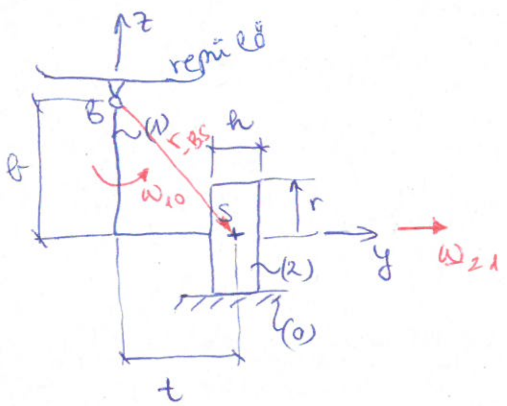
DINAMIKA – 9. gyakorlat

1.) Repülőgép futómű behúzása

szemből (felénk jön a repülő, földön gurul)



Adatok: $\underline{v}_S = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix}$ v ismert: ennyivel gurult (szállt fel) a repülő → kerék súlypontjának a sebessége is ennyi

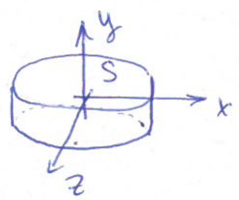
$v = \text{áll.}, \ \omega_{10}, \ m, \ r, \ h, \ t, \ b$

↑ (m) kerék tömege

Kérdés:
1.) $\underline{\underline{\Theta}}_S$, $\underline{\Pi}_S$
2.) $\underline{\underline{\Theta}}_B$, $\underline{\Pi}_B$
3.) T

Megoldás:

1.)



$$\underline{\underline{\Theta}}_S = \begin{bmatrix} \Theta_{Sx} & 0 & 0 \\ 0 & \Theta_{Sy} & 0 \\ 0 & 0 & \Theta_{Sz} \end{bmatrix}$$

$$\Theta_{Sx} = \Theta_{Sz} = \frac{1}{4} m r^2 + \frac{1}{12} m h^2$$

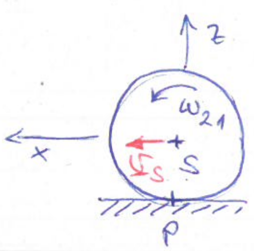
$$\Theta_{Sy} = \frac{1}{2} m r^2$$

$$\underline{\Pi}_S = \underline{\underline{\Theta}}_S \cdot \underline{\omega} \qquad (\underline{I} = m \underline{v})$$

$$\underline{\omega} = \underline{\omega}_{20} = \underline{\omega}_{10} + \underline{\omega}_{21}$$

→ $\underline{\omega}_{10} = \begin{bmatrix} \omega_{10} \\ 0 \\ 0 \end{bmatrix}$ (x tengely körül hajtja fel a kereket)

→ $\underline{\omega}_{21}$ a kerék forgásából:



$$\underline{v}_S = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix} (*) \qquad \underline{v}_S = \underbrace{\underline{v}_P}_{\underline{0}} + \underline{\omega}_{21} \times \underline{r}_{PS} = \begin{bmatrix} 0 \\ \omega_{21} \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix} = \begin{bmatrix} \omega_{21} \cdot r \\ 0 \\ 0 \end{bmatrix} (**)$$

$(*) = (**)$ $\begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \omega_{21} \cdot r \\ 0 \\ 0 \end{bmatrix} \rightarrow \omega_{21} = \frac{v}{r}$

$$\Longrightarrow \quad \underline{\omega}_{20} = \underline{\omega}_{10} + \underline{\omega}_{21} = \begin{bmatrix} \omega_{10} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_{21} \\ 0 \end{bmatrix} = \begin{bmatrix} \omega_{10} \\ v/r \\ 0 \end{bmatrix}$$

Tehát:

$$\underline{\Pi}_S = \underline{\underline{\Theta}}_S \, \underline{\omega} = \begin{bmatrix} \Theta_{Sx} & 0 & 0 \\ 0 & \Theta_{Sy} & 0 \\ 0 & 0 & \Theta_{Sz} \end{bmatrix} \begin{bmatrix} \omega_{10} \\ \omega_{21} \\ 0 \end{bmatrix} = \begin{bmatrix} \Theta_{Sx}\omega_{10} \\ \Theta_{Sy}\omega_{21} \\ 0 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{4} m r^2 + \frac{1}{12} m h^2\right) \omega_{10} \\ \frac{1}{2} m r v \\ 0 \end{bmatrix} = \underline{\Pi}_S$$

