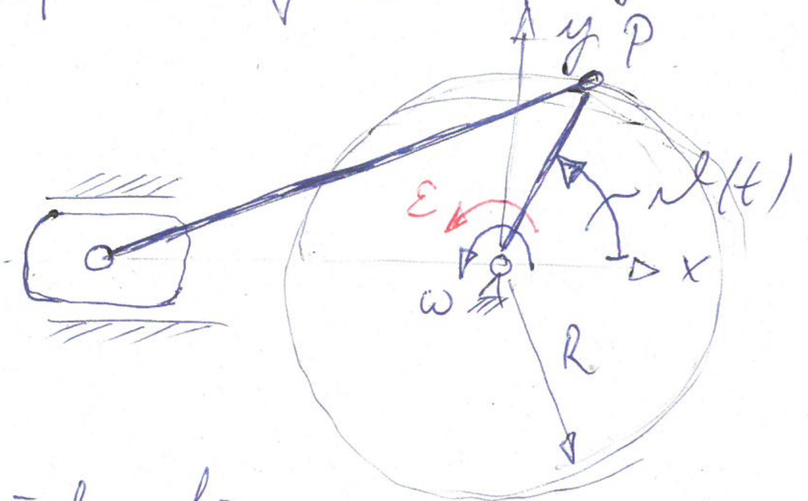
Anyagi pont mozgása körpályán (P)



forgattyús tengely

dekopírfűrész (jigsaw)

reciprocating pump

pályagörbe: kör

$t_0 = 0\,[s]$, $t_1 = 7\,[s]$, $\vartheta_0 = \vartheta(t_0) = 30\,[°]$, $\omega_0 = \omega(t_0) = \dot{\vartheta}(t_0) = 10\,[\frac{rad}{s}]$

$\ddot{\vartheta}(t) = \varepsilon(t) = -2\,[\frac{rad}{s^2}]$

a, $\vartheta(t_1) = ?$, b, milyen a mozgás iránya t_1-ben? $\mathrm{sgn}(\omega(t_1))$

c, $a_t(t)$, $v(t)$, $s(t)$ foronómiai görbék, $+\, a_n(t)$

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a, $\varepsilon :=$ állandó

$\omega(t) = \omega_0 + \varepsilon \cdot t = \ldots\ [\frac{rad}{s}]$

$\vartheta(t) = \vartheta_0 + \omega_0 \cdot t + \frac{1}{2}\varepsilon t^2 = \frac{\pi}{6} + 10 \cdot t + \frac{1}{2}(-2)t^2\,[rad]$

$\vartheta(t_1) = \frac{\pi}{6} + 10 \cdot 7 + \frac{1}{2}(-2)\cdot 7^2 = \underline{21.52} = \underline{3.43 \cdot 2\pi}\,[rad]$

b, $\omega(t_1) = 10 - 2 \cdot 7 = \underline{-4\,[\frac{rad}{s}]}$

Ha $\varepsilon(t) \neq$ állandó!

