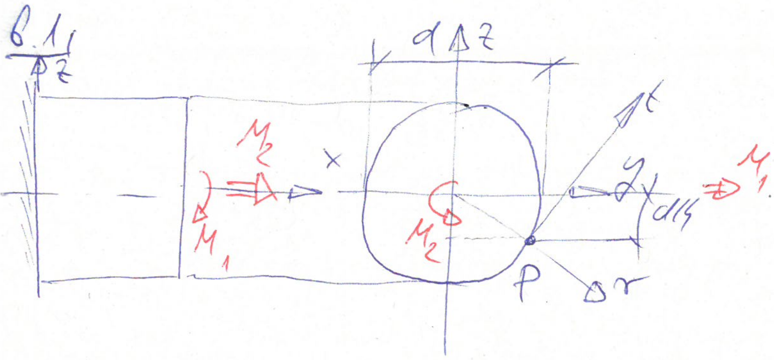
6.1.



$M_1 = M_2 = 530$ Nm, $d = 30$ mm

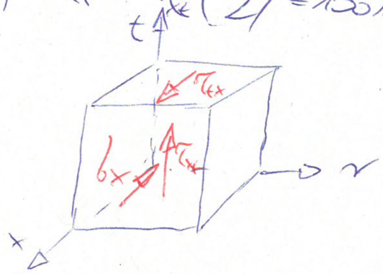
főfeszültségek, főirányok P-ben

sé'-sv // Mohr körök

$I_y = \frac{d^4\pi}{64}$, $I_p = \frac{d^4\pi}{32}$

$\sigma_x(z) = \frac{M_1}{I_y} z$, $\sigma_x^P = \sigma_x(-\frac{d}{2}) = -100\,\text{MPa}$, $\tau_{xt}(r) = \frac{M_2}{I_p} r$, $\tau_{xt}^P = \tau_{xt}(\frac{d}{2}) = 100\,\text{MPa}$

$$\underset{(x,r,t)}{\underline{\underline{\sigma}}} = \begin{bmatrix} \sigma_x & \tau_{rx} & \tau_{tx} \\ \tau_{xr} & \sigma_r & \tau_{tr} \\ \tau_{xt} & \tau_{rt} & \sigma_t \end{bmatrix} \qquad \underset{(x,r,t)}{\underline{\underline{\sigma}}^P} = \begin{array}{c} \begin{bmatrix} -100 & 0 & +100 \\ 0 & 0 & 0 \\ +100 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} X \\ R \\ T \end{array}$$



sé, sv

$(\underline{\underline{\sigma}}^P - \lambda \underline{\underline{E}})\underline{e} = \underline{0}$ $\quad \det(\underline{\underline{\sigma}}^P - \lambda \underline{\underline{E}}) = 0 = \det\begin{pmatrix} -100-\lambda & 0 & 100 \\ 0 & -\lambda & 0 \\ 100 & 0 & -\lambda \end{pmatrix} = (-100-\lambda)\lambda^2 - 0\cdot 0 + 100(100\lambda) =$

$= \lambda(-\lambda^2 - 100\lambda + 10000) = 0 \quad \lambda_1 = 0, \quad \lambda_{2,3} = \frac{100 \pm \sqrt{100^2 + 40000}}{-2} = \begin{cases} \lambda_2 = -161.8\,\text{MPa} \\ \lambda_3 = 61.8\,\text{MPa} \end{cases}$

$\sigma_1 = 61.8\,\text{MPa}$, $\sigma_2 = 0\,\text{MPa}$, $\sigma_3 = -161.8\,\text{MPa}$

$\underline{e}_1$ meghatározása $(\underline{\underline{\sigma}}^P - \sigma_1 \underline{\underline{E}})\underline{e}_1 = 0$

$$\begin{bmatrix} -161.8 & 0 & 100 \\ 0 & -61.8 & 0 \\ 100 & 0 & -61.8 \end{bmatrix} \begin{bmatrix} e_{1x} \\ e_{1r} \\ e_{1t} \end{bmatrix} = \underline{0} \quad \text{II}: e_{1r} = 0, \quad e_{1x} = 1, \quad \text{III}: e_{1t} = \frac{100}{61.8} = 1.618$$

