8.1, rugalmas test $\quad \underline{\underline{\sigma}} = \frac{E}{1+\nu}\left[\underline{\underline{\varepsilon}} + \frac{\nu}{1-2\nu}\varepsilon_I \underline{\underline{E}}\right]; \quad \underline{\underline{\varepsilon}} = \frac{1+\nu}{E}\left[\underline{\underline{\sigma}} - \frac{\nu}{1+\nu}\sigma_I \underline{\underline{E}}\right]$

$G = \frac{E}{2(1+\nu)}$

$\sigma_y = 100\,MPa$, $\sigma_z = -100\,MPa$; $\tau_{xy} = 120\,MPa$; $\gamma_{yz} = 10^{-4}\cdot 4$, $\gamma_{xz} = 8\cdot 10^{-4}\cdot(-1)$, $\varepsilon_z = 2\cdot 10^{-}$

$\underline{\underline{\sigma}}$, $\underline{\underline{\varepsilon}}$ teljes meghatározása $\quad E = 234\,GPa$, $\nu = 0.3$

$$\underset{(x,y,z)}{\underline{\underline{\sigma}}} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}; \quad \underset{(x,y,z)}{\underline{\underline{\varepsilon}}} = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix}$$

1, $\varepsilon_z = \frac{1+\nu}{E}\left(\sigma_z - \frac{\nu}{1+\nu}\left(\sigma_x + \underbrace{\sigma_y + \sigma_z}_{\oplus\; =0}\right)\right) \rightarrow \sigma_z = \underline{\underline{-589.3\,MPa}}$

2, $\varepsilon_y = \frac{1+\nu}{E}\left(\sigma_y - \frac{\nu}{1+\nu}\sigma_I\right) = \underline{\underline{1.31\cdot 10^{-3}}}$

3, $\varepsilon_x = \frac{1+\nu}{E}\left(\sigma_x - \frac{\nu}{1+\nu}\sigma_I\right) = \underline{\underline{-2.52\cdot 10^{-3}}}$

4, $\tau_{xz} = \frac{E}{1+\nu}\left(\frac{1}{2}\gamma_{xz} - \frac{\nu}{1-2\nu}\varepsilon_I\cdot 0\right) = \underline{\underline{36\,MPa}}$

5, $\tau_{yz} = \frac{E}{1+\nu}\left(\frac{1}{2}\gamma_{yz} - \frac{\nu}{1-2\nu}\varepsilon_I\cdot 0\right) = \underline{-72\,MPa}$

6, $\frac{1}{2}\gamma_{xy} = \frac{1+\nu}{E}\left(\tau_{xy} - \frac{\nu}{1+\nu}\sigma_I\cdot 0\right) = \underline{\underline{6.67\cdot 10^{-4}}}$

8.2, rugalmas test terheletlen felületi pontja, z normálisú sík

$\varepsilon_x = 350 \cdot 10^{-6}$, $\varepsilon_y = 50 \cdot 10^{-6}$; $\varepsilon_1 = 420 \cdot 10^{-6} \neq \varepsilon_z$ $\rightarrow$ $\sigma_z = 0$

$\gamma_{xy} = ?$, $\varepsilon_2 = ?$, $\varepsilon_3 = ?$

$$\underset{(x,y,z)}{\underline{\underline{\varepsilon}}} = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & 0 \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

$$(\underline{\underline{\varepsilon}} - \lambda \underline{\underline{E}})\,\underline{e} = \underline{0}$$

$$\det(\underline{\underline{\varepsilon}} - \lambda \underline{\underline{E}}) = 0 \rightarrow (\varepsilon_z - \lambda)\left[(\varepsilon_x - \lambda)(\varepsilon_y - \lambda) - \frac{1}{4}(\gamma_{xy})^2\right] = 0$$, mert $\lambda = \varepsilon_1$

mivel $\varepsilon_z - \varepsilon_1 \neq 0$ osztható vele

$$(\varepsilon_x - \varepsilon_1)(\varepsilon_y - \varepsilon_1) - \frac{1}{4}(\gamma_{xy})^2 = 0 \rightarrow \gamma_{xy} = 322 \cdot 10^{-6} \quad (((\text{vagy } \gamma_{xy} = -322 \cdot 10^{-6})))?$$

$$\sigma_z = 0 = \frac{E}{1+\nu}\left(\varepsilon_z + \frac{\nu}{1-2\nu}\underbrace{(\varepsilon_x + \varepsilon_y + \varepsilon_z)}_{\varepsilon_I} \cdot 1\right) \rightarrow \varepsilon_z = -171.4 \cdot 10^{-6}$$

