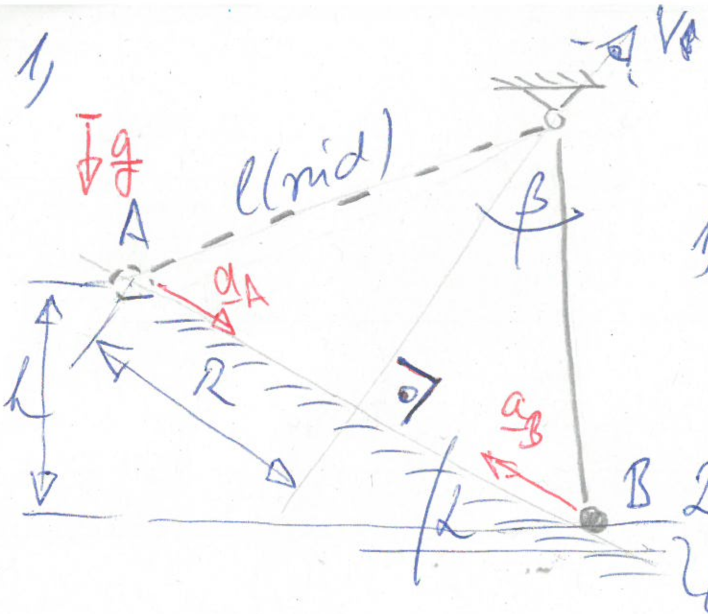
1,

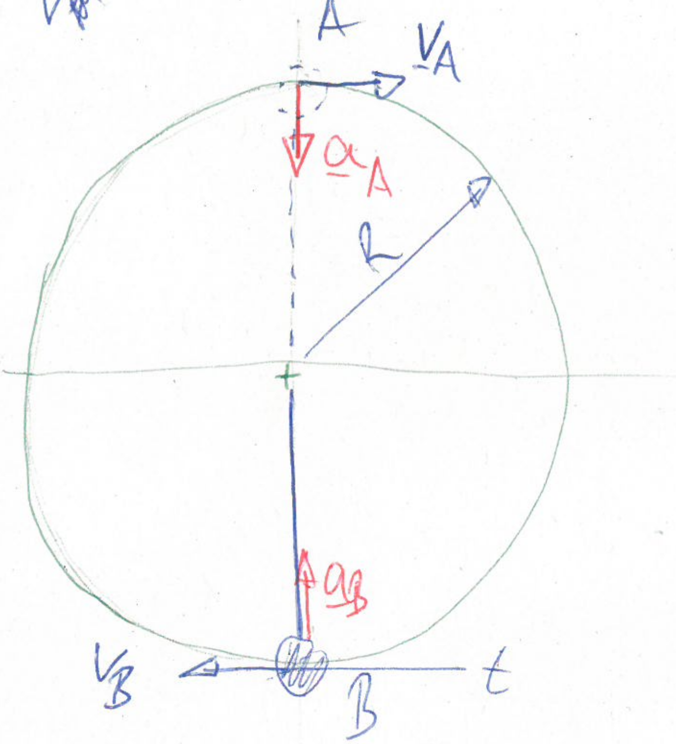



$m, l, \alpha, \beta$, ($\mu = 0$, sima felület)

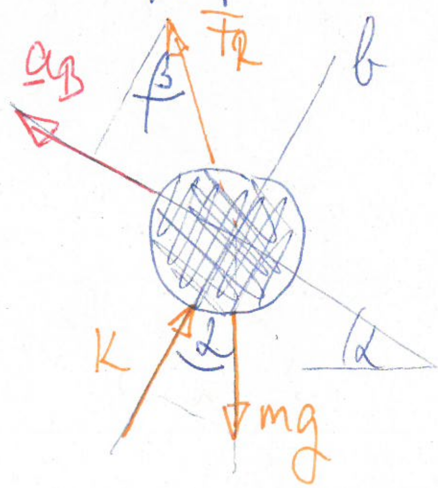
1, $V_A = ?$, hogy az „m" tömeg ne váljon el a kényszertől a B pontban?

2, Kényszererő az A pontban?

V⌀




1, SZTA a $\underline{B}$ pontban




éppen nem válik el $K = 0$!

