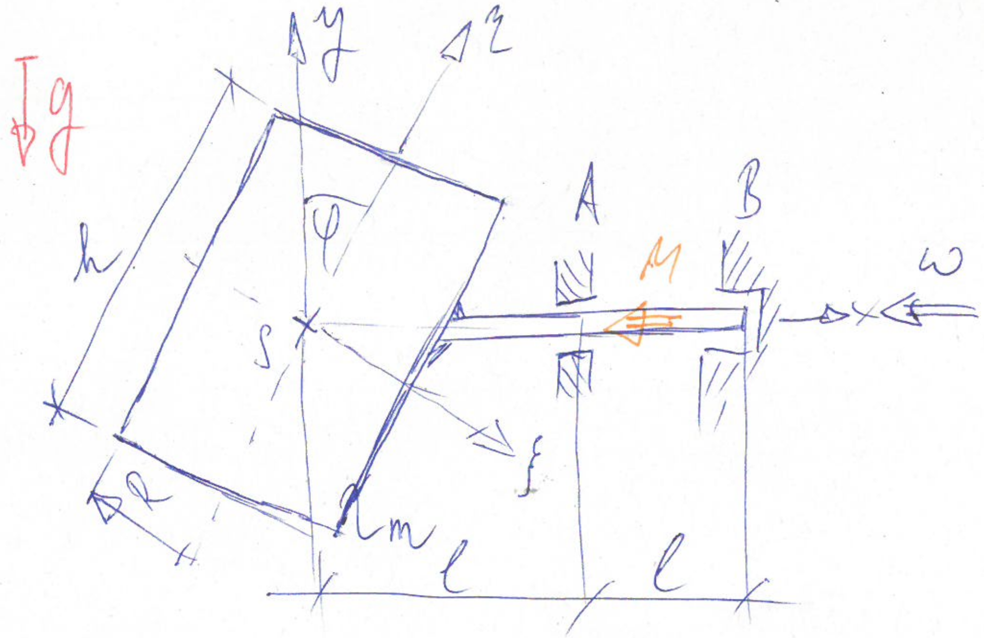

$m = 4\,[kg]$

$l = 0.25\,[m]$

$R = 0.1\,[m]$

$h = 0.3\,[m]$

$\omega = 10\,[rad/s]$

$M = 2\,[Nm]$

$\varphi = 30\,[°]$

$g = 9.81\,[m/s^2]$

1) $\underline{\Pi}_S$ parameteres (ξ, η, ζ)

2) SZTA'

3) DINAMIKA ALAPTÉTEL (x, y, z)

4) ε, $\underline{A}$, $\underline{B}$ numerikusan

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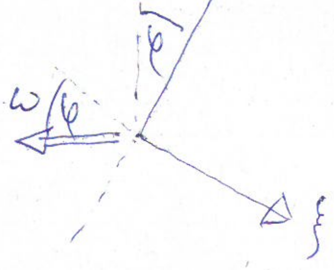
1) $\underline{\Pi}_S = \underline{\underline{\Theta}}_S \underline{\omega}$, $\underline{\underline{\Theta}}_S = \begin{bmatrix} \Theta_1 & 0 & 0 \\ \cdot & \Theta_2 & 0 \\ \cdot & \cdot & \Theta_1 \end{bmatrix}_{(\xi,\eta,\zeta)}$

