$$\underset{(x,y,z)}{\underline{\underline{\varepsilon}}} = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix}, \quad \underset{(x,y,z)}{\underline{\underline{\sigma}}} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

$$|\underline{n}| = 1$$

$$\underline{n}^T \cdot \underline{\underline{\varepsilon}} \cdot \underline{n} = \varepsilon_n$$

$$\underline{m}^T \cdot \underline{\underline{\varepsilon}} \cdot \underline{n} = \frac{1}{2}\gamma_{mn}$$

$$\underset{(x,y,z)}{\underline{\underline{\varepsilon}}} = \frac{1+\nu}{E}\left(\underset{(x,y,z)}{\underline{\underline{\sigma}}} - \frac{\nu}{1+\nu}\, tr(\underline{\underline{\sigma}}) \cdot \underline{\underline{E}}\right) \qquad tr(\underset{(x,y,z)}{\underline{\underline{\sigma}}}) = \sigma_x + \sigma_y + \sigma_z = \sigma_I$$

$$\underset{(x,y,z)}{\underline{\underline{\sigma}}} = \frac{E}{1+\nu}\left(\underset{(x,y,z)}{\underline{\underline{\varepsilon}}} + \frac{\nu}{1-2\nu}\, tr(\underline{\underline{\varepsilon}})\, \underline{\underline{E}}\right) \qquad \sigma_{II} = \frac{1}{2}\left((tr(\underline{\underline{\sigma}}))^2 - tr(\underline{\underline{\sigma}}^2)\right)$$ főminorok det. összege

$$\sigma_{III} = \det(\underline{\underline{\sigma}})$$

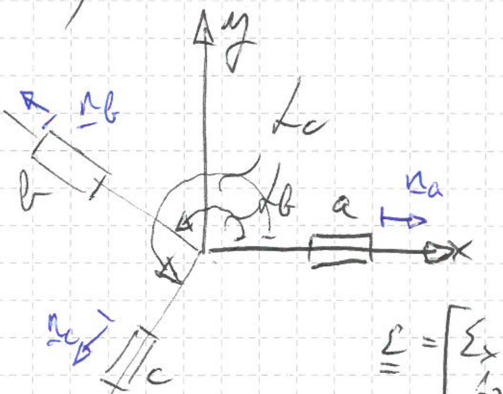
7.1.

Test felületi pontja $\alpha_a = 0°$, $\varepsilon_a = 130\,\frac{\mu m}{m}$, $\alpha_b = 135°$, $\varepsilon_b = -90\,\frac{\mu m}{m}$

$\alpha_c = 225°$, $\varepsilon_c = 260\,\mu m/m$ $\quad \varepsilon_x, \varepsilon_y, \gamma_{xy}$?, & $\varepsilon_I, \varepsilon_{II}, \varepsilon_{III}$ x,y,z-ben

$\sigma_1, \sigma_2, \sigma_3$? $\varepsilon_1, \varepsilon_2, \varepsilon_3$? $\underline{n}_1$? skalárinvariánsok krdsz függetlenek

$E = 200\,GPa$ $\quad \nu = 0.3$



$$\underline{n}_a = \begin{bmatrix} \cos\alpha_a \\ \sin\alpha_a \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \underline{n}_b = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \frac{1}{\sqrt{2}}$$

$$\underline{n}_c = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & 0 \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

$\underline{n}_a \cdot \underline{\underline{\varepsilon}} \cdot \underline{n}_a = \varepsilon_a = \boxed{\varepsilon_x = 130 \cdot 10^{-6}}$

$\underline{n}_b \cdot \underline{\underline{\varepsilon}} \cdot \underline{n}_b = \frac{1}{2}(\varepsilon_x + \varepsilon_y - \gamma_{xy}) = \varepsilon_b \quad \underline{n}_c \cdot \underline{\underline{\varepsilon}} \cdot \underline{n}_c = \frac{1}{2}(\varepsilon_x + \varepsilon_y + \gamma_{xy}) = \varepsilon_c$

$\varepsilon_y = 40 \cdot 10^{-6}, \ \gamma_{xy} = 350 \cdot 10^{-6}$

$\sigma_z = \frac{E}{1+\nu}\left(\varepsilon_z + \frac{\nu}{1-2\nu}(\varepsilon_x + \varepsilon_y + \varepsilon_z) \cdot 1\right) \rightarrow \boxed{\varepsilon_z = -72.86 \cdot 10^{-6}}$

