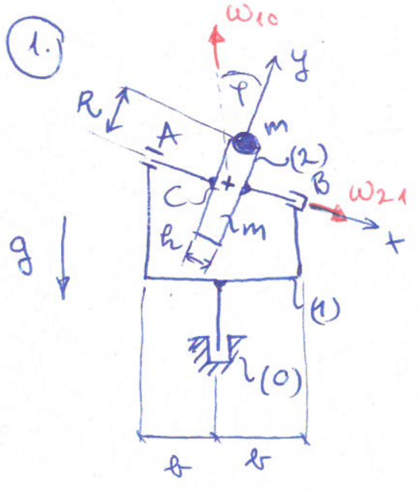
DINAMIKA

Térbeli feladatok

(1.)



$g = 9{,}81 \frac{m}{s^2}$

Adatok

$m = 0{,}6\ kg$

$R = 0{,}04\ m$

$h \approx 0$

$b = 0{,}04\ m$

$\omega_{10} = 20 \frac{rad}{s} =$ áll.

$\omega_{21} = 30 \frac{rad}{s} =$ áll.

$\varphi = 30°$

Feladat

a) $\underline{a}_{S2} = ?$ (minimum)

b) $\underset{=C}{\Theta}^{2}$ paraméteresen (x, y, z)

c) $\underline{\Pi}_C$ paraméteresen

d) SZTÁ (2)-re

e) din. alapt. par.

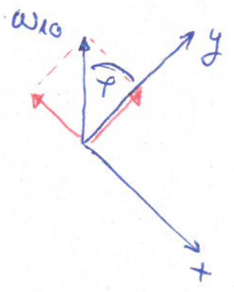
Megoldás

a) Mit tudunk?

- C pont: rajta van ω_{10} és ω_{21} által kijelölt forgástengelyen → tartósan álló pont → $\underline{v}_C = \underline{0}$, $\underline{a}_C = \underline{0}$

TEHÁT C-re MAJD FEL LEHET ÍRNI A DIN. ALAPT-T!

2-es test szögseb.:

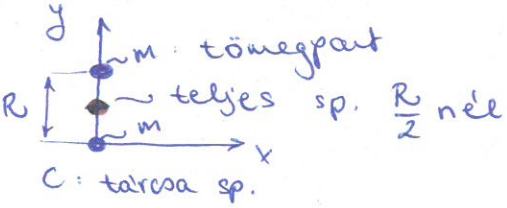


$$\underline{\omega}_2 = \underline{\omega}_{10} + \underline{\omega}_{21} = \begin{bmatrix} -\omega_{10}\sin\varphi \\ \omega_{10}\cos\varphi \\ 0 \end{bmatrix} + \begin{bmatrix} \omega_{21} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \omega_{21} - \omega_{10}\sin\varphi \\ \omega_{10}\cos\varphi \\ 0 \end{bmatrix}$$

2-es test szöggyorsulása.

$$\underline{\varepsilon}_2 = \underbrace{\underline{\varepsilon}_{10}}_{\underline{0}} + \underbrace{\underline{\varepsilon}_{21}}_{\underline{0}} + \underline{\omega}_{10} \times \underline{\omega}_{21} = \begin{bmatrix} -\omega_{10}\sin\varphi \\ \omega_{10}\cos\varphi \\ 0 \end{bmatrix} \times \begin{bmatrix} \omega_{21} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\omega_{10}\omega_{21}\cos\varphi \end{bmatrix}$$

Súlypont:



Súlypont a tömegek arányában osztik meg!

$$\underline{r}_{cs} = \begin{bmatrix} 0 \\ R/2 \\ 0 \end{bmatrix}$$

(2) -es test sp. -jainak gyorsulása:

$$\underline{a}_S = \underbrace{\underline{a}_C}_{\underline{0}} + \underline{\varepsilon}_2 \times \underline{r}_{cs} + \underline{\omega}_2 \times (\underline{\omega}_2 \times \underline{r}_{cs}) = \begin{bmatrix} 0 \\ 0 \\ -\omega_{10}\omega_{21}\cos\varphi \end{bmatrix} \times \begin{bmatrix} 0 \\ R/2 \\ 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} \omega_{21} - \omega_{10}\sin\varphi \\ \omega_{10}\cos\varphi \\ 0 \end{bmatrix} \times \left(\begin{bmatrix} \omega_{21} - \omega_{10}\sin\varphi \\ \omega_{10}\cos\varphi \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ R/2 \\ 0 \end{bmatrix} \right) = \dots = \begin{bmatrix} R\,\omega_{10}\omega_{21}\cos\varphi - \frac{R}{2}\omega_{10}^2\sin\varphi\cos\varphi \\ -\frac{R}{2}(\omega_{21} - \omega_{10}\sin\varphi)^2 \\ 0 \end{bmatrix}$$