$\Theta_1 = \frac{1}{4} m R^2 + \frac{1}{12} m h^2 = 0.04\,[k$

$\Theta_2 = \frac{1}{2} m R^2 = 0.02\,[kg m^2]$

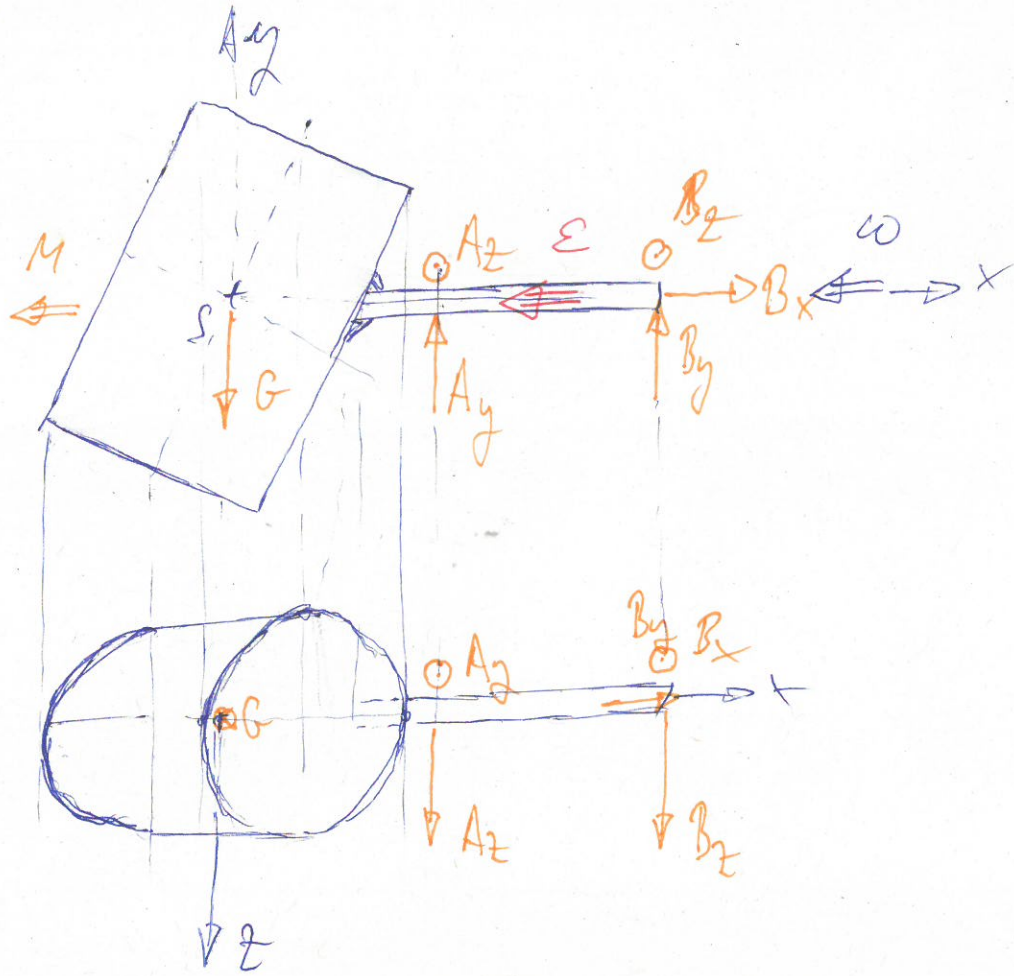
$\underset{(\xi,\eta,\zeta)}{\underline{\omega}} = -\omega [\cos\varphi \;\; \sin\varphi \;\; 0]^T$; $\underline{\Pi}_S = -\omega \begin{bmatrix} \cos\varphi\, \Theta_1 \\ \sin\varphi\, \Theta_2 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.346 \\ -0.1 \\ 0 \end{bmatrix} \left[kg \frac{m^2}{s}\right]$



4

14/01

2)



3)

