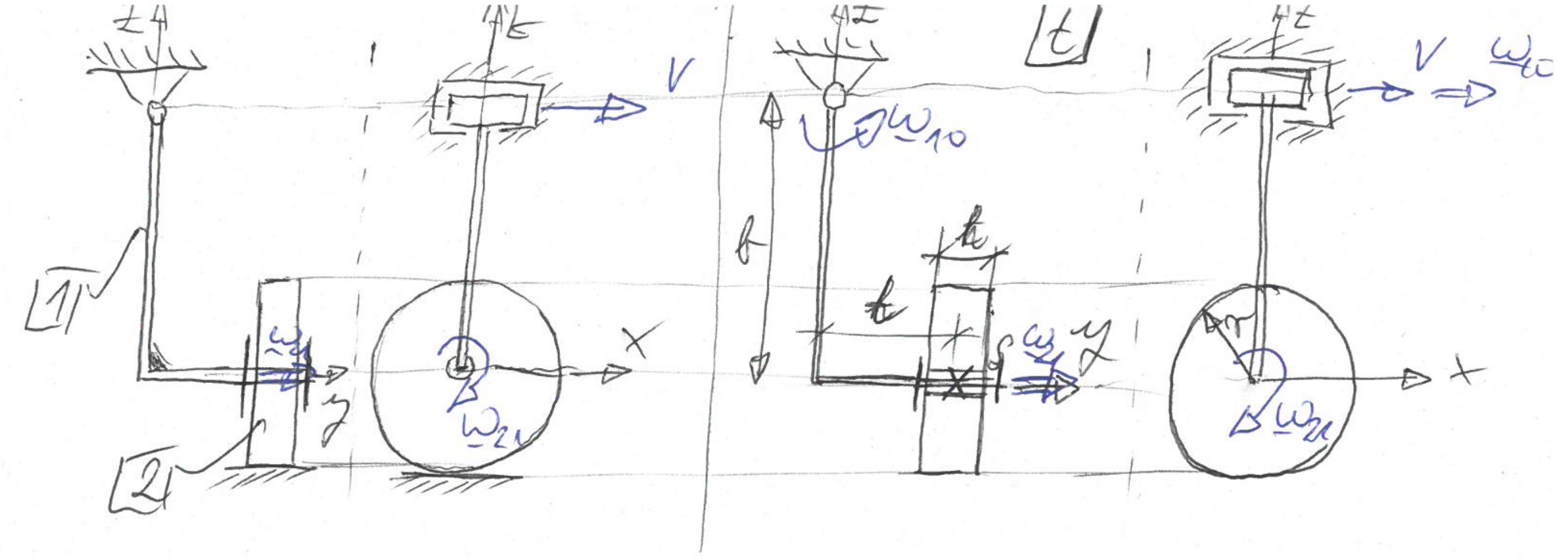


$V, \omega_{10}, m, r, t, h, b$ adott, $1,\ \underline{\underline{\Theta}}_S,\ \underline{\Pi}_S \,|\, 2,\ \underline{\underline{\Theta}}_B,\ \underline{\Pi}_B \,|\, 3,\ T$

$\underline{1,}$ $\underline{\underline{\Theta}}_S$, $y \rightarrow$ forgástengely . $\Theta_{S,y} = \frac{1}{2} m r^2$

$$\underline{\underline{\Theta}}_S = \begin{bmatrix} \Theta_{S,x} & 0 & 0 \\ 0 & \Theta_{S,y} & 0 \\ 0 & 0 & \Theta_{S,z} \end{bmatrix} \qquad \Theta_{S,x} = \Theta_{S,z} = \frac{1}{4} m r^2 + \frac{1}{2} m \ell^2$$

$\underline{\Pi}_S = \underline{\underline{\Theta}}_S \cdot \underline{\omega}_{20}$, $\underline{\omega}_{20} = \underline{\omega}_{10} + \underline{\omega}_{21}$; $\underline{\omega}_{10} = \begin{bmatrix} \omega_{10} \\ 0 \\ 0 \end{bmatrix}$, $\underline{\omega}_{21} = \begin{bmatrix} 0 \\ \omega_{21} \\ 0 \end{bmatrix}$

$\omega_{21} = \frac{V}{r}$, $\underline{\omega}_{20} = \begin{bmatrix} \omega_{10} \\ V/r \\ 0 \end{bmatrix}$, $\underline{\Pi}_S = \cancel{\Theta_{S,x} \cdot \omega_{10} + \Theta_{S,y} \cdot \frac{V}{r}}$ $\begin{bmatrix} \Theta_{S,x} \cdot \omega_{10} \\ \Theta_{S,y} \cdot V/r \\ 0 \end{bmatrix}$

2) $\underline{\underline{\Theta}}_B = \underline{\underline{\Theta}}_S + \underline{\underline{\Theta}}_{BS}$, $\underline{\underline{\Theta}}_{BS} = \left[(\underline{r}_{BS}^T \underline{r}_{BS}) \underline{\underline{I}} - \underline{r}_{BS} \underline{r}_{BS}^T \right] m$

$\underline{r}_{BS} = \begin{bmatrix} x_{BS} \\ y_{BS} \\ z_{BS} \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ -b \end{bmatrix}$, $\underline{\underline{\Theta}}_{BS} = m \begin{bmatrix} t^2+b^2 & 0 & 0 \\ \cdot & b^2 & tb \\ \cdot & \cdot & t^2 \end{bmatrix}$, ! $\underline{\underline{\Theta}}_{BS} = \underline{\underline{\Theta}}_{SB}$!