Numerikusan:

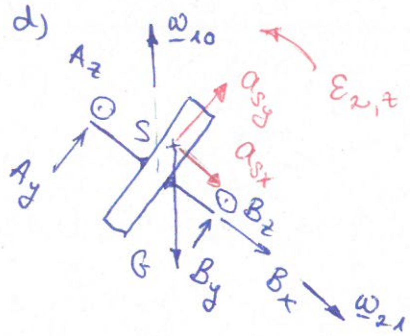
$$\underline{a}_S = \begin{bmatrix} 17{,}3 \\ -8 \\ 0 \end{bmatrix} \frac{m}{s^2}$$

b) $$\underline{\underline{\Theta}}_C = \underline{\underline{\Theta}}_C^{henger} + \underbrace{\underline{\underline{\Theta}}^{pont}}_{\underline{0}} + \underline{\underline{\Theta}}_C^{pont} = \begin{bmatrix} \frac{1}{2}mR^2 & 0 & 0 \\ 0 & \frac{1}{4}mR^2 & 0 \\ 0 & 0 & \frac{1}{4}mR^2 \end{bmatrix} + \begin{bmatrix} mR^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & mR^2 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{3}{2}mR^2 & 0 & 0 \\ 0 & \frac{1}{4}mR^2 & 0 \\ 0 & 0 & \frac{5}{4}mR^2 \end{bmatrix}$$

c) $$\underline{\Pi}_C = \underline{\underline{\Theta}}_C \cdot \underline{\omega}_2 = \begin{bmatrix} \frac{3}{2}mR^2(\omega_{21} - \omega_{10}\sin\varphi) \\ \frac{1}{4}mR^2\omega_{10}\cos\varphi \\ 0 \end{bmatrix}$$

d)



e) Álló pontra:

$$\left[\dot{\underline{I}}, \dot{\underline{\Pi}}_c\right]_c = \left[\underline{F}, \underline{M}_c\right]_c \qquad G = mg\cdot 2$$

$$\boxed{\dot{\underline{I}} = \underline{F}}$$

$x: \quad 2m\, a_{Sx} = B_x + 2mg\sin\varphi$

$y: \quad 2m\, a_{Sy} = A_y + B_y - 2mg\cos\varphi$

$z: \quad 0 = A_z + B_z$

$$\boxed{\dot{\underline{\Pi}}_c = \underline{M}_c}$$

$$\dot{\underline{\Pi}}_c = \underline{\underline{\Theta}}_c \cdot \underline{\varepsilon}_2 + \underline{\omega}_2 \times \underline{\Pi}_c = \begin{bmatrix} 0 \\ 0 \\ -\frac{5}{4} mR^2 \omega_{10}\omega_{21}\cos\varphi \end{bmatrix} + \begin{bmatrix} \omega_{21} - \omega_{10}\sin\varphi \\ \omega_{10}\cos\varphi \\ 0 \end{bmatrix} \times \begin{bmatrix} \frac{3}{2} mR^2 (\omega_{21} - \omega_{10}\sin\varphi) \\ \frac{1}{4} mR^2 \omega_{10}\cos\varphi \\ 0 \end{bmatrix} =$$

$$= \dots = \begin{bmatrix} 0 \\ 0 \\ -\frac{5}{4} mR^2 \omega_{10}\omega_{21}\cos\varphi + \frac{5}{4} mR^2 \omega_{10}^2 \sin\varphi\cos\varphi \end{bmatrix}$$

$$\underline{M}_c = \underline{r}_{CS} \times \underline{G} + \underline{r}_{CA} \times \underline{A} + \underline{r}_{CB} \times \underline{B} = \begin{bmatrix} 0 \\ R/2 \\ 0 \end{bmatrix} \times \begin{bmatrix} 2mg\sin\varphi \\ -2mg\cos\varphi \\ 0 \end{bmatrix} + \begin{bmatrix} -b/\cos\varphi \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ A_y \\ A_z \end{bmatrix} + \begin{bmatrix} b/\cos\varphi \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} =$$