$\dot{\underline{I}} = \underline{F}$

$x: \ m0 = B_x$ $\boxed{1}$

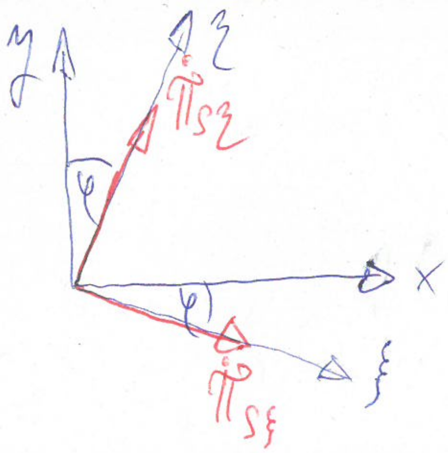
$y: \ m0 = A_y + B_y - G$ $\boxed{2}$

$z: \ m0 = A_z + B_z$ $\boxed{3}$

(ξ, η, ζ) - lm6

$\dot{\underline{\Pi}}_S = \underline{M}_S$; $\dot{\underline{\Pi}}_S = \underline{\Theta}_S \cdot \underline{\varepsilon} + \underline{\omega} \times \underline{\Pi}_S = -\varepsilon \begin{bmatrix} +\Theta_1 \cos\varphi \\ +\Theta_2 \sin\varphi \\ 0 \end{bmatrix} + \begin{vmatrix} i & j & k \\ -\omega c\varphi & -\omega s\varphi & 0 \\ -\Theta_1 \omega c\varphi & -\Theta_2 \omega s\varphi & 0 \end{vmatrix} =$

$$= \begin{bmatrix} -\varepsilon \Theta_1 \cos\varphi \\ -\varepsilon \Theta_2 \sin\varphi \\ \omega^2 \sin\varphi \cos\varphi (\Theta_2 - \Theta_1) \end{bmatrix} \to \begin{bmatrix} \dot{\Pi}_{S\xi} \\ \dot{\Pi}_{S\eta} \\ \dot{\Pi}_{S\zeta} \end{bmatrix}$$



$$\underset{(x,y,z)}{\mathring{\Pi}_S} = \begin{bmatrix} \sin\varphi\, \mathring{\Pi}_{S\eta} + \cos\varphi\, \mathring{\Pi}_{S\xi} \\ \cos\varphi\, \mathring{\Pi}_{S\eta} - \sin\varphi\, \mathring{\Pi}_{S\xi} \\ \mathring{\Pi}_{S\zeta} \end{bmatrix}_{(x,y,z)} =$$

$$= \begin{bmatrix} -\Theta_1 \varepsilon \cos^2\varphi - \Theta_2 \varepsilon \sin^2\varphi \\ (\Theta_1 - \Theta_2)\, \varepsilon \sin\varphi \cos\varphi \\ (\Theta_2 - \Theta_1)\, \omega^2 \sin\varphi \cos\varphi \end{bmatrix}_{(x,y,z)}$$

$$\underline{M}_S = \underline{M} + \underline{r}_{SA} \times \underline{A} + \underline{r}_{SB} \times \underline{B} = \begin{bmatrix} -M \\ 0 \\ 0 \end{bmatrix}_{(x,y,z)} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ l & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix}_{(x,y,z)} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2l & 0 & 0 \\ B_x & B_y & B_z \end{vmatrix}_{(x,y,z)} =$$

$$= \begin{bmatrix} -M \\ -A_z l - B_z 2l \\ A_y l + B_y 2l \end{bmatrix}_{(x,y,z)}$$

$$\underset{(x,y,z)}{\mathring{\Pi}_S} = \underset{(x,y,z)}{\underline{M}_S} \qquad x: \boxed{4} \quad y: \boxed{5} \quad z: \boxed{6}$$

Ismeretlenek: $A_y, A_z, B_x, B_y, B_z, \varepsilon$ (6 db)