2.) $\underline{\underline{\Theta}}_B = \underline{\underline{\Theta}}_S + \underline{\underline{\Theta}}_{BS}$

Ezt ne felejtsük le!

$$\underline{\underline{\Theta}}_{BS} = m \begin{bmatrix} y_{BS}^2 + z_{BS}^2 & -x_{BS} y_{BS} & -x_{BS} z_{BS} \\ -x_{BS} y_{BS} & x_{BS}^2 + z_{BS}^2 & -y_{BS} z_{BS} \\ -x_{BS} z_{BS} & -y_{BS} z_{BS} & y_{BS}^2 + x_{BS}^2 \end{bmatrix} \qquad \underline{r}_{BS} = \begin{bmatrix} x_{BS} \\ y_{BS} \\ z_{BS} \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ -b \end{bmatrix}$$

Behelyettesítés:

$$\underline{\underline{\Theta}}_{BS} = m \cdot \begin{bmatrix} t^2 + b^2 & 0 & 0 \\ 0 & b^2 & t \cdot b \\ 0 & t \cdot b & t^2 \end{bmatrix}$$

$$\underline{\underline{\Theta}}_B = \underline{\underline{\Theta}}_S + \underline{\underline{\Theta}}_{BS} = \begin{bmatrix} \Theta_{Sx} + m\left(t^2 + b^2\right) & 0 & 0 \\ 0 & \Theta_{Sy} + m b^2 & m t b \\ 0 & m t b & \Theta_{Sz} + m t^2 \end{bmatrix}$$

$\underline{\Pi}_B = \underline{\Pi}_S + \underline{r}_{BS} \times \underline{I}$ $\qquad (\underline{M}_B = \underline{M}_S + \underline{r}_{BS} \times \underline{F})$

$\underline{I} = m \underline{v}_S$ $\qquad \underline{v}_S$: már felszállás után!

(elkezdtük behajtani a kereket ω_{10}-val)

S pont sebessége: (S és B rajta vannak az (1)-es testen)

$$\underline{v}_S = \underline{v}_B + \underline{\omega}_{10} \times \underline{r}_{BS} \quad \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \omega_{10} \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ t \\ -b \end{bmatrix} = \begin{bmatrix} v \\ \omega_{10} b \\ \omega_{10} t \end{bmatrix}$$

$$\underline{r}_{BS} \times \underline{I} = \begin{bmatrix} 0 \\ t \\ -b \end{bmatrix} \times m \begin{bmatrix} v \\ \omega_{10} b \\ \omega_{10} t \end{bmatrix} = \begin{bmatrix} m\omega_{10}(t^2+b^2) \\ -m b v \\ -m t v \end{bmatrix}$$

$$\Longrightarrow \underline{\Pi}_B = \underline{\Pi}_S + \underline{r}_{BS} \times \underline{I} = \begin{bmatrix} \Theta_{Sx}\omega_{10} + m\omega_{10}(t^2+b^2) \\ \Theta_{Sy} \cdot \frac{v}{r} - m b v \\ -m t v \end{bmatrix}$$

3., $T = \underbrace{T_t}_{\text{transzláció}} + \underbrace{T_r}_{\text{rotáció}} = \frac{1}{2} m v_S^2 + \frac{1}{2} \underline{\omega}^T \underline{\underline{\Theta}}_S \underline{\omega}$