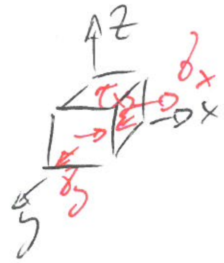
$\varepsilon_I = tr(\underline{\underline{\varepsilon}}) = \varepsilon_x + \varepsilon_y + \varepsilon_z = 97.14 \cdot 10^{-6}$

$\varepsilon_{II} = \varepsilon_y \cdot \varepsilon_z - 0 \cdot 0 + \varepsilon_x \cdot \varepsilon_z - 0 \cdot 0 + \varepsilon_x \cdot \varepsilon_y - \frac{1}{4}\gamma_{xy}^2 = -37.81 \cdot 10^{-6}$

$\varepsilon_{III} = 0 \ldots - 0 \ldots + \varepsilon_z\left(\varepsilon_x \cdot \varepsilon_y - \left(\frac{1}{2}\gamma_{xy}\right)^2\right) = 1.852 \cdot 10^{-12}$

Főfeszültségek, főnyúlások

$\underline{\underline{\sigma}} = \frac{E}{1+\nu}\left(\underline{\underline{\varepsilon}} + \frac{\nu}{1-2\nu} tr(\underline{\underline{\varepsilon}})\underline{\underline{E}}\right) \Rightarrow \underset{(x,y,z)}{\underline{\underline{\sigma}}} = \begin{bmatrix} 31.21 & 26.52 & 0 \\ 26.52 & 17.36 & 0 \\ 0 & 0 & 0 \end{bmatrix} [MPa]$



$(\underline{\underline{\sigma}} - \lambda \underline{\underline{E}})\underline{e} = \underline{0}, \quad \det(\underline{\underline{\sigma}} - \lambda \underline{\underline{E}}) = -\lambda(\lambda^2 - 48.57\lambda - 183.0) = 0$

$\lambda_1 = 0, \ \lambda_{2,3} = -3.513$ ~~MPa~~, $52.08 \rightarrow \sigma_1 = 52.08$ MPa, $\sigma_2 = 0$ MPa, $\sigma_3 = -3.513$ MPa

$\underset{(1,2,3)}{\underline{\underline{\sigma}}} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \qquad \underset{(1,2,3)}{\underline{\underline{\varepsilon}}} = \left[\frac{1+\nu}{E}\left(\underset{(1,2,3)}{\underline{\underline{\sigma}}} - \frac{\nu}{1+\nu} tr(\underset{(1,2,3)}{\underline{\underline{\sigma}}}) \cdot \underline{\underline{E}}\right) \right. = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix}$

$\varepsilon_1 = 265.7 \cdot 10^{-6}, \ \varepsilon_2 = -72.86 \cdot 10^{-6}, \ \varepsilon_3 = -35.65 \cdot 10^{-6}$ skalár invariáns!

$(\underline{\underline{\sigma}} - \sigma_1 \underline{\underline{E}}) \cdot \underline{e}_1 = \underline{0}, \quad (\underline{\underline{\varepsilon}} - \varepsilon_1 \underline{\underline{E}}) \cdot \underline{e}_1 = \underline{0} \qquad \underline{e}_1 = \begin{bmatrix} e_{1x} \\ e_{1y} \\ 0 \end{bmatrix} = \begin{bmatrix} 0.790 \\ 0.613 \\ 0 \end{bmatrix}$

7.2, Rugalmas test terheletlen felületi pontja

$\varepsilon_x = 350\cdot 10^{-6}$, $\varepsilon_y = 50\cdot 10^{-6}$, $\varepsilon_1 = 420\cdot 10^{-6}$, $\gamma_{xy} = ?$, $\varepsilon_2 = ?$, $\varepsilon_3 = ?$