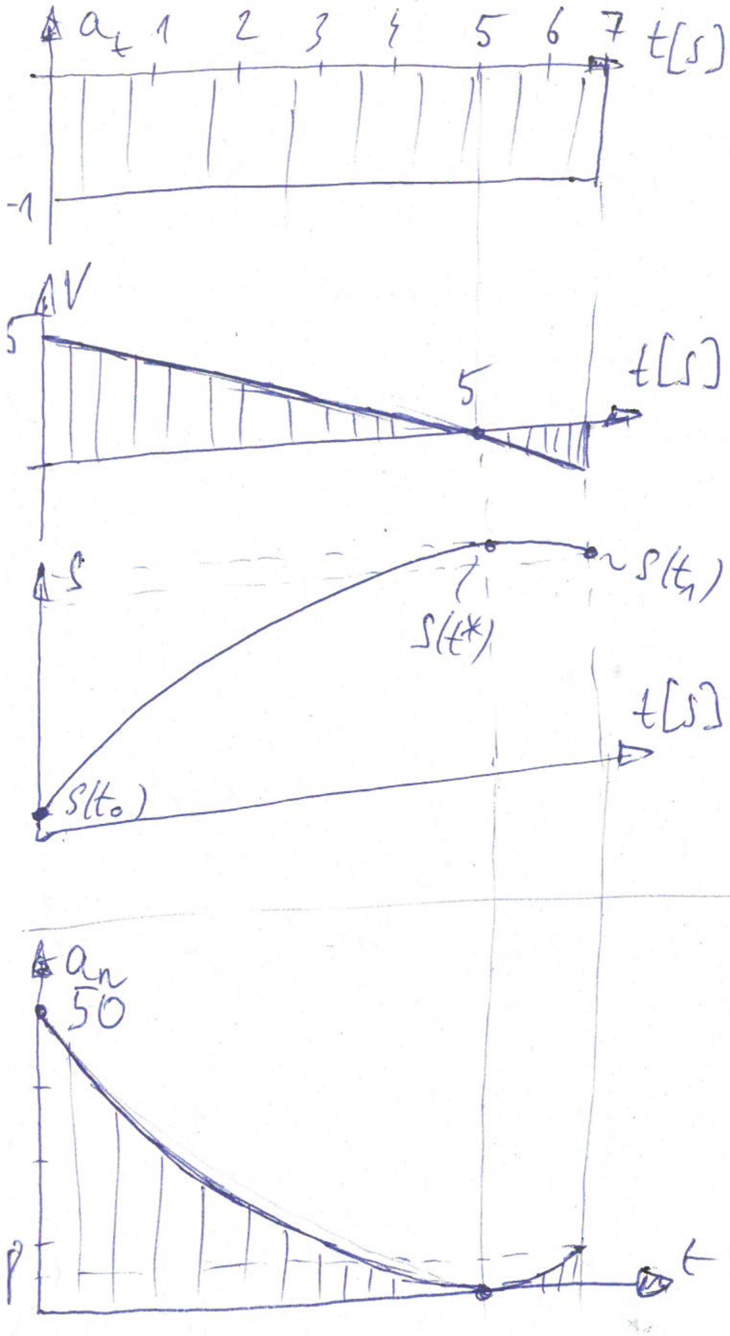
$\omega(t) = \int_{t_0}^{t} \varepsilon(\tau)\,d\tau + \omega_0$, $\vartheta(t) = \int_{t_0}^{t} \omega(\tau)\,d\tau + \vartheta_0$

b, + Hol vált előjelet a mozgás iránya?

$\omega(t^*) = 0 = -2 \cdot t^* + 10 \rightarrow t^* = 5\,[s]$, $\vartheta(t^*) = 4.06 \cdot 2\pi\,[rad]$

c) foronómiai görbék

$$a_t(t) = a_{t0} = \text{állandó} = R\cdot\varepsilon = -1\left[\frac{rad}{s^2}\right]$$



$$v(t) = v_0 + a_t \cdot t\,, \quad v_0 = R\cdot\omega_0 = 5\left[\frac{m}{s}\right]$$

$$v(t) = 5 - t\left[\frac{m}{s}\right]$$

irány!

$$s(t) = s_0 + v_0\cdot t + \frac{1}{2}a_t\cdot t^2$$

$$s(t) = R\cdot\vartheta(t)$$

$$s(t_0) = R\vartheta_0 = 0.262\,[m]$$

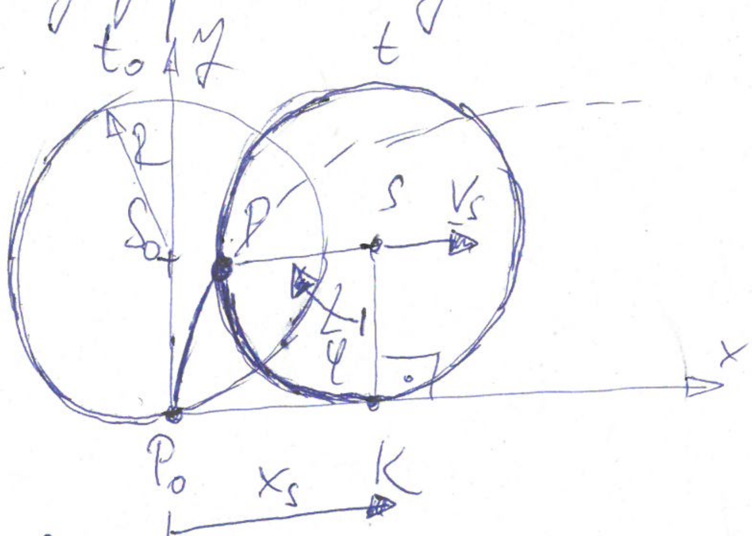
$$s(t^*) = 12.76\,[m]\,, \quad s(t_1) = 10.76\,[m]$$

