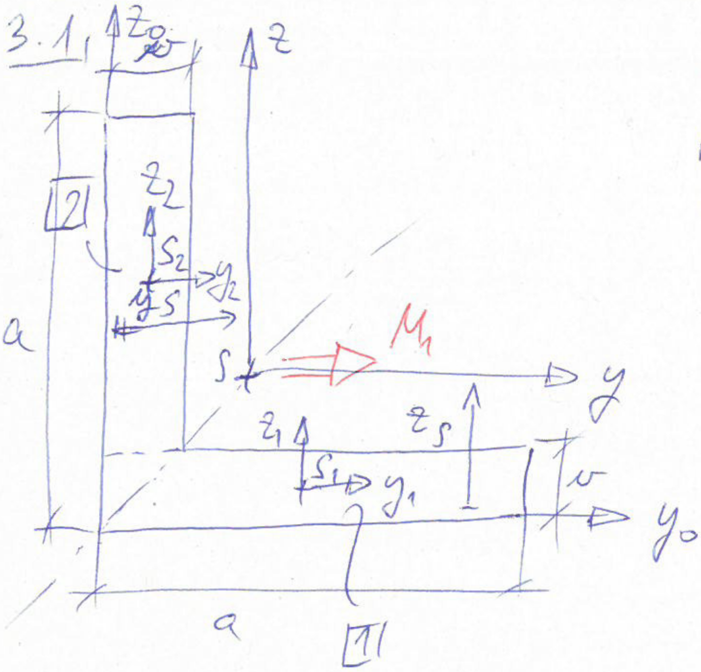
3.1,



Súlypont? másodrendű nyomatékok? Fő másodrendű nyomatékok, főirányok? Zérustengely y-nal? feszültség eloszlás?

$a = 50\,mm$, $v = 6\,mm$, $M_1 = 500\,Nm$

súlypont meghatározása

$A_1 = a \cdot v = 3\,cm^2$, $A_2 = v(a-v) = 2.64\,cm^2$

0-s koordinátarendszerben a súlypontok koordinátái

$y_S^1 = \frac{a}{2}$, $z_S^1 = \frac{v}{2}$; $y_S^2 = \frac{v}{2}$, $z_S^2 = v + \frac{a-v}{2}$

$S_y^0 = A_1 \cdot z_S^1 + A_2 \cdot z_S^2$; $S_z^0 = A_1 \cdot y_S^1 + A_2 \cdot y_S^2$

$z_S = \frac{S_y^0}{A_1 + A_2} = 14.70\,mm$, $y_S = \frac{S_z^0}{A_1 + A_2} = 14.70\,mm$

másodrendű nyomatékok a súlyponti tengelyre

$I_y^1 = \frac{a v^3}{12}$, $I_z^1 = \frac{v a^3}{12}$; $I_y^2 = \frac{v(a-v)^3}{12}$, $I_z^2 = \frac{(a-v)v^3}{12}$

$I_y = I_y^1 + A_1 \cdot (z_S - z_S^1)^2 + I_y^2 + A_2 (z_S - z_S^2)^2 = 13.13\,cm^4$

$I_z = I_z^1 + A_1 (y_S - y_S^1)^2 + I_z^2 + A_2 (y_S - y_S^2)^2 = 13.13\,cm^4$

$I_{yz} = 0 + A_1 (z_S - z_S^1)(y_S - y_S^1) + 0 + A_2 (z_S - z_S^2)(y_S - y_S^2) = -7.72\,cm^4$

$$\underset{(y,z)}{\underline{\underline{I}}} = \begin{bmatrix} I_y & -I_{yz} \\ -I_{yz} & I_z \end{bmatrix}$$

fő másodrendű nyomatékok és főirányok meghatározása

$$\underline{\underline{I}}\,\underline{e} = \lambda\,\underline{e} \qquad (\underline{\underline{I}} - \lambda \underline{\underline{E}})\,\underline{e} = \underline{0}$$

$$\det(\underline{\underline{I}} - \lambda \underline{\underline{E}}) = 0 \qquad \det\begin{pmatrix} I_y - \lambda & -I_{yz} \\ -I_{yz} & I_z - \lambda \end{pmatrix} = (I_y - \lambda)(I_z - \lambda) - I_{yz}^2 =$$

$$= \lambda^2 - 2.6252 \cdot 10^{-7}\lambda + 1.1264 \cdot 10^{-14} = 0$$

$$\lambda_{1,2} = \frac{2.6252 \cdot 10^{-7} \pm \sqrt{(2.6252)^2 \cdot 10^{-14} - 4 \cdot 1.1264 \cdot 10^{-14}}}{2}$$