$\nu = 0.3$

$$\underset{(x,y,z)}{\underline{\underline{\sigma}}} = \begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \underset{(x,y,z)}{\underline{\underline{\varepsilon}}} = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & 0 \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

$$\varepsilon_z = ? \quad \sigma_z = \frac{E}{1+\nu}\left(\varepsilon_z + \frac{\nu}{1-2\nu}\,tr(\underline{\underline{\varepsilon}})\,E\right) = 0 \Rightarrow \boxed{\varepsilon_z = -171.4\cdot 10^{-6}}$$

$$(\underset{(x,y,z)}{\underline{\underline{\varepsilon}}} - \lambda \underline{\underline{E}})\,\underline{e} = \underline{0} \qquad det(\underline{\underline{\varepsilon}} - \lambda \underline{\underline{E}}) = 0$$

$$0\cdot\ldots - 0\cdot\ldots + (\varepsilon_z - \lambda)\left[(\varepsilon_x - \lambda)(\varepsilon_y - \lambda) - \frac{1}{4}\gamma_{xy}^2\right] = 0 \qquad \lambda_1 = \varepsilon_z = -171.4\cdot 10^{-6}$$

$$\lambda^2 - \lambda(\varepsilon_x + \varepsilon_y) + \varepsilon_x\varepsilon_y - \frac{1}{4}\gamma_{xy}^2 = 0$$

$$\lambda_{1,2} = \frac{\varepsilon_x + \varepsilon_y \pm \sqrt{\varepsilon_x^2 + 2\varepsilon_x\varepsilon_y + \varepsilon_y^2 - 4\varepsilon_x\varepsilon_y + \gamma_{xy}^2}}{2} = \frac{\varepsilon_x + \varepsilon_y \pm \sqrt{\overbrace{(\varepsilon_x - \varepsilon_y)^2}^{>0} + \overbrace{\gamma_{xy}^2}^{>0}}}{2}$$

$$\varepsilon_1 = \frac{\varepsilon_x + \varepsilon_y + \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}}{2} \rightarrow \boxed{\gamma_{xy} = \pm 321.9\cdot 10^{-6}}$$

$$\boxed{\varepsilon_2 = \frac{\varepsilon_x + \varepsilon_y - \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}}{2} = -20\cdot 10^{-6}}$$

$$\boxed{\varepsilon_3 = \varepsilon_z = -171.4\cdot 10^{-6}}$$

7.3, Rugalmas test pontjában $\sigma_y = 100\,MPa$, $\sigma_z = -100\,MPa$, $\tau_{xy} = 120\,MPa$,

$\gamma_{xz} = 4\cdot10^{-4}$, $\gamma_{yz} = -8\cdot10^{-4}$, $\varepsilon_z = 2\cdot10^{-4}$, $E = 234\ GPa$, $\nu = 0.3$

$$\underset{(x,y,z)}{\underline{\underline{\sigma}}} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}, \quad \underset{(x,y,z)}{\underline{\underline{\varepsilon}}} = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix}$$

$$\frac{1}{2}\gamma_{xy} = \frac{1+\nu}{E}\left(\tau_{xy} - \frac{\nu}{1+\nu}tr(\underline{\underline{\sigma}})\cdot 0\right) \rightarrow \boxed{\gamma_{xy} = 1333\cdot10^{-6}}$$

$$\tau_{xz} = \frac{E}{1+\nu}\left(\frac{1}{2}\gamma_{xz} + \frac{\nu}{1-2\nu}tr(\underline{\underline{\varepsilon}})\cdot 0\right) \rightarrow \boxed{\tau_{xz} = 36\ MPa}$$

$$\tau_{yz} = \frac{E}{1+\nu}\left(\frac{1}{2}\gamma_{yz} + \frac{\nu}{1-2\nu}tr(\underline{\underline{\varepsilon}})\cdot 0\right) \rightarrow \boxed{\tau_{yz} = -72\ MPa}$$

$$\varepsilon_z = \frac{1+\nu}{E}\left(\sigma_z - \frac{\nu}{1+\nu}\underbrace{tr(\underline{\underline{\sigma}})}_{(\sigma_x+\sigma_y+\sigma_z)}\cdot 1\right) \rightarrow \boxed{\sigma_x = -589.3\ MPa}$$

$$\varepsilon_y = \frac{1+\nu}{E}\left(\sigma_y - \frac{\nu}{1+\nu}tr(\underline{\underline{\sigma}})\cdot 1\right) \rightarrow \boxed{\varepsilon_y = 1311\cdot10^{-6}}$$

$$\varepsilon_x = \frac{1+\nu}{E}\left(\sigma_x - \frac{\nu}{1+\nu}tr(\underline{\underline{\sigma}})\cdot 1\right) \rightarrow \boxed{\varepsilon_x = -2519\cdot10^{-6}}$$

$$\frac{\Delta V}{V} = \varepsilon_I = \varepsilon_x + \varepsilon_y + \varepsilon_z = -1007\cdot10^{-6}$$