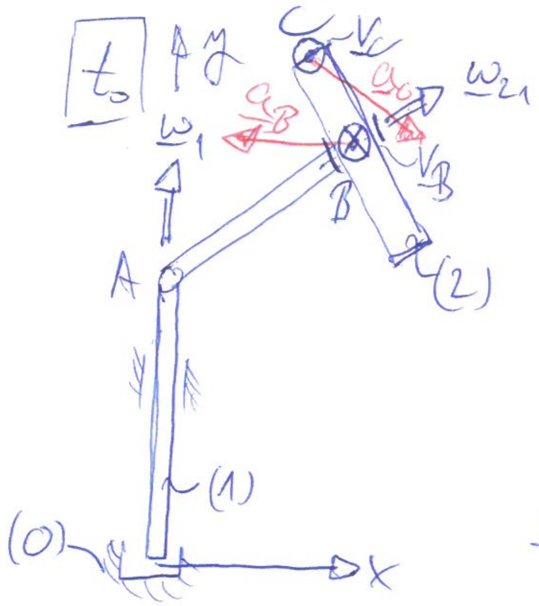
robotkar mozgásának vizsgálata



$$\underline{\omega}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \left[\frac{rad}{s}\right], \quad |\underline{\omega}_{21}| = 2 \left[\frac{rad}{s}\right] = \text{áll.}$$

$$\underline{r}_{AB}(t_0) = \begin{bmatrix} 0.8 \\ 0.6 \\ 0 \end{bmatrix} [m], \quad \underline{r}_{BC}(t_0) = \begin{bmatrix} -0.3 \\ 0.4 \\ 0 \end{bmatrix} [m]$$

Kérdés:

a) $\underline{v}_C = ?$ t_0-ban

b) $\underline{a}_C = ?$ t_0-ban

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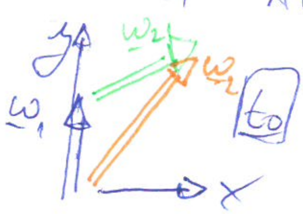
(2)-es test pontjaira: $\underline{v}_C = \underline{v}_B + \underline{\omega}_2 \times \underline{r}_{BC}$

$$\underline{v}_B = \underline{v}_A + \underline{\omega}_1 \times \underline{r}_{AB}$$

$$\underline{v}_B = \underline{0} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 1 & 0 \\ 0.8 & 0.6 & 0 \end{vmatrix} = \begin{bmatrix} +(0-0) \\ -(0-0) \\ +(0-0.8) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -0.8 \end{bmatrix} \left[\frac{m}{s}\right]$$

$$\underline{\omega}_2 = \underline{\omega}_1 + \underline{\omega}_{21}, \quad \underline{\omega}_{21} = \omega_{21} \cdot \underline{e}_{AB} = \omega_{21} \cdot \underline{r}_{AB} =$$

$$\underline{\omega}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1.6 \\ 1.2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.6 \\ 2.2 \\ 0 \end{bmatrix} \left[\frac{rad}{s}\right]$$



$$\underline{v}_C = \begin{bmatrix} 0 \\ 0 \\ -0.8 \end{bmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.6 & 2.2 & 0 \\ -0.3 & 0.4 & 0 \end{vmatrix} = \begin{bmatrix} +(0-0) \\ -(0-0) \\ +(1.6 \cdot 0.4 - 2.2 \cdot (-0.3)) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -0.8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix} \left[\frac{m}{s}\right]$$