14/02

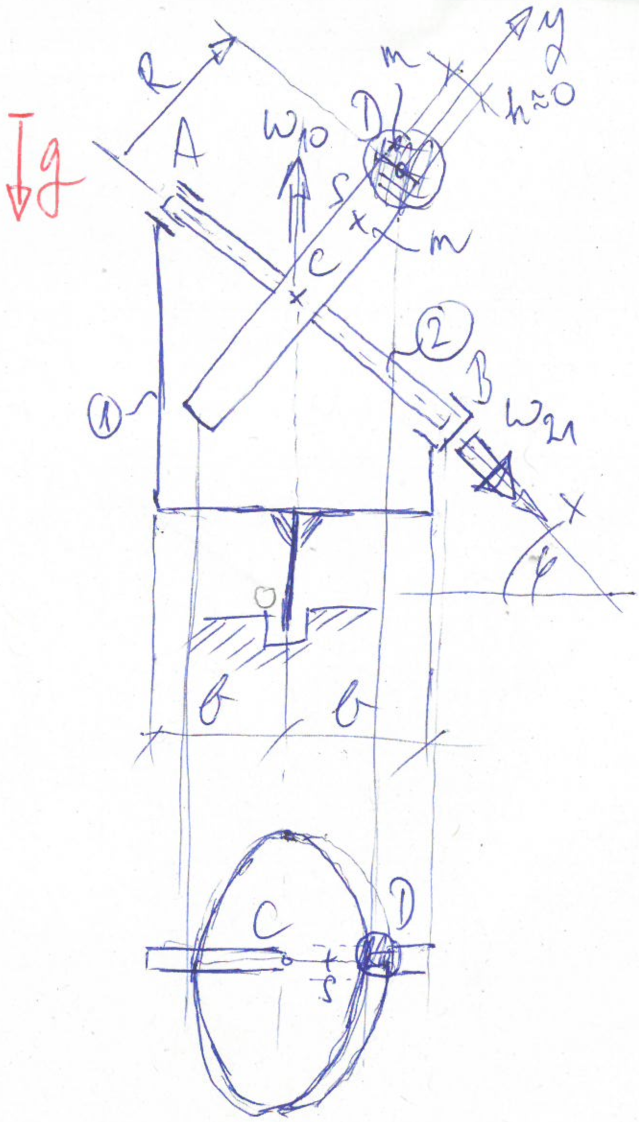
$$\underline{x} = [A_y, A_z, B_x, B_y, B_z, \varepsilon]^T, \quad \underline{\underline{A}} \cdot \underline{x} = \underline{b}$$

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\Theta_1 c^2\varphi - \Theta_2 s^2\varphi \\ 0 & l & 0 & 0 & 2l & (\Theta_1 - \Theta_2) c\varphi s\varphi \\ -l & 0 & 0 & -2l & 0 & 0 \end{bmatrix}$$

$$\underline{b} = [0 ; -gm ; 0 ; -M ; 0 ; (\Theta_1 - \Theta_2)\omega^2 (c\varphi s\varphi)]^T$$

$$\underline{x} = \begin{bmatrix} 2gm + \dfrac{(\Theta_1 - \Theta_2)\omega^2 c\varphi s\varphi}{l} \\ \dfrac{M(\Theta_1 - \Theta_2) c\varphi s\varphi}{l(\Theta_1 c^2\varphi + \Theta_2 s^2\varphi)} \\ 0 \\ -gm + \dfrac{(\Theta_1 - \Theta_2)\omega^2 c\varphi \sin\varphi}{l(\Theta_1 c^2\varphi + \Theta_2 s^2\varphi)} \\ -M \dfrac{(\Theta_1 - \Theta_2) c\varphi s\varphi}{l(\Theta_1 c^2\varphi + \Theta_2 s^2\varphi)} \\ \dfrac{M}{\Theta_1 c^2\varphi + \Theta_2 s^2\varphi} \end{bmatrix} = \begin{bmatrix} 81.94\,[N] \\ 1.98\,[N] \\ 0\,[N] \\ -42.70\,[N] \\ -1.98\,[N] \\ 57.14\,[\mathrm{rad}/s^2] \end{bmatrix}$$

$$; \underline{A} = \begin{bmatrix} 0 \\ 81.94 \\ 1.98 \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} 0 \\ -42.70 \\ -1.98 \end{bmatrix}$$



$$\underline{r}_{CS} = \frac{1}{m+m}\left(m \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + m \begin{bmatrix} 0 \\ R \\ 0 \end{bmatrix} \right) = \frac{1}{2} R \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$m = 0.6\,[kg]$

$R = 0.04\,[m]$

$b = 0.04\,[m]$

$\omega_{10} = 20\,[rad/s]$

$\omega_{21} = 30\,[rad/s]$

$\varphi = 30\,[°]$

$g = 9.81\,[m/s^2]$

1/ $\underline{a}_S = ?$, minimum kérdés!