$m\underline{a} = \underline{F}$

t: $m a_{Bt} = 0 \to a_{Bt} = 0$ max sebesség

(1) n: $m a_{Bn} = F_R \cancel{\cos}\sin\beta - mg\sin\alpha$

(2) b: $m0 = F_R\cos\beta - mg\cos\alpha + \cancel{K}$

$a_{Bn} = \dfrac{V_B^2}{R}$, $R = l\sin\beta$

$V_A$ munkatétellel számítjuk

$A: t_1, \quad B: t_2$

$$T_2 - T_1 = W_{12} = \int_{t_1}^{t_2} \underline{F}\,\underline{v}\,dt \qquad \underline{F} = m\underline{g} + \underline{K} + \underline{F}_R$$

$$\underline{K}\cdot\underline{v} = 0, \ \underline{F}_R\cdot\underline{v} = 0 \quad \text{ideális kényszer}$$

$$T_2 - T_1 = W_{12}^{g} = -(U_2 - U_1)$$

$$\underline{G} = m\underline{g} = -\frac{\partial U_g}{\partial \underline{r}}, \ U_g = mgz$$

$$\frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = -(0 - 2l\sin\beta\sin\alpha\, mg)$$

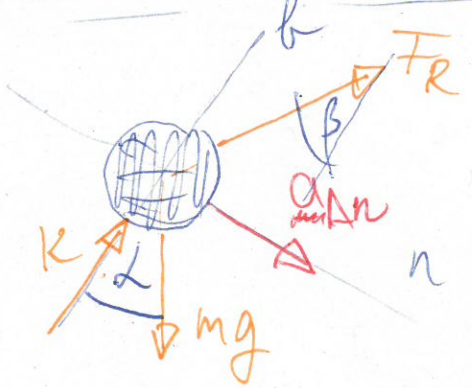
$$v_A^2 = v_B^2 - 4gl\sin\beta\sin\alpha \quad (3)$$

$$F_R = mg\frac{\cos\alpha}{\cos\beta}, \quad a_{Bn} = g\left(\frac{\cos\alpha\sin\beta}{\cos\beta} - \sin\alpha\right)$$

$$v_B^2 = R\,a_{Bn}$$

$$\boxed{v_A^2 = gl\sin\beta(\cos\alpha\,\mathrm{tg}\beta - 5\sin\alpha)}$$

2) SZTA az $\underline{A}$ pontban




$$m\underline{a} = \underline{F}$$

$$t: m\, a_{At} = 0 \rightarrow v_A \text{ minimális}$$

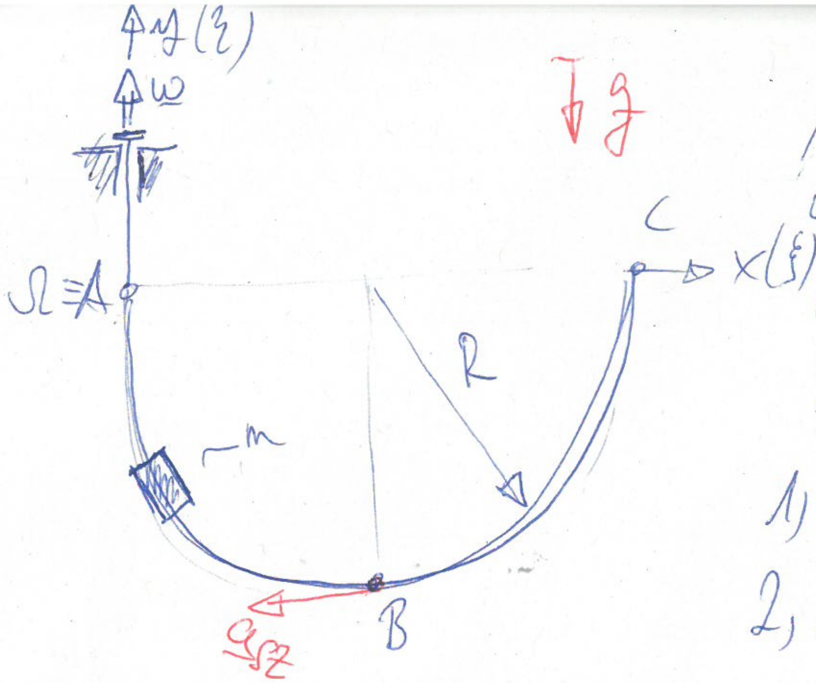
$$(1)\, n: m a_{An} = F_R\sin\beta + mg\sin\alpha$$