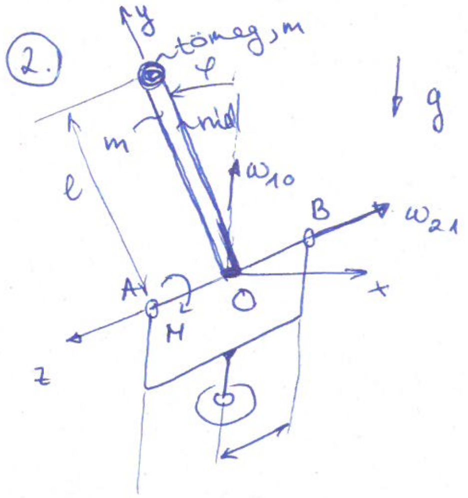
$$= \begin{bmatrix} 0 \\ A_z \dfrac{b}{\cos\varphi} - B_z \dfrac{b}{\cos\varphi} \\ -mgR\sin\varphi - A_y \dfrac{b}{\cos\varphi} + B_y \dfrac{b}{\cos\varphi} \end{bmatrix}.$$

Tehát: $x: \quad 0 = 0$

$y: \quad 0 = A_z \dfrac{b}{\cos\varphi} - B_z \dfrac{b}{\cos\varphi}$

$z: \quad -\frac{5}{4} mR^2 \dots = -mgR\sin\varphi - \dots$

$\boxed{M}$ Ismeretlenek: A_y, A_z, B_x, B_y, B_z (lenne a megoldáshoz 5 egyenlet)



Adatok

$m = 0{,}36\ kg$

$l = 0{,}6\ m$

$b = 0{,}1\ m$

$\omega_{10} = 2\ \frac{rad}{s} = \text{áll.}$

$\omega_{21} = 5\ \frac{rad}{s} = \text{áll.}$

$\varphi = 30°$

$g = 9{,}81\ \frac{m}{s^2}$

Feladat

a, $\underline{\underline{\Theta}}_O^2$ paraméteresen (x,y,z)

b, $\underline{\Pi}_O$ —"—

c, a_{S2} (minimum)

d, SZTÁ (2)-re

e, din. alapt. vetületi egyenletek param.

Megoldás

a, O tartósan álló pont! → Din. alapt. jó lesz ide.
Erre a feladat is rávezet. (Mindent O-ra kell felírni.)

$$\underline{\underline{\Theta}}_O^2 = \underline{\underline{\Theta}}_O = \underline{\underline{\Theta}}_O^{rúd} + \underline{\underline{\Theta}}_O^{pont}$$

$$\underline{\underline{\Theta}}_O^{rúd} = \underbrace{\begin{bmatrix} \frac{1}{12}ml^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{12}ml^2 \end{bmatrix}}_{\text{saját sp-ra}} + \underbrace{\begin{bmatrix} ml^2/4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & ml^2/4 \end{bmatrix}}_{\text{Steiner-tag}} = \begin{bmatrix} \frac{1}{3}ml^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}ml^2 \end{bmatrix}$$

$$\underline{\underline{\Theta}}_O^{pont} = \underbrace{\underline{\underline{0}}}_{\text{saját sp-ra}} + \underbrace{m\begin{bmatrix} l^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & l^2 \end{bmatrix}}_{\text{Steiner-tag}}$$

$$\underline{\underline{\Theta}}_O = \begin{bmatrix} \frac{4}{3}ml^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{4}{3}ml^2 \end{bmatrix}$$

b) $\underline{\Pi}_O = \underline{\underline{\Theta}}_O \cdot \underline{\omega}_2$ (O pont tartósan áll)

$$\underline{\omega}_2 = \underline{\omega}_{10} + \underline{\omega}_{21} = \begin{bmatrix} \omega_{10}\sin\varphi \\ \omega_{10}\cos\varphi \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\omega_{21} \end{bmatrix}$$