2/ $\underline{\underline{\Theta}}_{B}$ (x, y, z) $\underline{v}_C = \underline{0}$

3/ ~~(2) SZTA~~ $\underline{\Pi}_C$ (x, y, z)

4/ (2) SZTA

5/ DINAMIKA ALAPTÉTELE PARAMÉTERESEN

1)
$$\underline{a}_S = \underbrace{\underline{a}_C}_{=0} + \underline{\varepsilon}_2 \times \underline{r}_{CS} + \underline{\omega}_2 \times (\underline{\omega}_2 \times \underline{r}_{CS})$$

$$\underline{\varepsilon}_2 = \underline{\omega}_{10} \times \underline{\omega}_2 = \cancel{\begin{vmatrix} i & j & k \\ -\omega_{10}\sin\varphi & \omega_{10}\cos\varphi & 0 \\ \omega_{21} & 0 & 0 \end{vmatrix}} = \begin{vmatrix} i & j & k \\ -\omega_{10} S\varphi & \omega_{10} C\varphi & 0 \\ -\omega_{10} S\varphi + \omega_{21} & \omega_{10} C\varphi & 0 \end{vmatrix}$$

$$\underline{\omega}_2 = \underline{\omega}_{10} + \underline{\omega}_{21} = \begin{bmatrix} -\omega_{10}\sin\varphi \\ \omega_{10}\cos\varphi \\ 0 \end{bmatrix} + \begin{bmatrix} \omega_{21} \\ 0 \\ 0 \end{bmatrix}$$

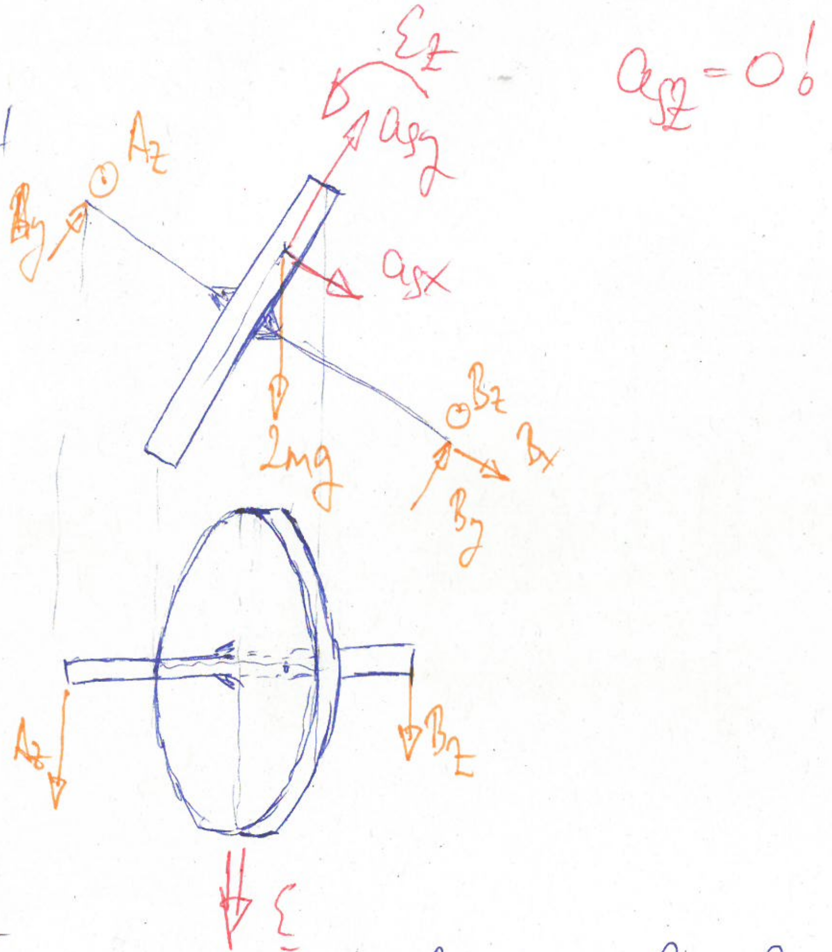
$$\underline{\varepsilon}_2 = \begin{bmatrix} 0 \\ 0 \\ -\omega_{10}\omega_{21} C\varphi \end{bmatrix}$$

$$\underline{a}_S = R\begin{bmatrix} \omega_{10}\omega_{21}\cos\varphi - \frac{1}{2}\omega_{10}^2 \sin\varphi\cos\varphi \\ -\frac{1}{2}(\omega_{21} - \omega_{10}\sin\varphi)^2 \\ 0 \end{bmatrix} = \begin{bmatrix} 17.32 \\ -8 \\ 0 \end{bmatrix} \left[m/s^2 \right]$$