$$a_n(t) = \frac{v^2(t)}{R} = \frac{(5-t)^2}{0.5}\left[\frac{m}{s^2}\right]$$

$$a_n(t_0) = 50\left[\frac{m}{s^2}\right],\ a_n(t^*) = 0\left[\frac{m}{s^2}\right],$$

$$a_n(t_1) = 8\left[\frac{m}{s^2}\right]$$

Anyagi pont mozgása ciklois pályán



$R = 0.3\,[m]$

$\underline{v}_s$ = állandó ~~[illegible]~~ $v_s = 5\,[\frac{m}{s}]$

a) $\underline{v}_P(\varphi)$

b) $\underline{a}_P(\varphi)$

c) $\underline{v}_P(\varphi_1)$, $\underline{a}_P(\varphi_1)$, $\varphi_1 = 75°$

d) $\underline{a}_P(\varphi_1) \nearrow \underline{a}_{Pn}$, $\searrow \underline{a}_{Pt}$

e) $\rho(\varphi_1)$

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vonatkoztatási rendszer: talaj, koordinátarendszer: talajhoz kötött

kezdeti helyzet $\underline{r}_P(t_0) = \underline{0}$

$$x_s = \overline{P_0K} = \widehat{PK} = R\cdot\varphi \,,\quad \underline{r}_P(\varphi) = \begin{bmatrix} R\cdot\varphi - R\sin(\varphi) \\ R - R\cos(\varphi) \end{bmatrix} = R\begin{bmatrix} \varphi - \sin(\varphi) \\ 1 - \cos(\varphi) \end{bmatrix}$$

$$\frac{d}{dt}\underline{r}_P(\varphi(t)) = R\begin{bmatrix} \dot\varphi(t) - \cos(\varphi(t))\cdot\dot\varphi(t) \\ \sin(\varphi(t))\cdot\dot\varphi(t) \end{bmatrix} = \underbrace{R\dot\varphi(t)}_{v_s}\begin{bmatrix} 1 - \cos(\varphi(t)) \\ \sin(\varphi(t)) \end{bmatrix}$$

$$\boxed{\dot x_s = v_s = R\cdot\dot\varphi}$$

$$\underline{a}_P = \frac{d}{dt}\underline{v}_P = \underline{\dot v}_P = v_s\cdot\begin{bmatrix} \sin(\varphi(t))\cdot\dot\varphi(t) \\ \cos(\varphi(t))\cdot\dot\varphi(t) \end{bmatrix} = \frac{v_s^2}{R}\begin{bmatrix} \sin(\varphi) \\ \cos(\varphi) \end{bmatrix}$$

megjegyzés

$$|\underline{a}_P| = \frac{v_S^2}{R}\sqrt{\sin^2(\varphi)+\cos^2(\varphi)} = \frac{v_S^2}{R} = \text{állandó, de az iránya változik}$$

$\varphi = k\cdot 2\pi$ esetben $\underline{v}_P = \underline{0}$ $\quad P = K$

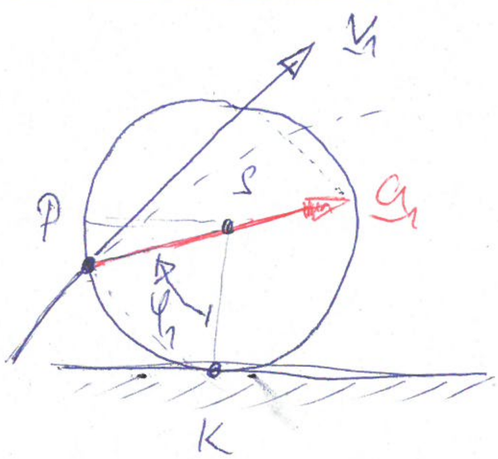
$$\underline{v}_1 = \underline{v}_P(\varphi = 75°) = 5\begin{bmatrix}1-\cos(75°)\\ \sin(75°)\end{bmatrix} = \begin{bmatrix}3.71\\ 4.83\end{bmatrix}\left[\mathrm{m/s}\right]$$

