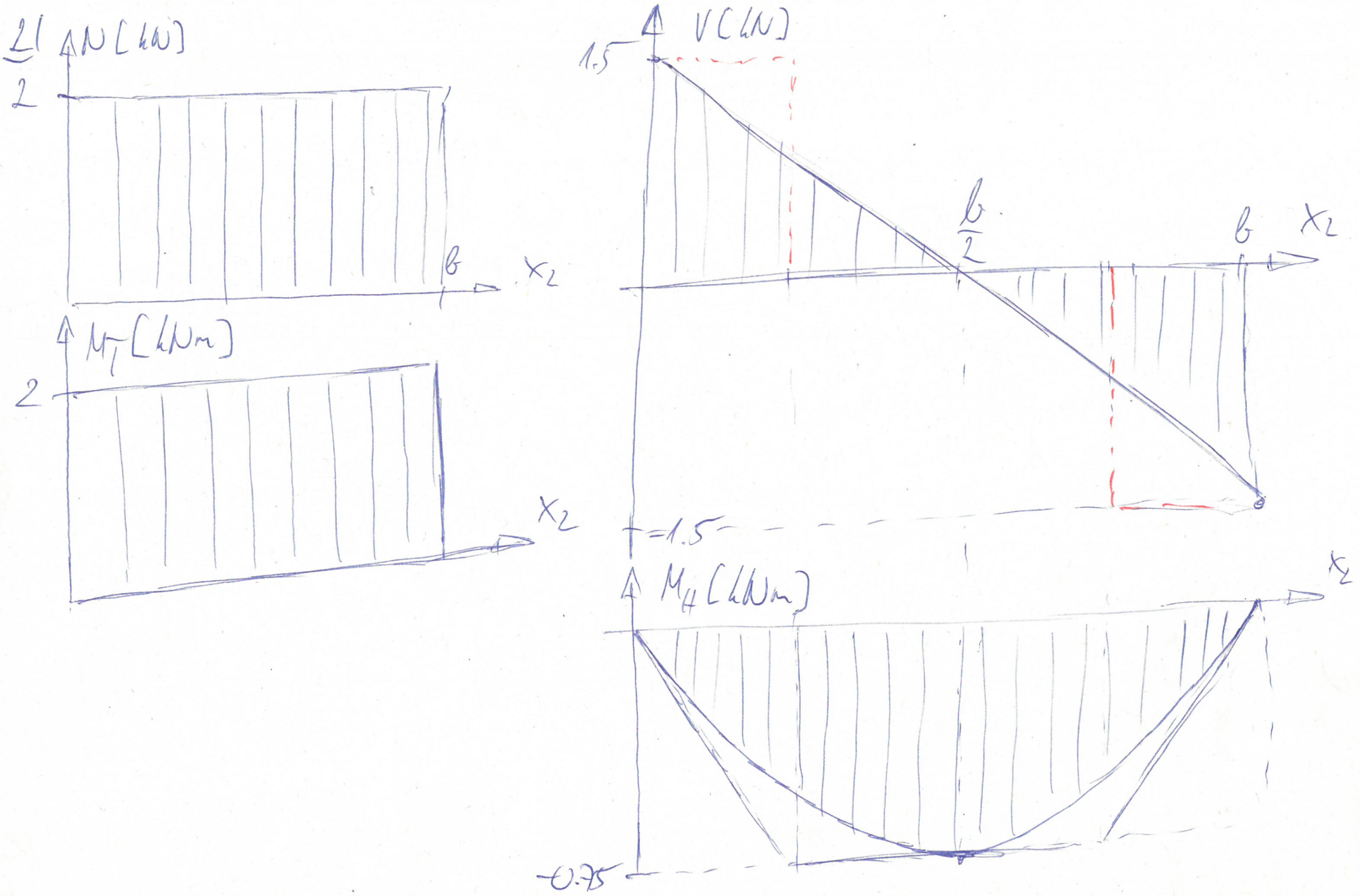
11 ismeretlen, 4+4+3 egyenlet

$F_{AX} = -F_0 = -2kN$, $F_{AZ} = \frac{pb}{2} = 1.5kN$, $M_{HA} = M_0 + p \cdot a \cdot b = 5 kNm$, $M_{TA} = -M_1 = -2kNm$

$F_{B1Z} = -\frac{bp}{2} = -1.5kN$, $F_{B1X} = F_0 = 2kN$, $M_{T1B} = M_1 = 2kNm$, $F_{B2Z} = \frac{pb}{2} = 1.5kN$, $F_{B2X} = -F_0 = -2kN$