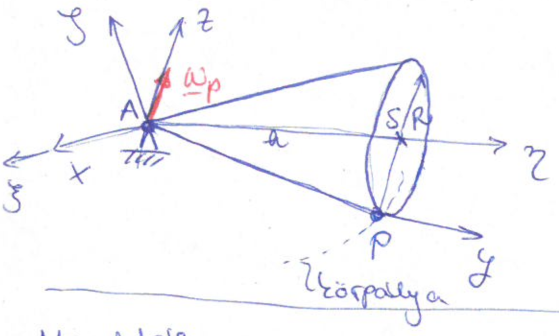
$$T_t = \frac{1}{2} m v_S^2 = \frac{1}{2} m \left(v^2 + \omega_{10}^2 (b^2+t^2)\right)$$

$$T_r = \frac{1}{2} \underline{\omega}^T \underline{\underline{\Theta}}_S \underline{\omega} = \frac{1}{2} \underline{\omega}^T \cdot \underline{\Pi}_S = \frac{1}{2} \begin{bmatrix} \omega_{10} \\ v/r \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \Theta_{Sx}\omega_{10} \\ \Theta_{Sy} \frac{v}{r} \\ 0 \end{bmatrix} = \frac{1}{2}\left(\Theta_{Sx}\omega_{10}^2 + \Theta_{Sy}\frac{v^2}{r^2}\right)$$

$$\Longrightarrow T = m v^2 + \left[\frac{1}{8} m r^2 + \frac{1}{24} m h^2 + \frac{1}{2}(t^2+b^2)\right]\omega_{10}^2$$

$\boxed{M}$ T: csak súlypontra / tartósan álló pontra. B-re itt NEM!

2.) Gördülő kúp: élén gördül!



Adatok: $m = 6\ kg$

$R = 0{,}2\ m$

$h = 0{,}3\ m$

$\omega_p = 2\ rad/s$

Kérdés:

1.) T

2.) $\tilde{\Pi}_S, \underline{\Pi}_A$

Megoldás:

1.) $T = \underbrace{\frac{1}{2} m v_S^2 + \frac{1}{2} \underline{\omega}^T \underline{\underline{\Theta}}_S \underline{\omega}}_{\text{súlypontra}} = \underbrace{\frac{1}{2} \underline{\omega}^T \underline{\underline{\Theta}}_A \underline{\omega}}_{\text{álló pontra}}$ A pont. tartósan álló pont!

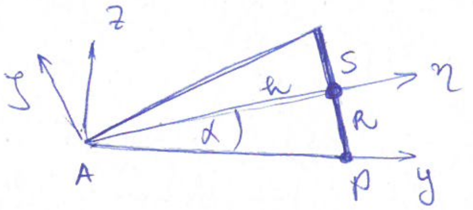
$$\underset{(\xi,\eta,\zeta)}{\underline{\underline{\Theta}}_A} = \begin{bmatrix} \Theta_\xi & 0 & 0 \\ 0 & \Theta_\eta & 0 \\ 0 & 0 & \Theta_\zeta \end{bmatrix}$$

Forgástím.: $\Theta_\zeta = \Theta_\xi = \frac{3}{80} m \left(4R^2 + h^2\right) + m\left(\frac{3}{4} h\right)^2$

$\Theta_\eta = \frac{3}{10} m R^2$ (η: forgástengely)

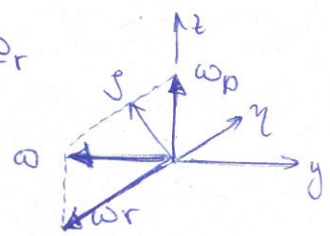
$\Longrightarrow \underline{\omega}$ (ξ, η, ζ) koord. rendszerben?

Külső szemlélőként:



$\underline{v}_A = \underline{0}$; $\underline{v}_P = \underline{0}$ $\longrightarrow$ $\overline{AP}$-re illesztett egyenes a forgástengely, ez megmutatja $\underline{\omega}$ irányát

$\underline{\omega} = \underline{\omega}_p + \underline{\omega}_r$



$\underline{\omega} \parallel \underline{r}_{AP} \longrightarrow \omega_x = \omega_z = 0$

$$\underset{(x,y,z)}{\underline{\omega}} = \begin{bmatrix} 0 \\ -\omega \\ 0 \end{bmatrix} ; \quad \underset{(x,y,z)}{\underline{\omega}_p} = \begin{bmatrix} 0 \\ 0 \\ \omega_p \end{bmatrix}$$

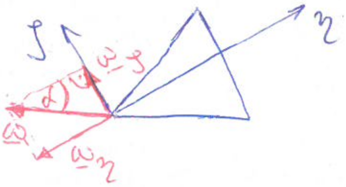
Relatív kinematika: ráülünk η tengelyre és nézzük a körlapot. Azt látom, hogy a kúp előttem forog.