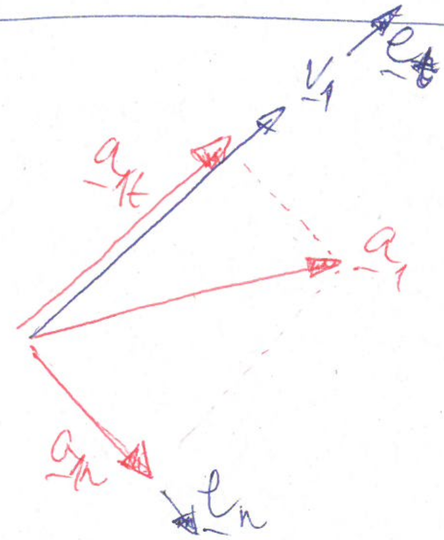
$$v_1 = |\underline{v}_1| = \sqrt{3.71^2 + 4.83^2} = 6.09\left[\mathrm{m/s}\right]$$

$\cancel{\underline{a}_1 = \underline{a}_P(\varphi = 75°) =}$

$$\underline{a}_1 = \underline{a}_P(\varphi = 75°) = \frac{5^2}{0.3}\begin{bmatrix}\sin(75°)\\ \cos(75°)\end{bmatrix} = \begin{bmatrix}80.5\\ 21.9\end{bmatrix}\left[\mathrm{m/s^2}\right]$$

$$a_1 = |\underline{a}_1| = 83.41\left[\mathrm{m/s^2}\right]$$





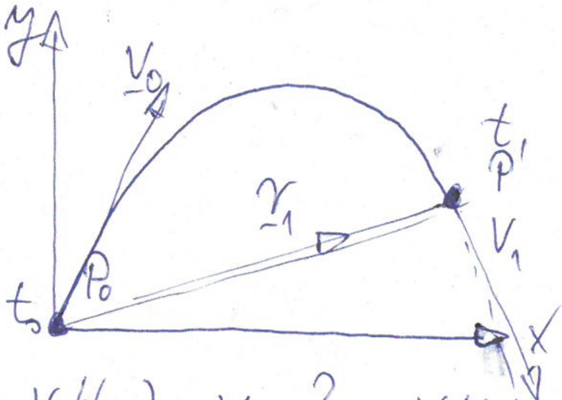
$$\underline{a}_{1t} = \underline{e}_t\cdot(\underline{a}_1\cdot\underline{e}_t) \quad , \quad \underline{e}_t = \frac{\underline{v}_1}{|\underline{v}_1|}$$

$$\underline{a}_{1t} = \frac{\underline{v}_1}{v_1}\cdot\left(\underline{a}_1\cdot\frac{\underline{v}_1}{v_1}\right) = \begin{bmatrix}40.3\\ 52.5\end{bmatrix}\left[\mathrm{m/s^2}\right] \quad , \quad a_{1t} = |\underline{a}_{1t}| = 66.2\left[\mathrm{m/s^2}\right]$$

$$\underline{a}_{1n} = \underline{a}_1 - \underline{a}_{1t} = \begin{bmatrix}40.2\\ -30.9\end{bmatrix}\left[\mathrm{m/s^2}\right] \quad , \quad a_{1n} = |\underline{a}_{1n}| = 50.6\left[\mathrm{m/s^2}\right]$$

$$a_{1n} = \frac{v_1^2}{\rho_1} \Rightarrow \rho_1 = \frac{v_1^2}{a_{1n}} = 0.73\,[m]$$

Anyagi pont mozgása ferde hajításkor



$$\underline{r}(t_0) = \underline{r}_0 = \underline{0}, \quad \underline{r}(t_1) = \underline{r}_1 = \begin{bmatrix} 4.6 \\ 1 \end{bmatrix} [m]$$

$$t \in [t_0, t_1] = [0, 1]\,[s]$$

a, $\underline{v}(t_0) = \underline{v}_0 = ?$, $\underline{v}(t_1) = \underline{v}_1 = ?$, b, pályagörbe $y(x) = f(x) = ?$

c, hodográf $t \in [t_0, t_1]$ $\underline{v}(t)$, d, $\underline{a}_t(t_1)$, $\underline{a}_n(t_1)$

e, $\rho(t_1)$ pálya görbületi sugara

állandó gyorsulással mozgó anyagi pont mozgástörvénye

*) $\underline{r}(t) = \underline{r}_0 + \underline{v}_0(t - t_0) + \frac{1}{2}\underline{a}(t - t_0)^2, \quad t_0 = 0\,[s]$