$M_{T2B} = -M_1 = -2kNm$, $F_{CZ} = p\frac{b}{2} = 1.5kN$





'A' ~~pont~~ a veszélyes keresztmetszet $M_H^A = 5\,\text{kNm}$, $M_T^A = 2\,\text{kNm}$, $N^A = 2\,\text{kN}$

méretezés hajlítás + csavarás

$$\sigma_x^{MAX} = \frac{M_H^A}{\frac{d^4\pi}{64}}\frac{d}{2}, \quad \tau_{xt}^{MAX} = \frac{M_T^A}{\frac{d^4\pi}{32}}\frac{d}{2} \qquad \underset{(x,z,t)}{\underline{\underline{\sigma}}} = \begin{bmatrix} \sigma_x^{MAX} & 0 & \tau_{xt}^{MAX} \\ 0 & 0 & 0 \\ \tau_{xt}^{MAX} & 0 & 0 \end{bmatrix}$$

Mohr-elmélet

$\sigma_{egy}^M = \sigma_1 - \sigma_3$ $\sigma_1, \sigma_2, \sigma_3$ Mohr körök vagy sé-ér probléma

$$(\underline{\underline{\sigma}} - \lambda \underline{\underline{E}})\underline{e} = \underline{0} \rightarrow \det(\underline{\underline{\sigma}} - \lambda \underline{\underline{E}}) = 0 \quad \sigma_1 = \frac{16}{d^3\pi}\left(M_H^A + \sqrt{(M_H^A)^2 + (M_T^A)^2}\right)$$

$$\sigma_2 = 0, \quad \sigma_3 = \frac{16}{d^3\pi}\left(M_H^A - \sqrt{(M_H^A)^2 + (M_T^A)^2}\right)$$

$$\sigma_{egy}^M = \sigma_1 - \sigma_3 = \frac{32}{d^3\pi}\sqrt{(M_H^A)^2 + (M_T^A)^2} \leq \sigma_{meg} \rightarrow d^M \geq 81.86\,\text{mm}$$

HMH elmélet

$$\underline{\underline{\sigma}}_d = \underline{\underline{\sigma}} - \frac{1}{3}\underbrace{\sigma_I}_{\sigma_x^{MAX}}\underline{\underline{E}} \rightarrow \sigma_{egy}^{HMH} = \sqrt{\frac{3}{2}\underline{\underline{\sigma}}_d : \underline{\underline{\sigma}}_d} = \frac{32}{d^3\pi}\sqrt{(M_H^A)^2 + \frac{3}{4}(M_T^A)^2} \leq \sigma_{meg}$$

$$d^{HMH} \geq 81.38\,\text{mm}$$

$d = 82\,mm$

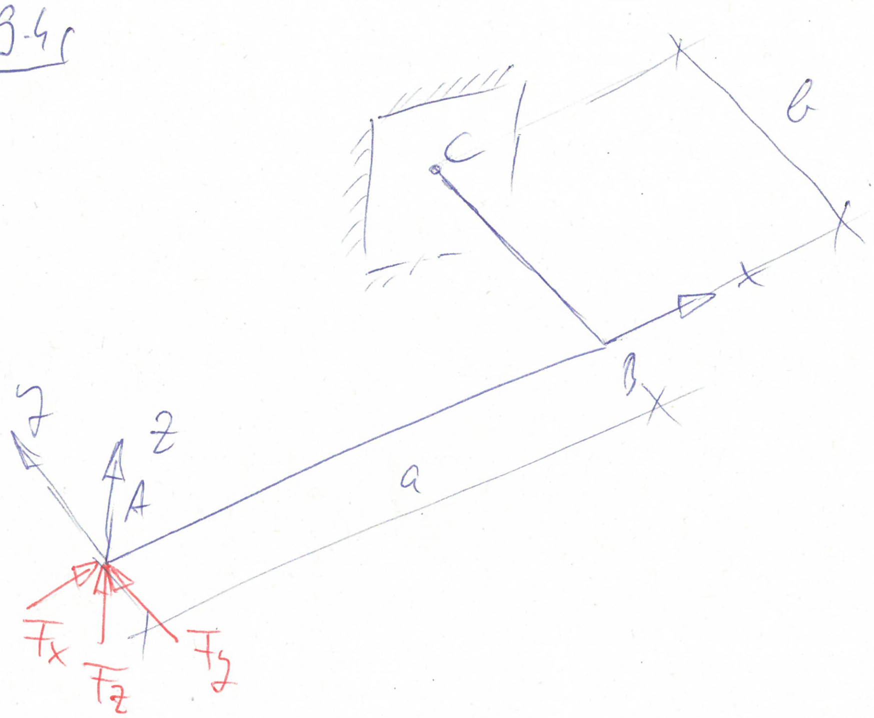
ellenőrzés

$$\sigma_x^{MAX} = \frac{N^A}{\frac{d^2\pi}{4}} + \frac{M_H^A}{d^4\pi/64}\frac{d}{2}, \quad \tau_{xt}^{MAX} = \frac{M_T^A}{d^4\pi/32}\frac{d}{2}$$

$$\underline{\underline{\sigma}} = \dots \quad 96.29 \quad -0.354$$

$$\sigma_{egy}^{M} = \sigma_1 - \sigma_3 = 99.84\,MPa < \sigma_{meg} \quad \sigma_{egy}^{HMH} = \sqrt{\frac{3}{2}\underline{\underline{\sigma}}_d : \underline{\underline{\sigma}}_d} = 98.\,MPa < \sigma_{meg}$$