ε_2 és ε_3 meghatározása

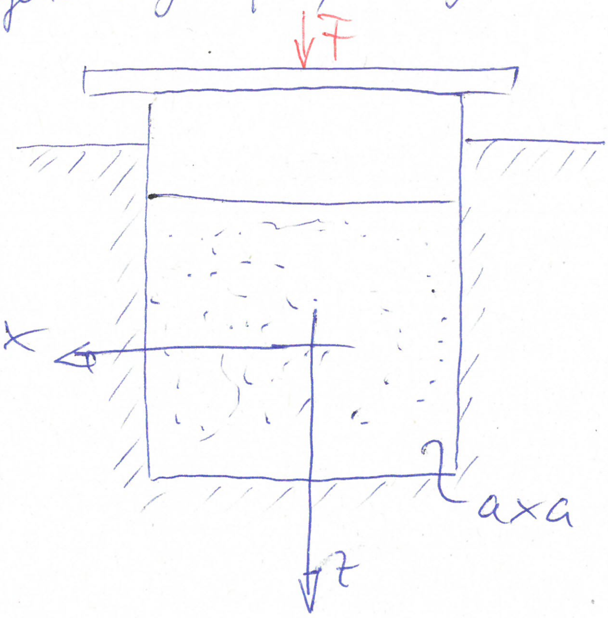
$$\det(\underline{\underline{\varepsilon}} - \lambda \underline{\underline{E}}) = (\varepsilon_z - \lambda)\left[\lambda^2 - \lambda(\varepsilon_x + \varepsilon_y) + \varepsilon_x\varepsilon_y - \frac{1}{4}(\gamma_{xy})^2\right] = 0$$, $\lambda_1 = \varepsilon_z = -171.4 \cdot 10^{-6}$

$$\lambda_{2,3} = \frac{\varepsilon_x + \varepsilon_y \pm \sqrt{(\varepsilon_x + \varepsilon_y)^2 - 4\varepsilon_x\varepsilon_y + (\gamma_{xy})^2}}{2} = \begin{cases} \lambda_2 = 420 \cdot 10^{-6} \\ \lambda_3 = -20 \cdot 10^{-6} \end{cases}$$

$\varepsilon_1 = 420 \cdot 10^{-6}$, $\varepsilon_2 = -20 \cdot 10^{-6}$, $\varepsilon_3 = \varepsilon_z = -171.4 \cdot 10^{-6}$

Poisson tényező $(-\infty, 0.5)$, negative Poisson ratio, auxetic material structure

8.31 axa állandó keresztmetszet, rugalmas hasáb, merev fal, feszültségállapot, zsugorodása a h magasságú hasábnak



$a = 200\,mm$, $F = 160\,kN$, $E = 50\,GPa$, $\nu = 0.35$, $h = 1\,m$

merev oldalsó megtámasztás

$$\underline{\underline{\varepsilon}}_{x,y,z} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

$$\underline{\underline{\sigma}}_{(x,y,z)} = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} \qquad \sigma_z = -\frac{F}{a \cdot a} = -4\,MPa$$

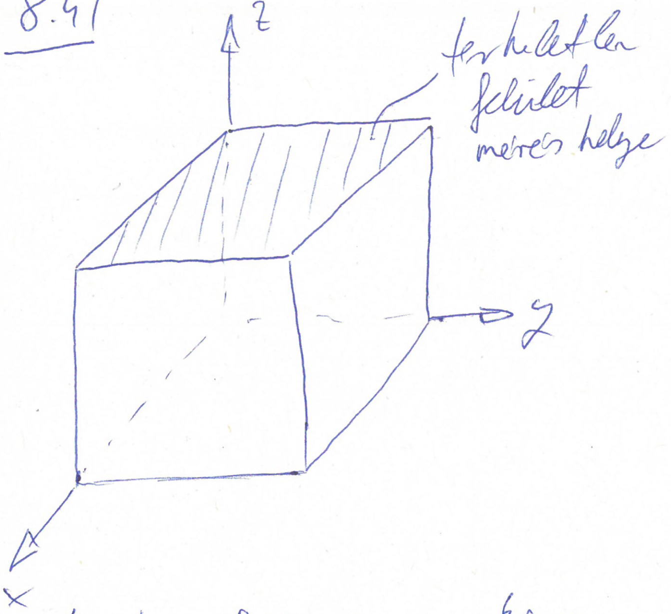
$$\sigma_z = \frac{E}{1+\nu}\left(\varepsilon_z + \frac{\nu}{1-2\nu} \overbrace{\varepsilon_z}^{\varepsilon_I}\right) \to \varepsilon_z = -4.985 \cdot 10^{-5} \to \Delta h = \varepsilon_z \cdot h = -4.98 \cdot 10^{-2}\,mm$$