$\underline{\Pi}_B = \underline{\underline{\Theta}}_B \cdot \underline{\omega}_{20} + \underline{r}_{BS} \times (m \underline{v}_{B2})$

$\underline{\Pi}_B = \cancel{\underline{\Pi}_S} + \cancel{\underline{r}_{BS}} \times \underline{I}_{S2}$, $\underline{I}_{S2} = m \underline{v}_{S2}$

a) $\underline{v}_{B1} = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix}$, $B1 \xrightarrow{[1]} S1 \xrightarrow{=} S2 \xrightarrow{[2]} B2$

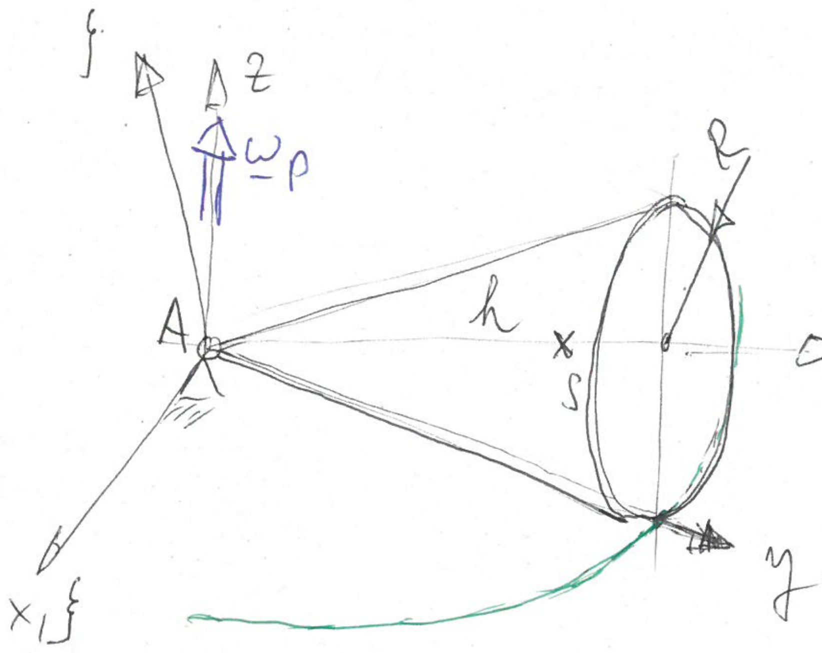
HF...

b) $\underline{v}_{S1} = \underline{v}_{B1} + \underline{\omega}_{10} \times \underline{r}_{BS} = \begin{bmatrix} v \\ \omega_{10} b \\ \omega_{10} t \end{bmatrix}$, $\boxed{\underline{v}_{S1} = \underline{v}_{S2}}$

$\underline{r}_{BS} \times (m \underline{v}_{B2}) = m \begin{vmatrix} i & j & k \\ 0 & t & -b \\ v & \omega_{10} b & \omega_{10} t \end{vmatrix} = m \begin{bmatrix} \omega_{10}(t^2+b^2) \\ -bv \\ -tv \end{bmatrix}$

$\underline{\Pi}_B = \begin{bmatrix} \omega_{10}(\Theta_{S,x} \omega_{10} + m(t^2+b^2)) \\ v(\Theta_{S,y}/r - mb) \\ -mtv \end{bmatrix}$

$T = \frac{1}{2} m v_S^2 + \frac{1}{2} \underline{\omega}_{20}^T \overbrace{\underline{\underline{\Theta}}_S \underline{\omega}_{20}}^{\Pi_S} = \frac{1}{2} m \left(v^2 + \omega_{10}^2 (b^2 + t^2) \right) + \frac{1}{2} \left(\Theta_{S,x} \omega_{10}^2 + \Theta_{S,y} \frac{v^2}{r^2} \right)$