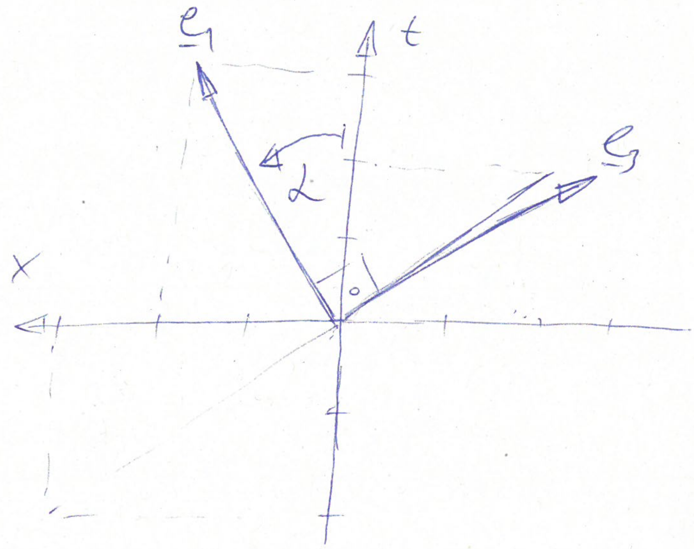
$\underline{e}_2$; $(\underline{\underline{\sigma}}^P - \sigma_2 \underline{\underline{E}})\underline{e}_2 = \underline{0}$

$$\begin{bmatrix} -100 & 0 & 100 \\ 0 & 0 & 0 \\ 100 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{2x} \\ e_{2r} \\ e_{2t} \end{bmatrix} = \underline{0} \quad \text{III}: e_{2x} = 0, \quad \text{I}: e_{2t} = 0, \quad e_{2r} = 1$$

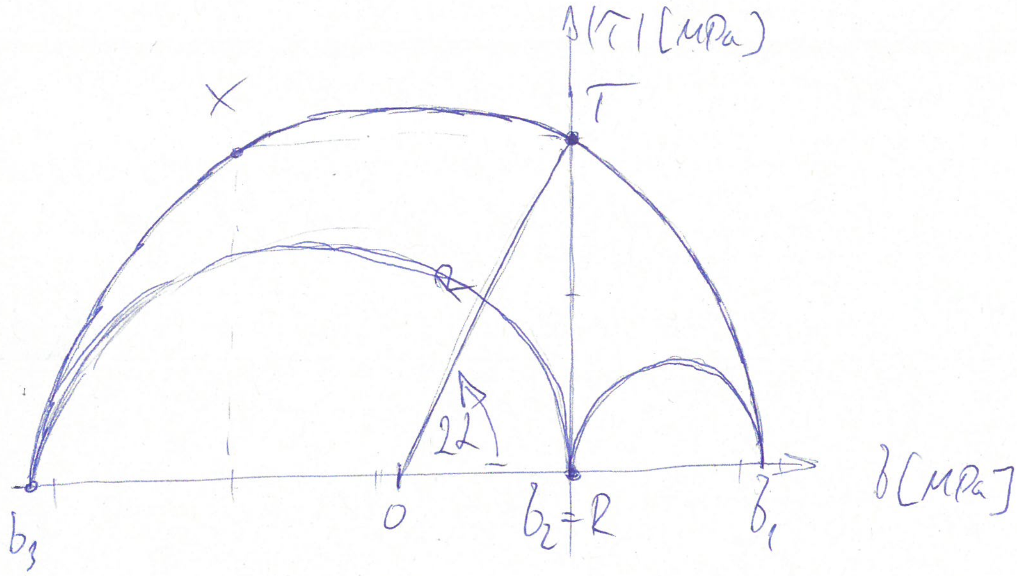
$\underline{e}_3$; $(\underline{\underline{\sigma}}^P - \sigma_3 \underline{\underline{E}})\underline{e}_3 = 0$

$$\begin{bmatrix} 61.8 & 0 & 100 \\ 0 & 161.8 & 0 \\ 100 & 0 & 161.8 \end{bmatrix} \begin{bmatrix} e_{3x} \\ e_{3r} \\ e_{3t} \end{bmatrix} = 0 \quad \text{II}: e_{3r} = 0, \quad e_{3t} = 1, \quad \text{III}: e_{3x} = -\frac{1.618}{100} = -1.618$$

$$\underline{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 1.618 \end{bmatrix}; \quad \underline{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \quad \underline{e}_3 = \begin{bmatrix} -1.618 \\ 0 \\ +1 \end{bmatrix}$$