9.4,



körkeresztmetszet, Mohr, HMH ellenőrzés, C km-ben

$a = 1m, \; b = 0.5\,m, \; d = 50\,mm$

$F_x = -100\,N, \; F_y = 500\,N$

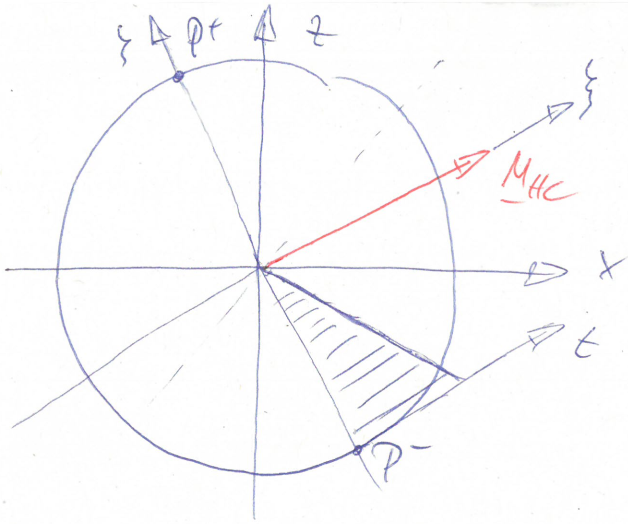
$F_z = 1000\,N \quad \sigma_{meg} = 100\,MPa$

$$\underline{F} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}, \quad r_{AC} = \begin{bmatrix} a \\ b \\ 0 \end{bmatrix}$$

$$\underline{M}_C = \underbrace{\underline{M}_A}_{= \underline{0}} + r_{AC} \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a & b & 0 \\ F_x & F_y & F_z \end{vmatrix} = \begin{bmatrix} F_z b \\ -F_z a \\ F_y a + F_x b \end{bmatrix} \quad \underline{M}_C = \underline{M}_{HC} + \underline{M}_{TC}$$

$$M_{TC} = \underline{j} \cdot \underline{M}_C = -F_z \cdot a = -1000\,Nm \quad \underline{M}_{HC} = \underline{M}_C - \underline{j} \cdot M_{TC} = \begin{bmatrix} F_z b \\ 0 \\ F_y a + F_x b \end{bmatrix} = \begin{bmatrix} 500 \\ 0 \\ 450 \end{bmatrix} Nm$$

$$N_C = -F_y = -500\,N$$



$$M_{HC}=\sqrt{(F_z b)^2+(F_y a+F_x b)^2}=743.3\,Nm$$

$$\sigma_y^{*}=\frac{M_{HC}}{I_\xi}\cdot\xi+\frac{N_C}{A}$$

negatív

$$|\sigma_y^-|>|\sigma_y^+|$$

p^- veszélyes pont

$$\sigma_y^-=\frac{N}{A}+\frac{M_{HC}}{d^4\pi/64}\cdot\left(-\frac{d}{2}\right)=-55.07\,MPa$$

$$\tau_{yt}^-=\frac{M_{TC}}{d^4\pi/32}\left(\frac{d}{2}\right)=-10.74\,MPa$$

$$\underline{\underline{\sigma}}_{(y,r,t)}=\begin{bmatrix}\sigma_y^- & 0 & \tau_{yt}^- \\ 0 & 0 & 0 \\ \tau_{yt}^- & 0 & 0\end{bmatrix}$$

HMH:

$$\underline{\underline{\sigma}}_d=\underline{\underline{\sigma}}-\frac{1}{3}\underbrace{\sigma_I}_{=\sigma_y^-}\underline{\underline{E}}\rightarrow\sigma_{egy}^{HMH}=\sqrt{\frac{3}{2}\,\underline{\underline{\sigma}}_d:\underline{\underline{\sigma}}_d}=93.17\,MPa<\sigma_{meg}\ \checkmark$$

Mohr:

$$\underline{\underline{\sigma}}\rightarrow\sigma_1,\sigma_2,\sigma_3\quad\sigma_1=20.43\,MPa,\ \sigma_2=0,\ \sigma_3=-81.25\,MPa$$

$$\sigma_{egy}^{M}=\sigma_1-\sigma_3=101.7\,MPa>\sigma_{meg}\ \times$$