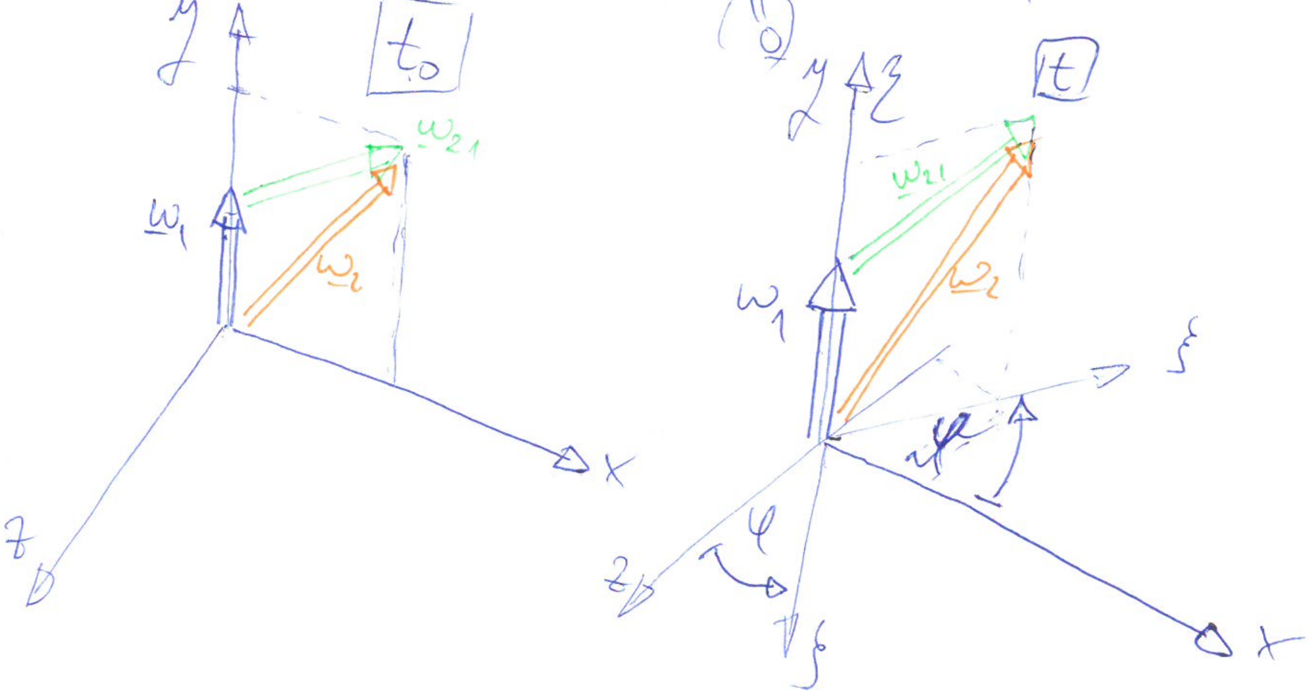
$\underline{a}_C$

(2)-es test $\quad \underline{a}_C = \underline{a}_B + \underline{\varepsilon}_2 \times \underline{r}_{BC} + \underline{\omega}_2 \times (\underline{\omega}_2 \times \underline{r}_{BC})$

(1)-es test $\quad \underline{a}_B = \underline{a}_A + \underline{\varepsilon}_1 \times \underline{r}_{AB} + \underline{\omega}_1 \times (\underline{\omega}_1 \times \underline{r}_{AB})$, $\underline{\omega}_1 =$ áll!

$$\underline{a}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 1 & 0 \\ 0 & 0 & -0.8 \end{vmatrix} = \begin{bmatrix} +(-0.8-0) \\ -(0-0) \\ +(0-0) \end{bmatrix} = \begin{bmatrix} -0.8 \\ 0 \\ 0 \end{bmatrix} \left[\frac{m}{s^2}\right] \quad \underline{a}_A = \underline{0}$$

$$\underline{\omega}_2 = \underline{\omega}_1 + \underline{\omega}_{21} \rightarrow \underline{\varepsilon}_2 = \underline{\dot{\omega}}_2 = \underbrace{\underline{\dot{\omega}}_1}_{(=0)} + \underline{\dot{\omega}}_{21}$$, $|\underline{\omega}_{21}| =$ áll, de az iránya!