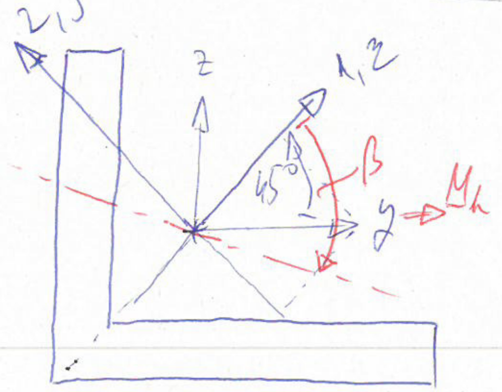
$$\lambda_1 = I_1 = 2.085 \cdot 10^{-7}\,m^4$$

$$\lambda_2 = I_2 = 5.402 \cdot 10^{-8}\,m^4$$

$$(\underline{\underline{I}} - \lambda_1 \underline{\underline{E}})\,\underline{e}_1 = \underline{0} \qquad \underline{e}_1 = \begin{bmatrix} 1 \\ e_{1z} \end{bmatrix} \qquad \begin{bmatrix} I_y - \lambda_1 & -I_{yz} \\ \cdot & \cdot \end{bmatrix}\begin{bmatrix} 1 \\ e_{1z} \end{bmatrix} = \begin{bmatrix} 0 \\ \cdot \end{bmatrix}$$

$$I_y - \lambda_1 - I_{yz} \cdot e_{1z} = 0 \Rightarrow e_{1z} = \frac{I_y - \lambda_1}{I_{yz}} = 1$$

$$\underline{e}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \xrightarrow{\times(-1)} \underline{e}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$



feszültségeloszlás

$$M_{h1} = \cos 45° \cdot M_h = 353.5\ Nm$$

$$M_{h2} = -\sin 45° \cdot M_h = -353.5\ Nm$$

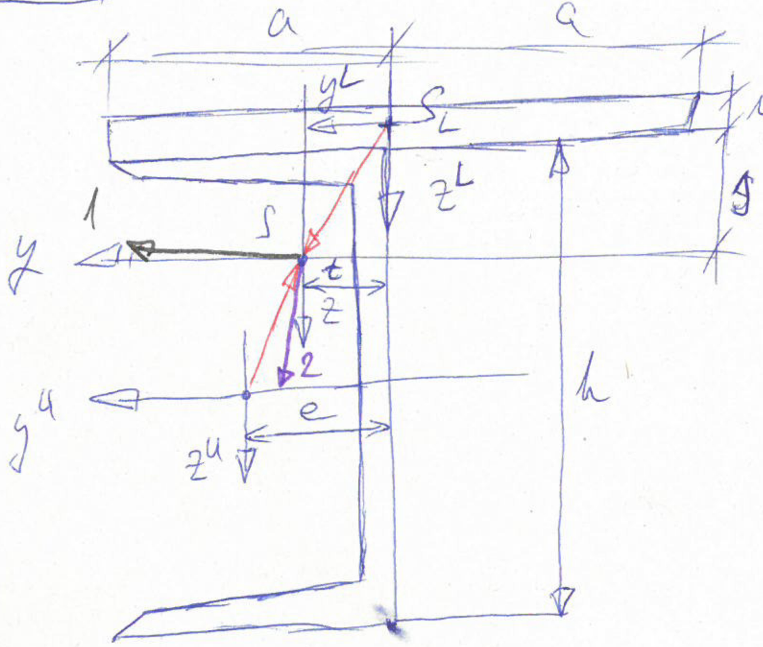
$$\sigma_x(\xi,\zeta) = \frac{M_{h1}}{I_1}\cdot\zeta - \frac{M_{h2}}{I_2}\,\xi$$