$\alpha = \text{arctg}\left(\frac{e_{1t}}{e_{1x}}\right) = 31.7°$



$$\sigma^0 = \frac{\sigma^x + \sigma^T}{2} = -50 \text{ MPa}$$

$$R = \sqrt{(\sigma^T - \sigma^0)^2 + (\tau^T)^2} = 111.8 \text{ MPa}$$

$$\sigma_1 = \sigma^0 + R = 61.8 \text{ MPa}$$

$$\sigma_2 = 0 \text{ MPa}$$

$$\sigma_3 = \sigma^0 - R = -161.8 \text{ MPa}$$

$$2\alpha = \operatorname{arctg}\left(\frac{\tau^T}{\sigma_2 - \sigma^0}\right) = 63.4^\circ \qquad \underline{\alpha = 31.7^\circ}$$

t irányt csúsztató feszültség irányába forgatjuk

6.2.1 Főfeszültségek és 1-es főirány megadása

a) $\underline{\underline{\sigma}}_{(x,y,z)} = \begin{bmatrix} 25 & 0 & -2.5 \\ 0 & -10 & 4 \\ -2.5 & 4 & 30 \end{bmatrix}$ [MPa]

hatványmódszer → σ_1, e_1
inverz hatványmódszer → σ_3, e_3
QR dekompozíció $\sigma_1, e_1, \sigma_2, e_2, \sigma_3, e_3$

$$\det(\underline{\underline{\sigma}} - \lambda \underline{\underline{E}}) = -(\lambda - \sigma_1)(\lambda - \sigma_2)(\lambda - \sigma_3)$$

wiki: Cubic function
algebrai, trigonometrikus, geometriai

$\sigma_1 = 31.4$ MPa, $\sigma_2 = 24.0$ MPa, $\sigma_3 = -10.4$ MPa

$$(\underline{\underline{\sigma}} - \sigma_1 \underline{\underline{E}})\underline{e}_1 = \underline{0} \qquad \begin{bmatrix} -6.4 & 0 & -2.5 \\ 0 & -41.4 & 4 \\ -2.5 & 4 & -1.4 \end{bmatrix} \begin{bmatrix} e_{1x} \\ e_{1y} \\ e_{1z} \end{bmatrix} = \underline{0}$$

$e_{1x} = 1$

I: $e_{1z} = \dfrac{-6.4}{2.5} = -2.56$

II: $e_{1y} = 4 \cdot (-2.56) \cdot \dfrac{1}{41.4} = -0.247$

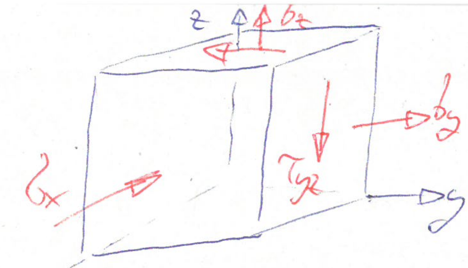
b) $\underline{\underline{\sigma}}_{(x,y,z)} = \begin{bmatrix} 0 & 10 & -2.5 \\ 10 & 0 & 4 \\ -2.5 & 4 & 0 \end{bmatrix}$ [MPa]

Wolfram Alpha, Mathematica Eigensystem[ ]
MATLAB eig()

$\sigma_1 = 28.1$ MPa, $\sigma_2 = -7.1$ MPa, $\sigma_3 = -21.0$ MPa

$\underline{e}_1^T = [0.632, 0.443, 0.632]$

c) $\underset{(x,y,z)}{\underline{\underline{\sigma}}} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 60 & -10 \\ 0 & -10 & 30 \end{bmatrix}$ [MPa] $\sigma^e - \sigma_0$, Mohr



$$\det(\underline{\underline{\sigma}} - \lambda \underline{\underline{E}}) = 0 = \det\begin{pmatrix} -2-\lambda & 0 & 0 \\ 0 & 60-\lambda & -10 \\ 0 & -10 & 30-\lambda \end{pmatrix} = (-2-\lambda)\left((60-\lambda)(30-\lambda)-100\right) =$$

$$= -(\lambda+2)(\lambda^2 - 90\lambda + 1700) = 0, \quad \lambda_1 = -2\,\text{MPa}, \quad \lambda_{2,3} = \frac{90 \pm \sqrt{90^2 - 6800}}{2} =$$

$\lambda_2 = 63.0$ MPa, $\lambda_3 = 27.0$ MPa, $\sigma_1 = 63.0$ MPa, $\sigma_2 = 27.0$ MPa, $\sigma_3 = -2$ MPa.