$m = 6\,[kg]$

$R = 0.2\,[m]$

$h = 0.3\,[m]$

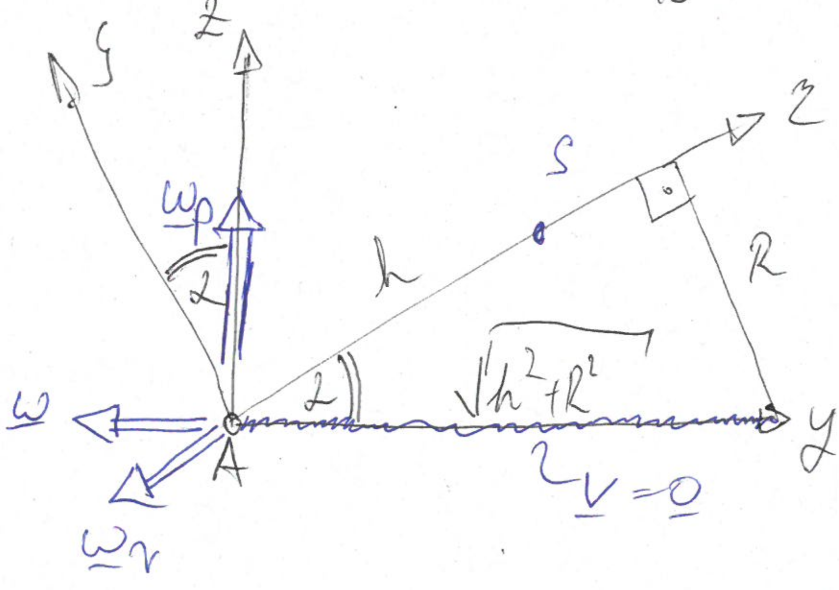
$\omega_p = 2\,[rad/s]$

1, $T = ?$, 2, Π_S, 3, Π_A

$$T = \frac{1}{2} m v_S^2 + \frac{1}{2} \underline{\omega}^T \underline{\underline{\Theta}}_S \underline{\omega} = \frac{1}{2} \underline{\omega}^T \underline{\underline{\Theta}}_A \underline{\omega}$$ Álló pontra!

$$\underset{(\xi,\eta,\zeta)}{\underline{\underline{\Theta}}_A} = \begin{bmatrix} \Theta_\zeta & 0 & 0 \\ \cdot & \Theta_\eta & 0 \\ \cdot & \cdot & \Theta_\zeta \end{bmatrix}, \quad \Theta_\zeta = \Theta_\xi = \cancel{\frac{3}{80} m(4R^2 + h^2) + m\left(\frac{3}{4}h\right)^2}$$

$$\Theta_\eta = \frac{3}{10} m R^2 \qquad \boxed{\Theta_\zeta = \Theta_\xi = \frac{3}{20}(4h^2 + R^2)\, m}$$



$$\underset{(\xi,\eta,\zeta)}{\underline{\omega}_p} = \begin{bmatrix} 0 \\ \omega_p \sin\alpha \\ \omega_p \cos\alpha \end{bmatrix}$$

$$\underset{(\xi,\eta,\zeta)}{\underline{\omega}_r} = \begin{bmatrix} 0 \\ -\omega_r \\ 0 \end{bmatrix}, \quad \underset{(\xi,\eta,\zeta)}{\underline{\omega}} = \begin{bmatrix} 0 \\ -\omega \cos\alpha \\ \omega \sin\alpha \end{bmatrix}, \quad \underline{\omega} = \underline{\omega}_r + \underline{\omega}_p$$

$\xi: \ \omega = \omega_p \frac{\cos\alpha}{\sin\alpha} = \omega_p \frac{h}{R} \Rightarrow \underset{(\xi,\eta,\zeta)}{\underline{\omega}} = \omega_p \frac{h}{R\sqrt{R^2+h^2}} \begin{bmatrix} -R \\ -h \\ R \end{bmatrix} \begin{matrix} \\ \leftarrow \omega_\eta \\ \leftarrow \omega_\zeta \end{matrix}$

$T = \frac{1}{2} \underline{\omega}^T \underline{\underline{\Theta}}_A \underline{\omega} = \frac{1}{2}\left(\Theta_\eta \omega_\eta^2 + \Theta_\zeta \cdot \omega_\zeta^2\right) = \cancel{2.35}\,[J]$

$0.723\,[J]$

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3) $\underline{\Pi}_A = \underline{\underline{\Theta}}_A \underline{\omega} + r_{AS} \times (m \underbrace{V_A}_{=0}) = \underline{\underline{\Theta}}_A \underline{\omega} = \begin{bmatrix} 0 \\ \Theta_\eta \omega_\eta \\ \Theta_\zeta \omega_\zeta \end{bmatrix} = \begin{bmatrix} 0 \\ -0.1797 \\ 0.5990 \end{bmatrix} \left[\frac{kgm^2}{s}\right]$

$[\underline{\underline{\Theta}}_A] = 1\, kgm^2$

4) $\underline{\Pi}_S = \underline{\underline{\Theta}}_S \cdot \underline{\omega}$, $\underline{\underline{\Theta}}_A = \underline{\underline{\Theta}}_S + \underline{\underline{\Theta}}_{AS}$, $r_{AS} = \begin{bmatrix} 0 \\ \frac{3}{4}h \\ 0 \end{bmatrix}$

$\underline{\underline{\Theta}}_{AS} = m\left(\frac{3}{4}h\right)^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightsquigarrow \underline{\underline{\Theta}}_S = \underline{\underline{\Theta}}_A - \underline{\underline{\Theta}}_{AS}$

$\underline{\Pi}_S = \begin{bmatrix} 0 \\ -0.1797 \\ 0.0936 \end{bmatrix} [kgm^2/s]$