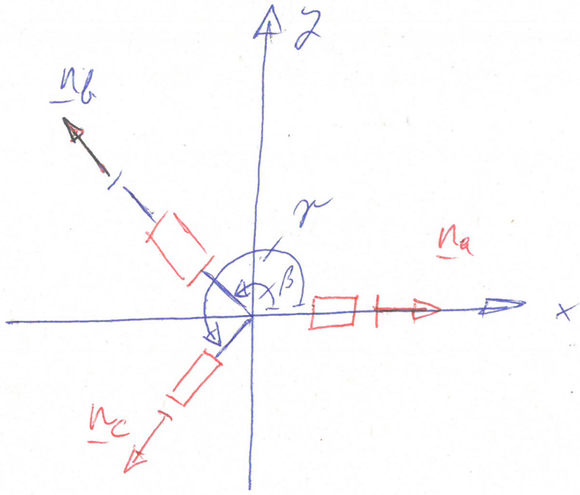
$$\sigma_x = \sigma_y = \frac{E}{1+\nu}\left(0 + \frac{\nu}{1-2\nu} \underbrace{\varepsilon_z}_{\varepsilon_I}\right) = -2.154\,MPa$$

Amennyiben az oldalak nincsenek mereven megtámasztva ≡ szabad perem

$$\sigma_z' = \sigma_z, \quad \sigma_x' = \sigma_y' = 0, \quad \varepsilon_z' = \frac{F}{AE} = -8 \cdot 10^{-5}$$

8.41





#	α [°]	$\varepsilon \cdot 10^6$ [1]
a	0	130
b	135	−90
c	225	260

$\sigma_1, \sigma_2, \sigma_3, \varepsilon_1, \varepsilon_2, \varepsilon_3, \underline{n}_1$

$\sigma_I, \sigma_{II}, \sigma_{III}$, $E = 200$ GPa, $\nu = 0.3$

$\sigma_z = 0$ terheletlen felület, $\tau_{zx} = \tau_{zy} = 0$

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & 0 \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}, \quad \underline{\underline{\sigma}} = \begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\varepsilon_a = \underline{n}_a^T \cdot \underline{\underline{\varepsilon}} \cdot \underline{n}_a \; ; \; \underline{n}_a = [1, 0, 0]^T \qquad \varepsilon_a = \underline{\underline{\varepsilon_x}} = \underline{130 \cdot 10^{-6}}$$

$$\varepsilon_b = \underline{n}_b^T \cdot \underline{\underline{\varepsilon}} \cdot \underline{n}_b \; , \; \underline{n}_b = [\cos\beta, \sin\beta, 0]^T = \frac{1}{\sqrt{2}}[-1, 1, 0]^T, \quad \boxed{\varepsilon_b = \frac{1}{2}(\varepsilon_x + \varepsilon_y - \gamma_{xy})}$$

$$\varepsilon_c = \underline{n}_c^T \cdot \underline{\underline{\varepsilon}} \cdot \underline{n}_c \; ; \; \underline{n}_c = [\cos\gamma, \sin\gamma, 0]^T = \frac{1}{\sqrt{2}}[-1, -1, 0]^T, \quad \boxed{\varepsilon_c = \frac{1}{2}(\varepsilon_x + \varepsilon_y + \gamma_{xy})}$$

$$\underline{\varepsilon_y} = \varepsilon_b + \varepsilon_c - \varepsilon_x = \underline{40 \cdot 10^{-6}} \qquad \underline{\gamma_{xy}} = \varepsilon_c - \varepsilon_b = \underline{350 \cdot 10^{-6}}$$

$$\sigma_z = \frac{E}{1+\nu}\Big(\varepsilon_z + \frac{\nu}{1-2\nu}\underbrace{(\varepsilon_x + \varepsilon_y + \varepsilon_z)}_{\varepsilon_I} \cdot 1\Big) = 0 \rightarrow \underline{\varepsilon_z} = -\underline{72.86 \cdot 10^{-6}}$$