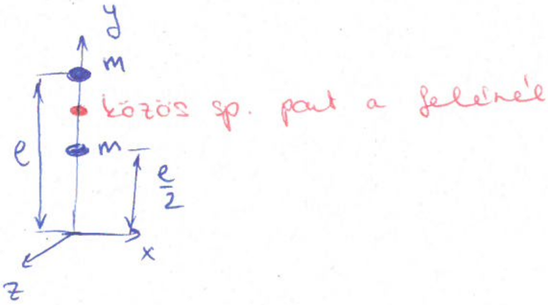
$$\underline{\Pi}_O = \begin{bmatrix} \frac{4}{3}ml^2\omega_{10}\sin\varphi \\ 0 \\ -\frac{4}{3}ml^2\omega_{21} \end{bmatrix}$$

c) $\underline{a}_S = \underline{a}_0 + \underline{\varepsilon}_2 \times \underline{r}_{0S} + \underline{\omega}_2 \times (\underline{\omega}_2 \times \underline{r}_{0S})$

$\underline{\omega}_2$: b, feladatban kijött

$$\underline{\varepsilon}_2 = \underbrace{\underline{\varepsilon}_{10}}_{\underline{0}} + \underbrace{\underline{\varepsilon}_{21}}_{\underline{0}} + \underline{\omega}_{10} \times \underline{\omega}_{21} = \begin{bmatrix} \omega_{10}\sin\varphi \\ \omega_{10}\cos\varphi \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -\omega_{21} \end{bmatrix} = \begin{bmatrix} -\omega_{10}\omega_{21}\cos\varphi \\ \omega_{10}\omega_{21}\sin\varphi \\ 0 \end{bmatrix}$$

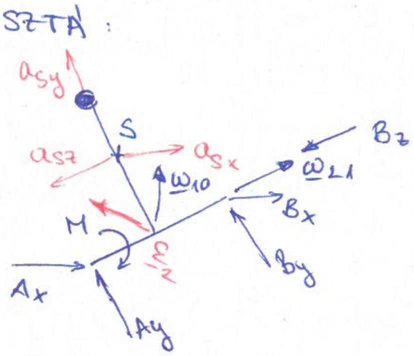
$$\underline{r}_{0S} = \begin{bmatrix} 0 \\ \frac{3}{4}l \\ 0 \end{bmatrix}$$



Behelyettesítések után:

$$\underline{a}_S = \begin{bmatrix} \frac{3}{4} l\,\omega_{10}^2 \sin\varphi\cos\varphi \\ -\frac{3}{4} l\,(\omega_{10}^2 \sin^2\varphi + \omega_{21}^2) \\ -\frac{3}{4} l\,\omega_{10}\omega_{21}\cos\varphi \cdot 2 \end{bmatrix} = \begin{bmatrix} 0{,}779 \\ -11{,}1 \\ -7{,}79 \end{bmatrix} \frac{m}{s^2}$$

d, SZTA:



e, Din. alapt.:

$$[\dot{\underline{I}}, \dot{\underline{\Pi}}_0]_0 = [\underline{F}, \underline{M}_0]_0$$

$$\boxed{\dot{\underline{I}} = \underline{F}}$$

x: $2m\,a_{Sx} = A_x + B_x - 2mg\sin\varphi$

y: $2m\,a_{Sy} = A_y + B_y - 2mg\cos\varphi$

z: $2m\,a_{Sz} = B_z$

$$\boxed{\dot{\underline{\Pi}}_0 = \underline{M}_0}$$

$$\dot{\underline{\Pi}}_0 = \underline{\underline{\Theta}}_0\,\underline{\varepsilon}_2 + \underline{\omega}_2 \times \underline{\Pi}_0 = \ldots = \begin{bmatrix} -\frac{8}{3} m l^2 \omega_{10}\omega_{21}\cos\varphi \\ 0 \\ -\frac{4}{3} m l^2 \omega_{10}^2 \sin\varphi\cos\varphi \end{bmatrix}$$

!

$$\underline{M}_0 = \underline{M} + \underline{r}_{0A} \times \underline{A} + \underline{r}_{0B} \times \underline{B} + \underline{r}_{0S} \times \underline{G} = \begin{bmatrix} 0 \\ 0 \\ -M \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} \times \begin{bmatrix} A_x \\ A_y \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -b \end{bmatrix} \times \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{3}{4}l \\ 0 \end{bmatrix} \times \begin{bmatrix} -2mg\sin\varphi \\ -2mg\cos\varphi \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} -A_y b + B_y b \\ A_x b - B_x b \\ -M + \frac{3}{2} mgl\sin\varphi \end{bmatrix} \rightarrow \text{szét szedni vektoriális egyenletekre} \ldots$$