Parameter identification of periodic systems by impulse dynamic subspace description

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Summary. In this contribution, a parameter identification technique is introduced for delayed time-periodic differential equation, which is based on the so-called impulse dynamic subspace (IDS). The description is tested on the delayed Mathieu equation. Then, a possible application is discussed for chatter detection technique in machine tool vibrations, and its limitation due to arising nonlinear behaviour is summarized.

Introduction

The main goal of this work is to characterize the dynamical behaviour of delayed periodic systems without substantial knowledge of the underlying model. Our assumptions are that the system is a linearized version of a nonlinear phenomenon and the time-period is known. The main idea is to capture the dominant spectral properties of the system from its impulse response. In order to achieve this, the so-called impulse dynamic subspace (IDS) is used [1]. This is an efficient description for possible automatic parameter fitting for mechanical structures. Originally, the method is developed for linear time-invariant systems. It is based on the evaluation of frequency response functions by using Green function representation of the homogeneous dynamics. However, in this work, we slightly modify the original ideas of the method to make it suitable for periodic systems. In this way, here, we apply this method for periodic non-autonomous systems and we characterize its the Floquet multipliers (characteristic multiplier), consequently, we characterize the stability properties of the underlying system without knowing its parameters. The method and its applicability is tested on a time-periodic delay-differential equation (DDE), the so-called delayed Mathieu equation by using time signal generated by numerical integration. Finally, one possible application field is discussed from the topic of machine tool vibrations, namely quantitative chatter detection and stability prediction method in milling operations [2].

Impulse dynamic subspace description for periodic non-autonomous case

The IDS can describe the relevant dynamics of a structure by means of the singular value decomposition applied on the Green function representation of the vibration signal $x(t)$ of a periodic non-autonomous system with time-period T. For detailed derivation on the IDS description, the reader is referred to [1]. The so-called block-Hankel matrix representation of the Green function for time-periodic system is

$$\mathbf{G} = \begin{bmatrix} \mathbf{x}_0 & \mathbf{x}_1 & \dots & \mathbf{x}_N \\ \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_{N+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}_N & \mathbf{x}_{N+1} & \dots & \mathbf{x}_{2N} \end{bmatrix}, \tag{1}$$

where $\mathbf{x}_i$ is the discretized state vector along a period T. It form as

$$\mathbf{x}_i = [x(iT)\ x(iT+\Delta t)\ \dots\ x(iT+(n-1)\Delta t)] = \mathrm{row}_{j=0}^{n-1} x(iT+j\Delta t), \tag{2}$$

where the number of sampled points in one period is n and the discretized time step is $\Delta t = T/n$. Note that in case of time-periodic systems, the blocks are shifted by 1 time-period in the Hankel matrix. By using singular value decomposition (SVD) of the Hankel matrix $\mathbf{G} = \mathbf{V\Sigma W}^{\mathrm{H}}$, the dominance of subsystems can be identified. Here $\mathbf{V}$ and $\mathbf{W}$ are the stroboscopic- and sampled-singular-responses (SR) (alike singular-IRF in [1]), respectively. By taking an appropriate singular set, as a result, using the truncated versions of the stroboscopic-SR $\tilde{\mathbf{V}}$, a discrete map can be defined as

$$\mathbf{B} = \tilde{\mathbf{V}}^{\mathrm{H}}\mathbf{S}\tilde{\mathbf{V}}, \tag{3}$$

where $\mathbf{S}$ is a shift matrix with non-zero elements $S_{i+1,i} = 1$. The eigenvalues of $\mathbf{B}$ are the Floquet multipliers of the periodic non-autonomous (time-variant) system.