$$\underset{(x,y,z)}{\underline{\underline{\varepsilon}}} = \begin{bmatrix} 130 & 350/2 & 0 \\ 350/2 & 40 & 0 \\ 0 & 0 & -72.86 \end{bmatrix} \cdot 10^{-6} \to \underline{\underline{\sigma}} = \frac{E}{1+\nu}\left(\underline{\underline{\varepsilon}} + \frac{\nu}{1-2\nu}\,\varepsilon_I\,\underline{\underline{E}}\right) =$$

$$\underline{\underline{\sigma}} = \begin{bmatrix} 31.21 & 26.9 & 0 \\ 26.9 & 17.3 & 0 \\ 0 & 0 & 0 \end{bmatrix} [MPa]$$

főnyúlások meghatározása sajátérték-sajátvektor probléma

$$(\underline{\underline{\varepsilon}} - \lambda \underline{\underline{E}})\underline{e} = \underline{0} \to \det(\underline{\underline{\varepsilon}} - \lambda \underline{\underline{E}}) = 0 = (-72.86 - \lambda)\left[(130-\lambda)(40-\lambda) - \left(\frac{350}{2}\right)^2\right]$$

$$\lambda_1 = -72.86, \quad \lambda_{2,3} = \frac{170 \pm \sqrt{170^2 - 4\left(130 \cdot 40 - \left(\frac{350}{2}\right)^2\right)}}{2} = \begin{cases} 265.7 \\ -95.69 \end{cases}$$

$$\underline{\underline{\varepsilon_1 = 265.7 \cdot 10^{-6}}}, \quad \underline{\underline{\varepsilon_2 = -72.86 \cdot 10^{-6}}}, \quad \underline{\underline{\varepsilon_3 = -95.69 \cdot 10^{-6}}}$$

$$(\underline{\underline{\varepsilon}} - \varepsilon_1 \underline{\underline{E}})\underline{e}_1 = \underline{0} \to \underline{e}_1 = \begin{bmatrix} 0.79 \\ 0.613 \\ 0 \end{bmatrix}$$

$$\underset{(1,2,3)}{\underline{\underline{\varepsilon}}} = \begin{bmatrix} 265.7 & 0 & 0 \\ 0 & -72.86 & 0 \\ 0 & 0 & -95.69 \end{bmatrix} \cdot 10^{-6} \to \underset{(1,2,3)}{\underline{\underline{\sigma}}} = \frac{E}{1+\nu}\left(\underset{(1,2,3)}{\underline{\underline{\varepsilon}}} + \frac{\nu}{1-2\nu}\,\varepsilon_I\,\underline{\underline{E}}\right)$$

$$\underset{(1,2,3)}{\underline{\underline{\sigma}}} = \begin{bmatrix} \overset{\sigma_1}{52.08} & 0 & 0 \\ 0 & \overset{\sigma_2}{0} & 0 \\ 0 & 0 & \underset{\sigma_3}{-3.51} \end{bmatrix} [MPa] \qquad \begin{aligned} &\underline{\underline{\sigma_1 = 52.08\ MPa}} \\ &\underline{\underline{\sigma_2 = 0\ MPa}} \\ &\underline{\underline{\sigma_3 = -3.51\ MPa}} \end{aligned}$$

$$\underline{\underline{\varepsilon_I}} = \varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \underline{\underline{97.14 \cdot 10^{-4}}}$$

$$\underline{\underline{\varepsilon_{II}}} = (\varepsilon_y \cdot \varepsilon_z - 0^2) + (\varepsilon_x \cdot \varepsilon_z - 0^2) + \left(\varepsilon_x \cdot \varepsilon_y - \left(\tfrac{1}{2}\gamma_{xy}\right)^2\right) = \varepsilon_2 \cdot \varepsilon_3 + \varepsilon_1 \cdot \varepsilon_3 + \varepsilon_1 \cdot \varepsilon_2 = \underline{\underline{-37.1 \cdot 10^{-9}}}$$

$$\underline{\underline{\varepsilon_{III}}} = \det\left(\underset{(x,y,z)}{\underline{\underline{\varepsilon}}}\right) = \det\left(\underset{(1,2,3)}{\underline{\underline{\varepsilon}}}\right) = \varepsilon_1 \cdot \varepsilon_2 \cdot \varepsilon_3 = \underline{\underline{1.85 \cdot 10^{-12}}}$$