$$\underline{a} = \underline{g} = \begin{bmatrix} 0 \\ -9.81 \end{bmatrix} [m/s^2], \quad \underline{r}_0 = \underline{0} \qquad \underline{a} = \underline{g} = \begin{bmatrix} 0 \\ -g \end{bmatrix} = \begin{bmatrix} 0 \\ -9.81 \end{bmatrix} [m/s^2]$$

$$\underline{r}(t_1) = \underline{v}_0 \cdot t_1 + \frac{1}{2}\underline{g} \cdot t_1^2 = \underline{r}_1$$

$$\left.\begin{aligned} x&: v_{0x} \cdot 1 + \frac{1}{2} \cdot 0 \cdot 1^2 = 4.6 \Rightarrow v_{0x} = 4.6 \left[\frac{m}{s}\right] \\ y&: v_{0y} \cdot 1 + \frac{1}{2} \cdot (-9.81) \cdot 1^2 = 1 \Rightarrow v_{0y} = 5.9 \left[\frac{m}{s}\right] \end{aligned}\right\} \quad \underline{v}_0 = \begin{bmatrix} 4.6 \\ 5.9 \end{bmatrix} [m/s]$$

$$v_0 = |\underline{v}_0| = 7.48 \,[m/s]$$

a) $\underline{v}(t) = \dot{\underline{r}}(t) = \underline{v}_0 + \underline{g}\cdot t$, $\underline{v}(t_1) = \underline{v}_0 + \underline{g}\cdot t_1 = \begin{bmatrix}4.6+0\cdot 1\\ 5.9-9.81\cdot 1\end{bmatrix} = \begin{bmatrix}4.6\\ -3.9\end{bmatrix}\left[\frac{m}{s}\right]$

$v_1 = |\underline{v}(t_1)| = 6\left[\frac{m}{s}\right]$

b) pályagörbe $\underline{r}(t)$-ből kiindulva

$x(t) = x_0 + v_{0x}\cdot t \Rightarrow \boxed{t = \frac{x - x_0}{v_{0x}}} = \frac{x}{v_{0x}}$

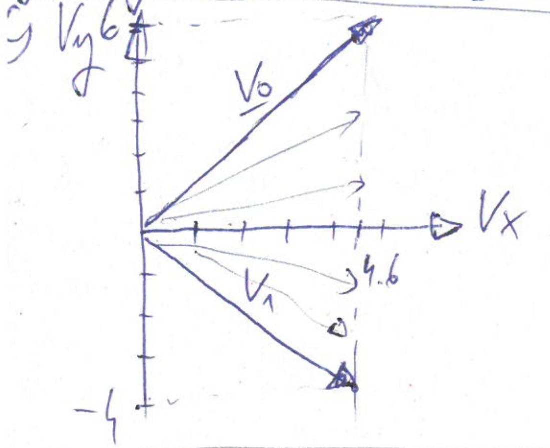
$y(t) = y_0 + v_{0y}\cdot t + \frac{1}{2}(-g)\cdot t^2 \Rightarrow \frac{v_{0y}}{v_{0x}}\cdot x - \frac{1}{2}\frac{g}{v_{0x}^2}\cdot x^2 = y(x)$

pálya csúcspontja

$y'(x) = 0 \Rightarrow \frac{v_{0y}}{v_{0x}} - \frac{g}{v_{0x}^2}x \Rightarrow x^* = \frac{v_{0y}v_{0x}}{g} = 2.77[m]$

$y^* = y(x^*) = 1.77[m]$

c)



d) $\underline{a}_t(t) = \cancel{a_t\,\underline{e}_t = a_t\frac{\underline{v}}{|\underline{v}|}}$ $\quad \underline{e}_t\cdot\big(\overbrace{\underline{a}\cdot\underline{e}_t}^{a_t}\big) = \frac{\underline{v}}{|\underline{v}|}\cdot\left(\underline{g}\cdot\frac{\underline{v}}{|\underline{v}|}\right)$

$\underline{a}_{t1} = \underline{a}_t(t_1) = \begin{bmatrix}4.83\\ -4.1\end{bmatrix}\left[m/s^2\right]$

$\underline{a}_{n1} = \underline{g} - \underline{a}_{t1} = \begin{bmatrix}-4.83\\ -5.71\end{bmatrix}\left[m/s^2\right]$ $\quad$ e) $\rho(t) = \frac{v^2(t)}{a_n(t)} \Rightarrow \rho(t_1) = \rho_1 = \frac{v_1^2}{a_{n1}} = 4.86\,[m]$