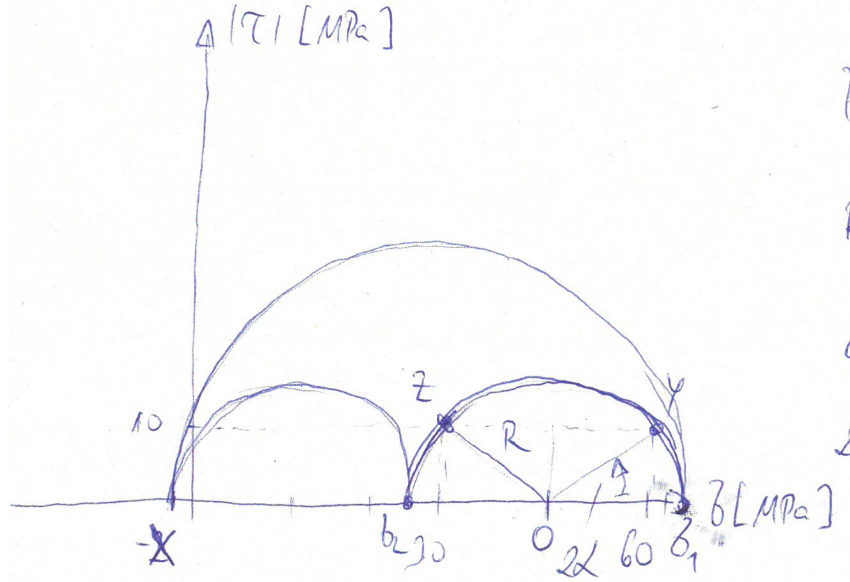
$$(\underline{\underline{\sigma}} - \sigma_1 \underline{\underline{E}})\underline{e}_1 = \underline{0}$$

$$\begin{bmatrix} -65 & 0 & 0 \\ 0 & -3 & -10 \\ 0 & -10 & -33 \end{bmatrix} \begin{bmatrix} e_{1x} \\ e_{1y} \\ e_{1z} \end{bmatrix} = \underline{0}$$

I $e_{1x} = 0$

$e_{1y} = 1$

II $e_{1z} = -\frac{3}{10}$



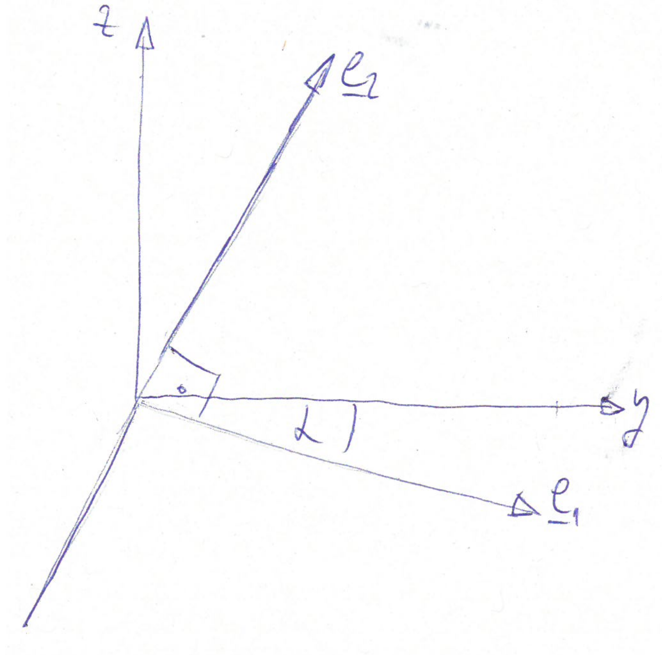
$$\sigma^0 = \frac{\sigma^y + \sigma^z}{2} = 45\,\text{MPa}$$

$$R = \sqrt{(\sigma^y - \sigma^0)^2 + (\tau^y)^2} = 18.0\,\text{MPa}$$

$$\sigma_1 = \sigma^0 + R = 63.0\,\text{MPa}$$

$$2\alpha = \operatorname{arctg}\left(\frac{\tau^y}{\sigma^y - \sigma^0}\right) = 33.65°$$