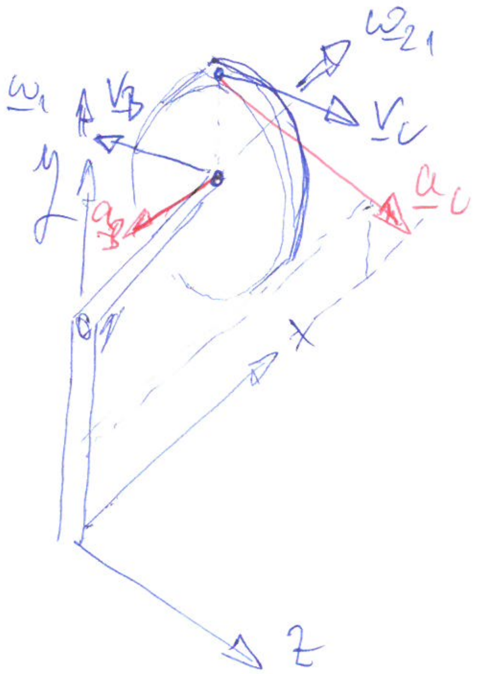
$\omega_{2\zeta} =$ állandó!, $\quad \omega_{2x} = \omega_{2\xi} = 1.6 \left[\frac{rad}{s}\right]$

$$\underline{\omega}_2(t)_{(x,y,z)} = \begin{bmatrix} \omega_{2\xi} \cdot \cos(\varphi(t)) \\ \omega_{2\zeta} \\ -\omega_{2\xi} \sin(\varphi(t)) \end{bmatrix} \qquad \varphi(t) = \omega_1 \cdot t$$

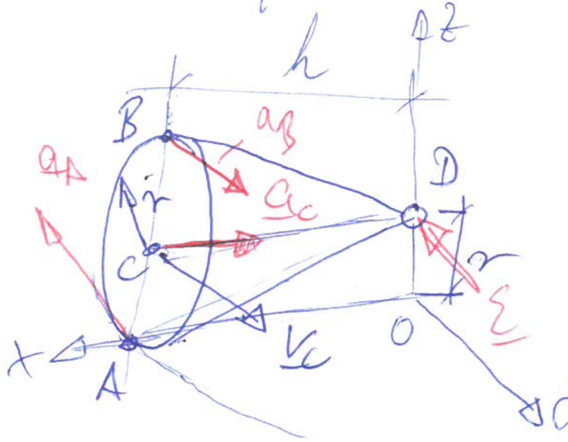
$$\underline{\dot{\omega}}_2(t) = \omega_{2\xi} \cdot \omega_1 \begin{bmatrix} -\sin(\omega_1 t) \\ 0 \\ -\cos(\omega_1 t) \end{bmatrix} \qquad \underline{\varepsilon}_2(t_0 = 0) = -\omega_{2\xi} \cdot \omega_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = -1.6\underline{k} \left[\frac{rad}{s^2}\right]$$

$$\underline{a}_C = \underline{a}_B + \underline{\varepsilon}_2 \times \underline{r}_{BC} + \underline{\omega}_2 \times (\underline{\omega}_2 \times \underline{r}_{BC})$$

$$\underline{a}_C = \begin{bmatrix} -0.8 \\ 0 \\ 0 \end{bmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & -1.6 \\ -0.3 & 0.4 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.6 & 2.2 & 0 \\ 0 & 0 & 1.3 \end{vmatrix} = \begin{bmatrix} 2.7 \\ -1.6 \\ 0 \end{bmatrix} \left[\frac{m}{s^2}\right]$$



Gördülő kúp



$r = 2[m]$, $h = 4[m]$, $|\underline{v}_C| = v_C = 10[\frac{m}{s}]$

t_0-ban kúp tengelye ∥ x-tengellyel

a) $\underline{\omega} = ?$ $\underline{\varepsilon} = ?$ t_0-ban

b) $\underline{v}_B = ?$, $\underline{a}_B = ?$

c) $\underline{a}_A = ?$

a) $\underline{v}_A = 0$, $\underline{v}_D = 0$ $\overline{AD}$ pillanatnyi forgástengely

$$\underline{\omega} = \omega \cdot \underline{e}_{AD}$$

$$\underline{v}_C = \underline{v}_D + \underline{\omega} \times \underline{r}_{DC} = \underline{0} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ h & 0 & 0 \end{vmatrix} = \begin{bmatrix} +0 \\ +h\omega_z \\ -h\omega_y \end{bmatrix} = \begin{bmatrix} 0 \\ v_C \\ 0 \end{bmatrix}$$