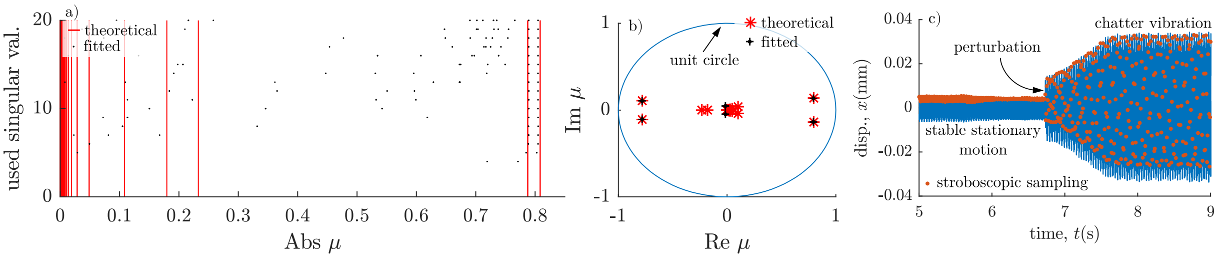

Figure 1: Effect of truncation on fitted multipliers (a), dominant multipliers in the complex plane (b); bistabe machining process (c).

Test case

In this section, the time periodic DDEs are detailed, on which IDS description was applied. The general form of the delayed Mathieu equation reads

$$\ddot{x}(t) + a_1\dot{x}(t) + (\delta + \varepsilon \cos t)\, x(t) = b_0 x(t - \tau) \ . \tag{4}$$

where the time-period is $T = 2\pi$. Here, the special case will be investigated, when the time delay is just equal to the principal period ($\tau = T$). For periodic systems, the stability condition is provided by the Floquet theory.
In order to validate the description by IDS, we test it on the delayed Mathieu equation, where the fitted dominant multipliers are compared directly to the theoretically ones calculated by the semidiscretization method (SDM) [3]. The IDS description is applied on numerically generated time signal, with parameters $\delta = 3.1605,\ \varepsilon = 1,\ a_1 = 0,\ b_0 = -0.6246$ and initial function $[x_0\ \ \dot{x}_0]^\top = \mathbf{0}$ if $t \in [-\tau, 0)$ and $[x_0\ \ \dot{x}_0]^\top = [0\ \ 1]^\top$ if $t = 0$. The number of the simulated periods are $2N = 120$ and the discretized number of points during 1 period is $n = 100$. Consequently, the size of the Hankel matrix is $N \times n\, N$. The size of the problem is decreased by truncating the Hankel matrix squarely, thus, considering only $N \times N$ elements. Then, the IDS description is applied with several truncated versions of the stroboscopic-SR $\tilde{\mathbf{V}}$, also (see Eq. 3).
The magnitude of the fitted multiplier are plotted in Fig. 1a with black dots together with the theoretical values (red vertical lines) as a function of the truncation of $\tilde{\mathbf{V}}$. For a few considered stroboscopic-SR, the IDS description can capture 2 complex conjugate pair of multipliers with large magnitude with good accuracy. On the other hand, taking into account more stroboscopic-SR, the fitted multipliers start to scatter. To select the appropriate number of stroboscopic-SR, the singular values of the Hankel matrix can be analysed, see more details in [1]. In this case, we select 6, since all the other singular values were merely close to zero. This case is illustrated in the complex plane in Fig. 1b. As shown in the figure, the IDS description is not only able to capture the magnitude of the multipliers, but its imaginary and real components as well. However, as cons of the proposed method, it could not identify multipliers with small magnitude, since solution segments relating to these terms are decaying very fast. Nevertheless, from the practical point of view, those are usually not relevant in engineering applications.

Stability prediction in machining

One possible application of the described method can be used in the operational stability prediction in milling processes [2]. During machining, an undesired phenomenon, the so-called chatter vibration can lead to unacceptable surface quality and possible damage in the machine components. Therefore, avoidance, or at least, the prediction of these vibrations is necessary.
During the operational stability prediction technique, the stability of the machining process can be characterized through the Floquet multipliers. They can be determined during machining by applying the above discussed method. By changing technological parameters, the variation of the modulus of the Floquet multipliers can be monitored, by which, precise stability limit can be extrapolated while the manufacturing parameters remain in the safe region.
On the other hand, according to practical observations, usually there is a bistable region near the linear stability boundary due to unmodelled nonlinear effects. This is also unsafe zone since two attractive motions coexist: stable cutting and large amplitude chatter, which are separated by an unstable periodic motion [4, 5]. In this range, the vibration can jump from the linear attraction zone to the chatter motion due to a large-enough perturbation, as shown in Fig. 1c. Consequently, one cannot excite a stable stationary solution in this range and thus one cannot determine the multipliers of the linearized system.

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