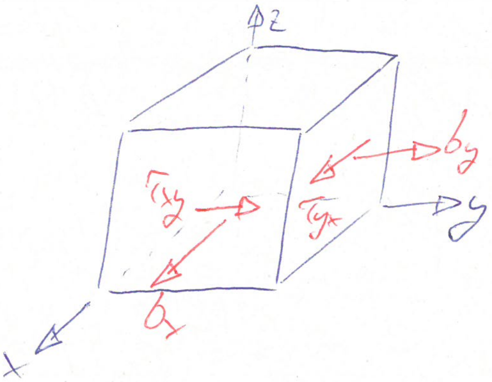
$$\alpha = 16.85°$$



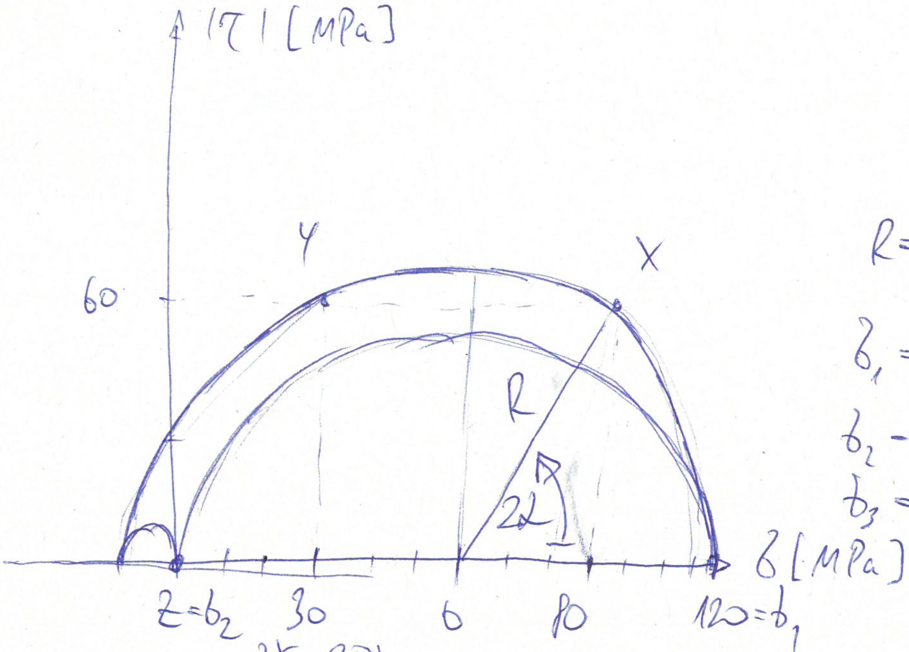
$$\underline{e}_1 \times \underline{e}_2 = \underline{e}_3$$

6.3,

$$\underline{\underline{\sigma}}_{(x,y,z)} = \begin{array}{cc} \begin{bmatrix} 80 & 60 & 0 \\ 60 & 30 & 0 \\ 0 & 0 & 0 \end{bmatrix} [MPa] & \begin{array}{c} x \\ y \\ z \end{array} \end{array}$$



Mohr körök $\sigma_1, \sigma_2, \sigma_3, \underline{e}_1, \underline{e}_2, \underline{e}_3$



$$\sigma^\circ = \frac{\sigma^x + \sigma^y}{2} = 55 \, MPa$$

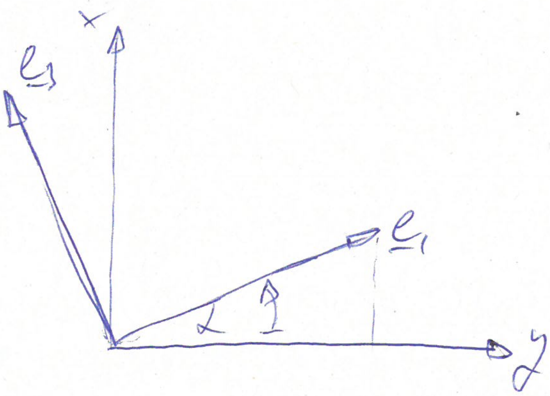
$$R = \sqrt{(\sigma^x - \sigma^\circ)^2 + (\tau^x)^2} = 65 \, MPa$$

$$\sigma_1 = \sigma^\circ + R = 120 \, MPa$$

$$\sigma_2 = \sigma^z = 0 \, MPa$$

$$\sigma_3 = \sigma^\circ - R = -10 \, MPa$$

$$2\alpha = arctg\left(\left(\frac{\sigma^x - \sigma^\circ}{|\tau^x|}\right)^{-1}\right) = 67.38^\circ \quad \alpha = 33.7^\circ$$



$$\underline{e}_1 = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \\ 0 & \end{bmatrix} = \begin{bmatrix} 0.555 \\ 0.832 \\ 0 \end{bmatrix}$$

$$\underline{e}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\underline{e}_3 = \begin{bmatrix} \cos\alpha \\ -\sin\alpha \\ 0 \end{bmatrix} = \begin{bmatrix} 0.832 \\ -0.555 \\ 0 \end{bmatrix}$$