$$\boxed{\omega_y = 0 [\frac{rad}{s}]} \qquad \boxed{\omega_z = \frac{v_C}{h} = 2.5 [\frac{rad}{s}]}$$

$$\underline{v}_C = \underline{v}_A + \underline{\omega} \times \underline{r}_{AC} = \underline{0} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & 0 & 2.5 \\ 0 & 0 & r \end{vmatrix} = \begin{bmatrix} 0 \\ -\omega_x r \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ v_c \\ 0 \end{bmatrix}$$

$$\boxed{\omega_x = -\frac{v_c}{r} = -5 \frac{rad}{s}}$$

$$\underline{\omega} = \begin{bmatrix} -5 \\ 0 \\ 2.5 \end{bmatrix} [\frac{rad}{s}]$$

$$\underline{a}_C = \underset{\underset{0}{\parallel}}{\underline{a}_D} + \underline{\varepsilon} \times \underline{r}_{DC} + \underline{\omega} \times (\underbrace{\underline{\omega} \times \underline{r}_{DC}}_{\underline{V}_C}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \varepsilon_x & \varepsilon_y & \varepsilon_z \\ h & 0 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & 0 & \omega_z \\ \cancel{0} & \cancel{V_C} & 0 \end{vmatrix} =$$

$$= \begin{bmatrix} 0 \\ \varepsilon_z h \\ -\varepsilon_y h \end{bmatrix} + \begin{bmatrix} -\omega_z V_C \\ 0 \\ \omega_x V_C \end{bmatrix} = \begin{bmatrix} -\frac{V_C^2}{h} \\ 0 \\ 0 \end{bmatrix}$$

$\cancel{\omega_z = \frac{V_C}{h}} \Rightarrow$ $\boxed{\varepsilon_z = 0 \left[\frac{rad}{s^2}\right]}$

$$\boxed{\varepsilon_y = \frac{\omega_x V_C}{h} = -12.5 \left[\frac{rad}{s^2}\right]}$$

$$\underline{a}_A = \underset{\underset{0}{\parallel}}{\underline{a}_D} + \underline{\varepsilon} \times \underline{r}_{DA} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{DA}) = \underline{0} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \varepsilon_x & \varepsilon_y & \cancel{\varepsilon_z} \\ h & 0 & -r \end{vmatrix} + \underline{0} =$$

$$= \begin{bmatrix} -\varepsilon_y r \\ -(-r\varepsilon_x - h\cancel{\varepsilon_z}) \\ -h\varepsilon_y \end{bmatrix} = \begin{bmatrix} a_{Ax} \\ 0 \\ a_{Ay} \end{bmatrix} \Rightarrow a_{Ax} = 25 \left[\frac{m}{s^2}\right]$$

$a_{Ay} = 0 \left[\frac{m}{s^2}\right]$

$\varepsilon_x = 0 \left[\frac{rad}{s^2}\right]$

$a_{Ay} = 50 \left[\frac{m}{s^2}\right]$

$$\underline{a}_A = \begin{bmatrix} 25 \\ 0 \\ 50 \end{bmatrix} \left[\frac{m}{s^2}\right]$$

$$\underline{\varepsilon} = \begin{bmatrix} 0 \\ -12.5 \\ 0 \end{bmatrix} \left[\frac{rad}{s^2}\right]$$

$$\underline{v}_B = \underbrace{\underline{v}_A}_{0} + \underline{\omega} \times \underline{r}_{AB} = \underline{\omega} \times (2\underline{r}_{AC}) = 2\underline{v}_C = \begin{bmatrix} 0 \\ 20 \\ 0 \end{bmatrix} \left[\frac{m}{s}\right]$$

$$\underline{a}_B = \underline{a}_C + \underline{\varepsilon} \times \underline{r}_{CB} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{CB}) = \begin{bmatrix} -v_C^2/h \\ 0 \\ 0 \end{bmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & r \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & 0 & \omega_z \\ 0 & v_C & 0 \end{vmatrix} =$$

$$= \begin{bmatrix} -75 \\ 0 \\ -80 \end{bmatrix} \left[m/s^2 \right]$$