Synthesizing Safety-Critical Controllers for Systems with Input Delay

Tamas G. Molnar *, **Adam K. Kiss** **, **Aaron D. Ames** *, and
Gábor Orosz ***

* *Department of Mechanical and Civil Engineering, California Institute of Technology, Pasadena, CA 91125, USA (e-mail: tmolnar@caltech.edu, ames@caltech.edu).*
** *MTA-BME Lendület Machine Tool Vibration Research Group, Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, H-1111, Hungary (e-mail: kiss_a@mm.bme.hu)*
*** *Department of Mechanical Engineering and Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, MI 48109, USA, (e-mail: orosz@umich.edu)*

Time delays often occur in the feedback loops of control systems due to sensing, signal processing, computation and actuation. If the delays are large, they may significantly affect the closed-loop dynamics and performance. This is crucial in safety-critical systems where unsafe behavior caused by delays is not tolerable. In this work, we establish formal safety guarantees for continuous-time control systems with input delay of the form:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t-\tau). \tag{1}$$

We show that controllers designed to guarantee safety for delay-free systems may become unsafe once time delay is introduced. Thus, to recover formal safety guarantees, we extend the theory of control barrier functions to systems with input delay. We establish safety through rendering a desired safe set in the state space forward invariant, and we propose an optimization-based method to synthesize provably safe controllers.

To achieve this goal, we leverage the idea of predictor feedback. The future state of the system is predicted over the delay interval by forward integration of a nominal model. Then, the predicted state is used for feedback in an optimization-based control strategy that relies on control barrier functions. The proposed approach ultimately leads to controllers that depend on the most recent state estimate and the history of the control input. Ideally, the predictor eliminates the effects of the time delay, however, this requires the predicted state to accurately match the future state of the system. Considering the inevitable mismatch between prediction and reality, we supplement the controller with robustifying terms to achieve safety even in the presence of bounded prediction errors. Special care is taken in cases where safety is affected not only by the state of the dynamical system but also by the state of the environment, whose future may be unknown or uncertain. The resulting controllers may possess some level of conservatism to achieve robustness, but they are provably safe.

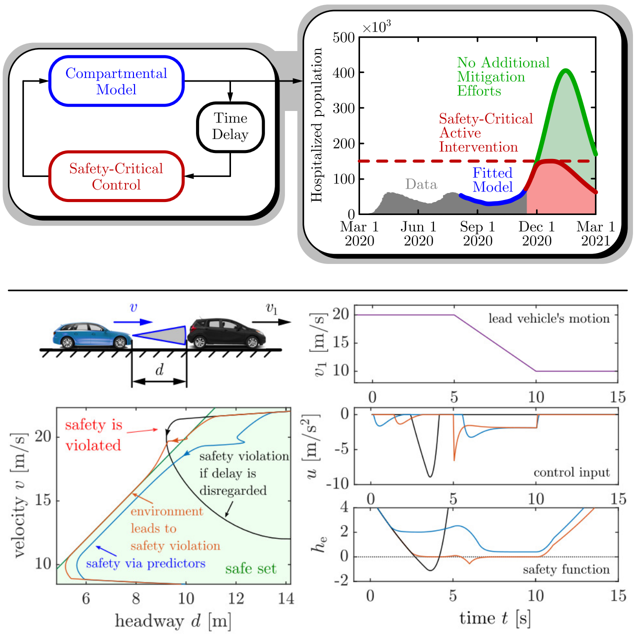

Fig. 1. Practical application of safety-critical control to systems with input delays. Top: control of epidemiological models describing the spread of COVID-19. Bottom: adaptive cruise control (ACC) system.

We demonstrate our method by practical examples, including the control of epidemiological models where significant delays arise due to incubation period and testing, and an adaptive cruise control (ACC) problem for automated vehicles where the motion of other vehicles creates an uncertain environment.

[★] This research is supported in part by the National Science Foundation, CPS Award #1932091, Dow and Aerovironment.