2)
$$\underline{\underline{\Theta}}_C = \begin{bmatrix} \frac{1}{2}mR^2 & 0 & 0 \\ 0 & \frac{1}{4}mR^2 & 0 \\ 0 & 0 & \frac{1}{4}mR^2 \end{bmatrix} + \begin{bmatrix} mR^2 & 0 & 0 \\ \cdot & 0 & 0 \\ \cdot & \cdot & mR^2 \end{bmatrix} = \begin{bmatrix} \frac{3}{2}mR^2 & 0 & 0 \\ \cdot & \frac{1}{4}mR^2 & 0 \\ \cdot & \cdot & \frac{5}{4}mR^2 \end{bmatrix}$$

3) $\underline{\Pi}_C = \underline{\underline{\Theta}}_C \circ \underline{\omega}_2 = mR^2 \begin{bmatrix} \frac{3}{2}(\omega_{21} - \omega_{10}\sin\varphi) \\ \frac{1}{4}\omega_{10}\cos\varphi \\ 0 \end{bmatrix}$

4)



5)

$\underline{\dot{I}} = \underline{F}$

$x: 2m\, a_{Sx} = B_x + 2mg\sin\varphi$

$y: 2m\, a_{Sy} = B_y + A_y - 2mg\cos\varphi$

$z: 2m\, 0 = A_z + B_z$

$\underline{\dot{\Pi}}_C = \underline{M}_C \;;\; \underline{\dot{\Pi}}_C = \underline{\underline{\Theta}}_C \cdot \underline{\varepsilon} + \underline{\omega}_2 \times \underline{\Pi}_C = \begin{bmatrix} 0 \\ 0 \\ -\frac{5}{4} mR^2 \omega_{10}\omega_{21}\cos\varphi \end{bmatrix} + \begin{bmatrix} \omega_{21} - \omega_{10}\sin\varphi \\ \omega_{10}\cos\varphi \\ 0 \end{bmatrix} \times$

14/04

$$\times \begin{bmatrix} \frac{3}{2} m R^2 (\omega_{21} - \omega_{10} \sin\varphi) \\ \frac{1}{4} m R^2 \omega_{10} \cos\varphi \\ 0 \end{bmatrix} = \frac{5}{4} m R^2 \begin{bmatrix} 0 \\ 0 \\ -2\omega_{10}\omega_{21}\cos\varphi + \omega_{10}^2 \sin\varphi\cos\varphi \end{bmatrix}$$

$$\underline{M}_C = \underline{r}_{CS} \times \underline{G} + \underline{r}_{CA} \times \underline{A} + \underline{r}_{CB} \times \underline{B}, \quad \underline{G} = \begin{bmatrix} 2mg\sin\varphi \\ -2mg\cos\varphi \\ 0 \end{bmatrix}, \quad \underline{r}_{CA} = -\underline{r}_{CB} = \begin{bmatrix} b/\cos\varphi \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{A} = \begin{bmatrix} 0 \\ A_y \\ A_z \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

$$\underline{M}_C = \begin{bmatrix} 0 \\ b/\cos\varphi \, (A_z - B_z) \\ -mgR\sin\varphi + b/\cos\varphi \, (B_y - A_y) \end{bmatrix}$$

$$0 = 0$$

$$0 = b/\cos\varphi \, (A_z - B_z)$$

$$\frac{5}{4} m R^2 (\omega_{10}^2 \sin\varphi\cos\varphi - 2\omega_{10}\omega_{21}\cos\varphi) = -mgR\sin\varphi + b/\cos\varphi \, (B_y - A_y)$$