$\sigma_x(\xi,\zeta) = 1696\,\zeta + 6544\,\xi$ [MPa] ξ,ζ [m]-ben behelyettesítve zérustengely $\sigma_x(z, y) = 0 \Rightarrow y = \frac{I_1}{I_2} \cdot \frac{M_{h2}}{M_{h1}} z \Rightarrow y = -3.859 z$

$\beta = \operatorname{arctg}(-3.859) = -75.47°$

$\angle(y, \text{zérustengely}) = 45° + \beta = \underline{-30.47°}$

3.2)



U140, merevítő lap

$a = 60\,mm, v = 10\,mm, \eta = 4.22\,cm$

$t = 1.1\,cm, A^u = 20.4\,cm^2, e = 1.75\,cm$

$I_1^u = 605\,cm^4, I_2^u = 62.7\,cm^4$

$h = 140\,mm$

fő másodrendű nyomatékok, helyek?

$A^L = 2a \cdot v; \quad I_y^L = \frac{2a v^3}{12}; \quad I_z^L = \frac{v(2a)^3}{12}$

$I_1^u = I_y^u, \quad I_2^u = I_z^u$

$I_y = I_y^L + A^L \cdot \left(\frac{v}{2} + \eta\right)^2 + I_y^u + A^u \left(\frac{h}{2} - \eta\right)^2 = \underline{1031\,cm^4}$

$I_z = I_z^L + A^L (t)^2 + I_z^u + A^u (e - t)^2 = \underline{229.8\,cm^4}$

$I_{yz} = 0 + A^L \left(\frac{v}{2} + \eta\right)(t) + 0 + A^u \left(\eta - \frac{h}{2}\right)(t - e) = \underline{99.17\,cm^4}$

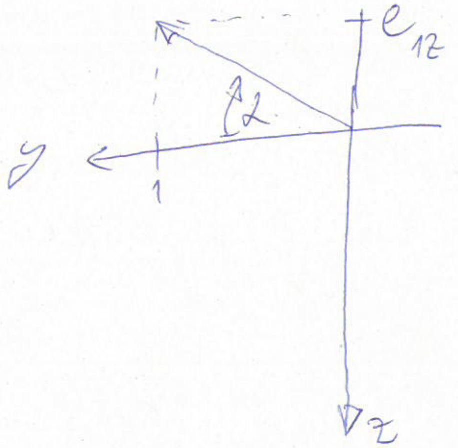
$\underline{\underline{I}} = \begin{bmatrix} I_y & -I_{yz} \\ -I_{yz} & I_z \end{bmatrix} \rightarrow I_{1,2}$ $\quad \det(\underline{\underline{I}} - \lambda \underline{\underline{E}}) = 0$

$(I_y - \lambda)(I_z - \lambda) - I_{yz}^2 = 0$

$\lambda^2 - \lambda(I_y + I_z) + I_y I_z - I_{yz}^2 = 0 \quad \begin{cases} \lambda_1 = 1043\,cm^4 \\ \lambda_2 = 217.7\,cm^4 \end{cases}$

$(\underline{\underline{I}} - \lambda_1 \underline{\underline{E}})\underline{e}_1 = \underline{0} \quad \underline{e}_1 = \begin{bmatrix} 1 \\ e_{1z} \end{bmatrix} \quad \begin{bmatrix} I_y - \lambda_1 & -I_{yz} \\ \cdot & \end{bmatrix} \begin{bmatrix} 1 \\ e_{1z} \end{bmatrix} = \begin{bmatrix} 0 \\ \cdot \end{bmatrix}$