$$(2)\, b: m 0 = F_R\cos\beta - mg\cos\alpha + K$$

$$(3)\, a_{An} = \frac{v_A^2}{R} \qquad v_A \text{ ismert}$$

$$(3) \rightarrow (1) \rightarrow F_R = \dots \ mg\left(\frac{\cos\alpha}{\cos\beta} - 6\frac{\sin\alpha}{\sin\beta}\right)$$

$$F_R \rightarrow (2) \rightarrow K = \frac{6\sin\alpha}{\mathrm{tg}\beta}\, mg$$




$\mu=0$, $m=0.2\,kg$, $R=0.1\,m$

$\omega=3.14\ rad/s$

m csúszik A pontból, álló helyzetben

1, B pontban a kényszererő?

2, $\underline{v}_C=?$

$\underline{\omega}=\begin{bmatrix}0\\0\\\omega\end{bmatrix}$

álló koord. $\{x, y, z, A\}$

mozgó $\{\xi, \eta, \zeta, \Omega\}$ (korong**gal**)

dinamika alaptétele B pontra

$\underline{\dot{I}}=\underline{F}$ $\quad m\underline{a}=\underline{F}$, $\quad \underline{a}=\underline{\alpha}+\underline{a}_{sz}+\underline{a}_{cor}$

$m\underline{\alpha}+m\underline{a}_{sz}+m\underline{a}_{cor}=\underline{F} \Rightarrow m\underline{\alpha}=\underline{F}+\underline{F}_{sz}+\underline{F}_{cor}=\underline{F}_{rel}$

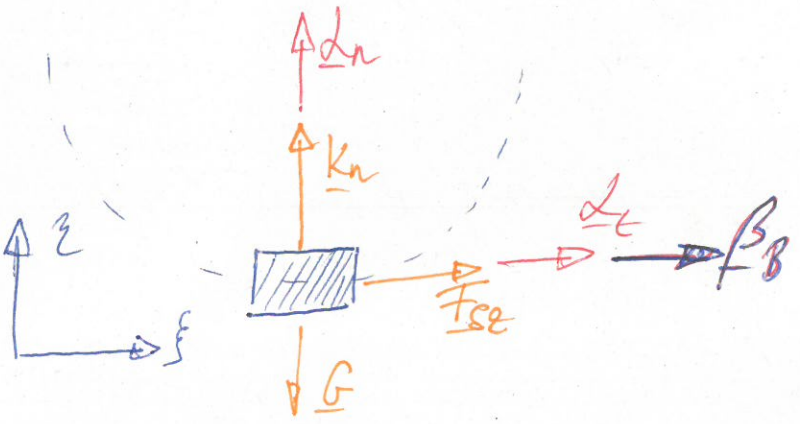
$-\underline{F}_{sz}$ $\quad -\underline{F}_{cor}$ $\quad \underline{K}+\underline{G}$

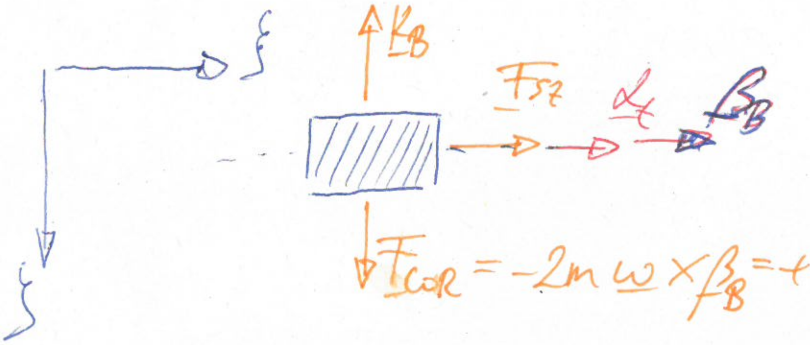
$\underline{a}_{sz}=\underline{a}_{\Omega}+\underline{\varepsilon}\times\underline{\rho}_{\Omega B}+\underline{\omega}\times(\underline{\omega}\times\underline{\rho}_{\Omega B})=\begin{bmatrix}-R\omega^2\\0\\0\end{bmatrix}$, $\quad \underline{a}_{\Omega}=\underline{0}$, $\underline{\varepsilon}=\underline{0}$