$\underline{\beta}_A = \underline{0}$; $\underline{\beta}_S = \underline{0}$ $\longrightarrow$ η körül forog $\longrightarrow$ $\underline{\omega}_r \parallel \underline{r}_{AS}$

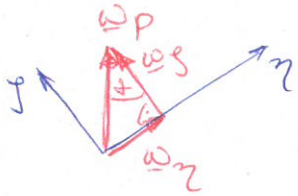
$$\underset{(\xi,\eta,\zeta)}{\underline{\omega}_r} = \begin{bmatrix} 0 \\ -\omega_r \\ 0 \end{bmatrix}$$

Mivel $\underline{\underline{\Theta}}_A$ a (ξ,η,ζ) koord. rsz-ben van felírva, $\underline{\omega}$-t is ebben kellene megkapni. $\underline{\omega}$-t és $\underline{\omega}_p$-t transzformáljuk ide (x,y,z)-ből!

Forgatás szöge: α



$$\underset{(\xi,\eta,\zeta)}{\underline{\omega}} = \begin{bmatrix} 0 \\ -\omega\cos\alpha \\ \omega\sin\alpha \end{bmatrix}$$



$$\underset{(\xi,\eta,\zeta)}{\underline{\omega}_p} = \begin{bmatrix} 0 \\ \omega_p\sin\alpha \\ \omega_p\cos\alpha \end{bmatrix}$$

Ezzel $\underline{\omega} = \underline{\omega}_p + \underline{\omega}_r$ (ξ,η,ζ)-ban:

$$\begin{bmatrix} 0 \\ -\omega\cos\alpha \\ +\omega\sin\alpha \end{bmatrix} = \begin{bmatrix} 0 \\ \omega_p\sin\alpha \\ \omega_p\cos\alpha \end{bmatrix} + \begin{bmatrix} 0 \\ -\omega_r \\ 0 \end{bmatrix} \longrightarrow \quad \omega = +\omega_p \underbrace{\frac{\cos\alpha}{\sin\alpha}}_{\mathrm{ctg}\,\alpha} = +\omega_p \frac{h}{R}$$

Kinetikus energia:

$$\underset{(\xi,\eta,\zeta)}{\underline{\omega}} = \begin{bmatrix} 0 \\ -\omega_p \frac{h}{R}\cdot\frac{h}{\sqrt{h^2+R^2}} \\ \omega_p \frac{h}{R}\cdot\frac{R}{\sqrt{h^2+R^2}} \end{bmatrix} = \begin{bmatrix} 0 \\ \omega_\eta \\ \omega_\zeta \end{bmatrix}$$

$$T = \frac{1}{2}\underline{\omega}^T \underline{\underline{\Theta}}_A \underline{\omega}$$

$$T = \frac{1}{2}\begin{bmatrix} 0 & \omega_\eta & \omega_\zeta \end{bmatrix} \begin{bmatrix} \Theta_\xi & 0 & 0 \\ 0 & \Theta_\eta & 0 \\ 0 & 0 & \Theta_\zeta \end{bmatrix} \begin{bmatrix} 0 \\ \omega_\eta \\ \omega_\zeta \end{bmatrix}$$

$$\begin{bmatrix} 0 & \omega_\eta\Theta_\eta & \omega_\zeta\Theta_\zeta \end{bmatrix} = \frac{1}{2}\left(\omega_\eta^2\Theta_\eta + \omega_\zeta^2\Theta_\zeta\right) =$$

$$= \frac{1}{2}\left(\frac{3}{10} m R^2 \left(\omega_p \frac{h}{R}\frac{h}{\sqrt{h^2+R^2}}\right)^2 + \left(\frac{3}{80} m\left(4R^2+h^2\right) + m\left(\frac{3}{4}h\right)^2\right)\left(\omega_p\frac{h}{R}\frac{R}{\sqrt{h^2+R^2}}\right)^2\right) = \underline{\underline{2{,}397}}$$