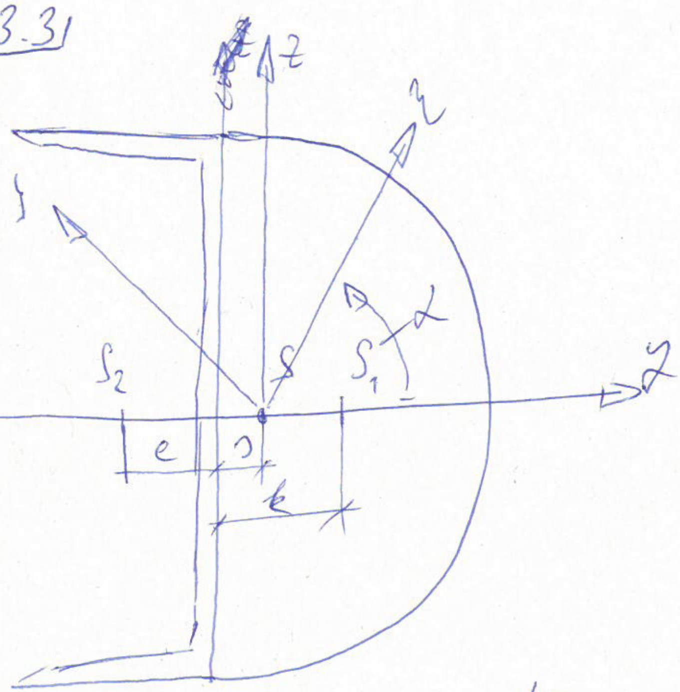
$e_{1z} = -0.1219 \qquad \underline{e}_1 = \begin{bmatrix} 1 \\ -0.1219 \end{bmatrix}$



$\alpha = \mathrm{arctg}\left(\frac{e_{1z}}{1}\right) = \cancel{\ldots} -6.95^\circ$

2-es főtengely $\alpha + 90^\circ$-ot zár be.

3.31



$I_y = ?$, $I_z = ?$ second moment of area

$I_\xi, I_\eta, I_{\xi\eta}$?

$d = 14\ \mathrm{cm}$, $k = \frac{2d}{3\pi} = 2.97\mathrm{cm}$

$r = 1.98\mathrm{cm}$, $\alpha = 60^\circ$, $A^u = 20.4\mathrm{cm}^2$

$I_1^u = I_y^u = 605\mathrm{cm}^4$, $I_2^u = I_z^u = 62.7\mathrm{cm}^4$

$e = 1.75\ \mathrm{cm}$

$$I_y^\circ = \frac{d^4\pi}{128}, \quad I_z^\circ = \frac{d^4\pi}{128} - \frac{d^4}{18\pi}; \quad A^\circ = \frac{d^2\pi}{8}$$

$$\left.\begin{aligned} I_y &= I_y^u + A^u \cdot 0^2 + I_y^\circ + A^\circ \cdot 0^2 = \underline{1547.9\ \mathrm{cm}^4} \\ I_z &= I_z^u + A^u (e+r)^2 + I_z^\circ + A^\circ (k-r)^2 = \underline{685.6\ \mathrm{cm}^4} \\ I_{yz} &= 0 + A^u \cdot 0 + 0 + A^\circ \cdot 0 = \underline{0} \end{aligned}\right\} \underline{\underline{I}}_{(y,z)} = \begin{bmatrix} I_y & 0 \\ 0 & I_z \end{bmatrix}$$