$\underline{a}_{cor}=2\underline{\omega}\times\underline{\beta}_B$

$\underline{v}_B=\underline{v}_{sz}+\underline{\beta}_B$, $\quad \underline{v}_{sz}=\underline{v}_{\Omega}+(\underline{\omega}\times\underline{\rho}_{\Omega B})=\begin{bmatrix}0\\0\\-R\omega\end{bmatrix}$, $\quad \underline{v}_{\Omega}=\underline{0}$, $\quad \underline{\beta}_B=\begin{bmatrix}\beta_B\\0\\0\end{bmatrix}$

$\beta_B$ munkatétel segítségével! . . .




$F_{COR} = -2m\,\omega\times\beta_B = +2m\,\omega\beta_B\,k$

munkatétel (relatív)

$$\frac{1}{2}m\beta_B^2 - \frac{1}{2}m\beta_A^2 = \int_{t_1}^{t_2} F_{REL}\,\beta\,dt = \int_{t_1}^{t_2}\left(F\beta + F_{SZ}\beta + F_{COR}\beta\right)dt$$

$$F = K + G \qquad K\circ\beta = 0,\ K\perp\beta,\qquad G = -\frac{\partial U_g}{\partial \rho} = -\frac{mg\zeta}{\partial\rho} \qquad \rho = \begin{bmatrix}\xi\\ \eta\\ \zeta\end{bmatrix}$$

$$-\left(U_g(r_B) - U_g(r_A)\right)$$

$$\int_{t_1}^{t_2} F\cdot\beta\,dt = \int_{r_A}^{r_B} -dU_g = U_g(r_A) - U_g(r_B) = mgR - 0$$

$$\int_{t_1}^{t_2} F_{SZ}\cdot\beta\,dt = \int_{r_A}^{r_B} -dU_{SZ} = \frac{1}{2}m\omega^2R^2 - 0$$

$$F_{SZ} = -m\,a_{SZ} = \begin{bmatrix} m\omega^2 R\\ 0\\ 0\end{bmatrix} \to U_{SZ} = -\frac{1}{2}m\omega^2\xi^2$$

# Coriolis erő munkája

1, $\underline{F}_{cor} \cdot \beta = (2\underline{\omega} \times \beta) \cdot \beta = 0$ !

$\beta_A = \phi$,

$\frac{1}{2} m \beta_B^2 = mgR + \frac{1}{2} m\omega^2 R^2 \Rightarrow \beta_B = \sqrt{2gR + \omega^2 R} = 1.528 [m/s]$

dinamika alaptétele

$m\underline{a} = \underline{F} + \underline{F}_{sz} + \underline{F}_{cor} = \underline{K} + \underline{G} + m \begin{bmatrix} R\omega^2 \\ 0 \\ 0 \end{bmatrix} + 2m\omega\beta_B \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\underline{K} = \begin{bmatrix} 0 \\ K_n \\ -K_b \end{bmatrix}$

$\xi$: $m a_t = mR\omega^2 \rightarrow a_t = R\omega^2 = 3.77 [m/s^2]$

$\xi$: $m a_n = -mg + K_n \qquad a_n = \frac{\beta_B^2}{R}$

$\xi$: $0 = -K_b + 2m\omega\beta_B \rightarrow K_b = 3.76 [N]$, $K_n = 6.64 [N]$

2) munkatétel relatív rendszerben A & C pont között

$\frac{1}{2} m \beta_C^2 - \frac{1}{2} m \beta_A^2 = \int_{t_A}^{t_C} \underline{F}\beta + \underline{F}_{sz}\beta + \underline{F}_{cor}\beta \, dt$

• valódi erők munkája $U_g(r_A) - U_g(r_C) = 0$ azonos pot. szinten

$= (-0 + (-\frac{1}{2} m\omega^2 (2R)^2))$

$\beta_C = 2R\omega = 1.228 [m/s]$

$\underline{v}_C = \beta_C + v_{Csz} = \begin{bmatrix} 0 \\ 2R\omega \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -2R\omega \end{bmatrix} = 2R\omega \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, $\underline{v}_C = \begin{bmatrix} 0 \\ 1.228 \\ -1.228 \end{bmatrix} [m/s]$, $v_C = 1.737 [m/s]$