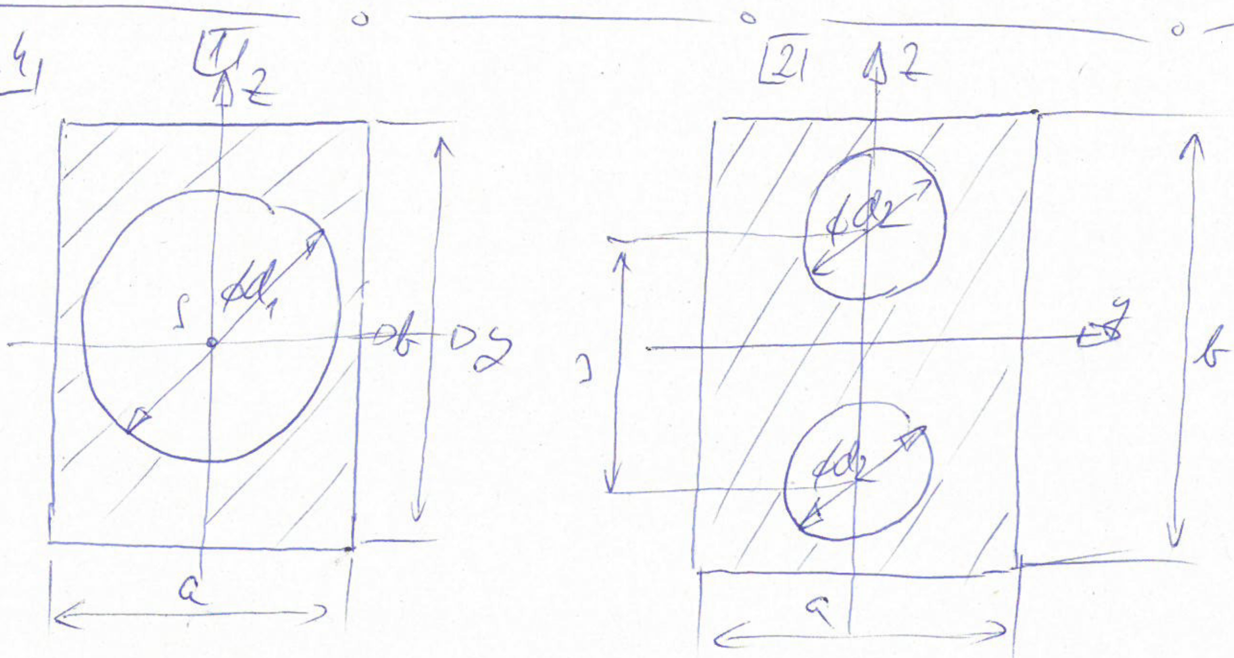
$$\underline{\underline{T}} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} c & -s \\ s & c \end{bmatrix} \qquad \cos\alpha = \frac{1}{2}, \ \sin\alpha = \frac{\sqrt{3}}{2}$$

$$\underline{\underline{I}}_{(\xi,\eta)} = \underline{\underline{T}}^T \cdot \underline{\underline{I}}_{(x,y)} \cdot \underline{\underline{T}} = \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \cdot \begin{bmatrix} I_y & 0 \\ 0 & I_z \end{bmatrix} \begin{bmatrix} c & -s \\ s & c \end{bmatrix} = \begin{bmatrix} c \cdot I_y & s I_z \\ -s I_y & c \cdot I_z \end{bmatrix} \begin{bmatrix} c & -s \\ s & c \end{bmatrix} =$$

$$\begin{bmatrix} c^2 \cdot I_y + s^2 I_z & c \cdot s (I_z - I_y) \\ c \cdot s (I_z - I_y) & c^2 I_z + s^2 I_y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} I_y + 3 I_z & \sqrt{3}(I_z - I_y) \\ \sqrt{3}(I_z - I_y) & I_z + 3 I_y \end{bmatrix} = \begin{bmatrix} I_\xi & -I_{\xi\eta} \\ -I_{\xi\eta} & I_\eta \end{bmatrix}$$

$\underline{I_z = 901.2\ cm^4}$, $\underline{I_y = 1332\ cm^4}$, $\underline{I_{zy} = 373.4\ cm^4}$

3.4,



meredtsmeny meggyőzése, hogy az 1 vagy a 2 lukas a jobb . y hajlítás

$a = 240\,mm$, $b = 300\,mm$, $d_1 = 200\,mm$, $d_2 = 100\,mm$, $s = 150\,mm$

$A^T = a \cdot b$, $I_y^T = \frac{a b^3}{12}$, $I_y^{o1} = \frac{d_1^4 \pi}{64} \Rightarrow I_y^1 = I_y^T - I_y^{o1} = \underline{46146\ cm^4}$

$A^{o1} = \frac{d_1^2 \pi}{4}$; $A^1 = A^T - A^{o1} = \cancel{314} = \underline{405.8\ cm^2}$

$A^{o2} = \frac{d_2^2 \pi}{4}$, $I_y^{o2} = \frac{d_2^4 \pi}{64}$ $\quad I_{ys}^{o2} = I_y^{o2} + A^{o2} \cdot \left(\frac{s}{2}\right)^2$

$A^2 = A^T - 2 \cdot A^{o2} = \underline{562.9\ cm^2}$

$I_y^2 = I_y^T - 2 \cdot I_{ys}^{o2} = \underline{44183}\ cm^4$

$I_y^1 \to I_y^2$ 4.3%-os csökkenés

$A^1 \to A^2$ 38.7%-os növekedés