Quantitative identification of chatter based on Floquet multipliers in milling operation

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Summary:
In this contribution, a chatter detection method is presented for general milling operations, which quantifies stability through the determination of the Floquet multipliers. The corresponding monodromy operator is approximated from the measured system's response during the machining operation. The precise stability limit can be extrapolated while the manufacturing parameters remain in the safe region. The presented approach is validated by laboratory tests.
Keywords: milling, stability, chatter detection, Floquet multiplier

1. Introduction

In the manufacturing industry, productivity is a key factor besides quality, efficiency and sustainability. However, productivity cannot be increased arbitrarily partly due to undesired vibrations that may arise during the cutting process [1]. The prediction and avoidance of these vibrations are still an active research area. This undesired phenomenon, so-called chatter, leads to unacceptable surface quality and possible damage in the machine components. From dynamical systems' point of view, chatter is associated with the loss of stability of the stationary (chatter-free) machining process. We introduce a method that is capable to provide a quantitative measure of stability based on measured vibrations, where there is no need for modal parameter identification and experimentally tuned threshold, which are usually defined empirically in case of traditional chatter detection methods

The main idea is to approximate the Floquet multipliers of the corresponding mechanical system based on response during the machining. By changing technological parameters, the variation of the modulus of the Floquet multipliers can be monitored. The stability limit can be interpolated precisely for the unitary multiplier, furthermore, the stability limit can be extrapolated while the manufacturing parameters remain only in the chatter-free region.

2. Determination of the Floquet multipliers

The stability of the stationary milling process of a test rig (see Fig. 1a) can be analysed through the variational delay differential equation (DDE) of the form (see in details [2])

$$\mathbf{M}\ddot{\boldsymbol{\eta}}(t) + \mathbf{C}\dot{\boldsymbol{\eta}}(t) + \mathbf{K}\boldsymbol{\eta}(t) = \mathbf{G}(t)\big(\boldsymbol{\eta}(t) - \boldsymbol{\eta}(t-\tau)\big), \tag{1}$$

where $\mathbf{M}, \mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matricies, respectively. $\mathbf{G}(t)$ is the τ-periodic directional force coefficient matrix and $\boldsymbol{\eta}(t)$ is the small perturbation around the stationary machining (Fig. 1b). According to the Floquet theory of DDEs [2], we can define the discretized state space for the i^{th} period ($t_i = t_0 + i\tau$) as

$$\boldsymbol{\eta}_i = \text{col}_{j=0}^{p-1}\boldsymbol{\eta}(t_i - j\Delta t), \tag{2}$$

where the number of sampled points is p, the discretized time step is $\Delta t = \tau/p$. The stability is determined by the corresponding monodromy matrix $\boldsymbol{\Phi}$, which maps the states $\boldsymbol{\eta}_i$ to $\boldsymbol{\eta}_{i+1}$. In our proposed method, we use the mapping of the measured states $\bar{\boldsymbol{\eta}}_i$ for each period:

$$\mathbf{X}_{i+1} = \bar{\boldsymbol{\Phi}}\mathbf{X}_i, \quad (3)$$

where $\mathbf{X}_i = [\bar{\boldsymbol{\eta}}_i \quad \bar{\boldsymbol{\eta}}_{i+1} \quad \cdots \quad \bar{\boldsymbol{\eta}}_{q-1}]$, $q > p$. Then $\bar{\boldsymbol{\Phi}}$ can be expressed as $\bar{\boldsymbol{\Phi}} = \mathbf{X}_{i+1}\mathbf{X}_i^\dagger$, where $\mathbf{X}_i^\dagger = \mathbf{X}_i^\intercal(\mathbf{X}_i\mathbf{X}_i^\intercal)^{-\mathbf{1}}$ is the generalized inverse (or Moore-Penrose pseudoinverse) of $\mathbf{X}_i$, which provides the solution in linear least square sense. The characteristic multipliers μ can be calculated from $\bar{\boldsymbol{\Phi}}$ based on the transient vibration of the measured and filtered acceleration (see Fig. 1ab), which gives a good approximation for the dynamical behaviour of the system (stable if $\max(|\mu| < 1)$. In Fig. 1c, we compare the determined multipliers to the theoretical ones for a set of different spindle speeds. The stability boundary can be determined precisely by means of extrapolation based on stable measurement points, only (see 7900, 8000, 8100 rpm in Fig. 1d).

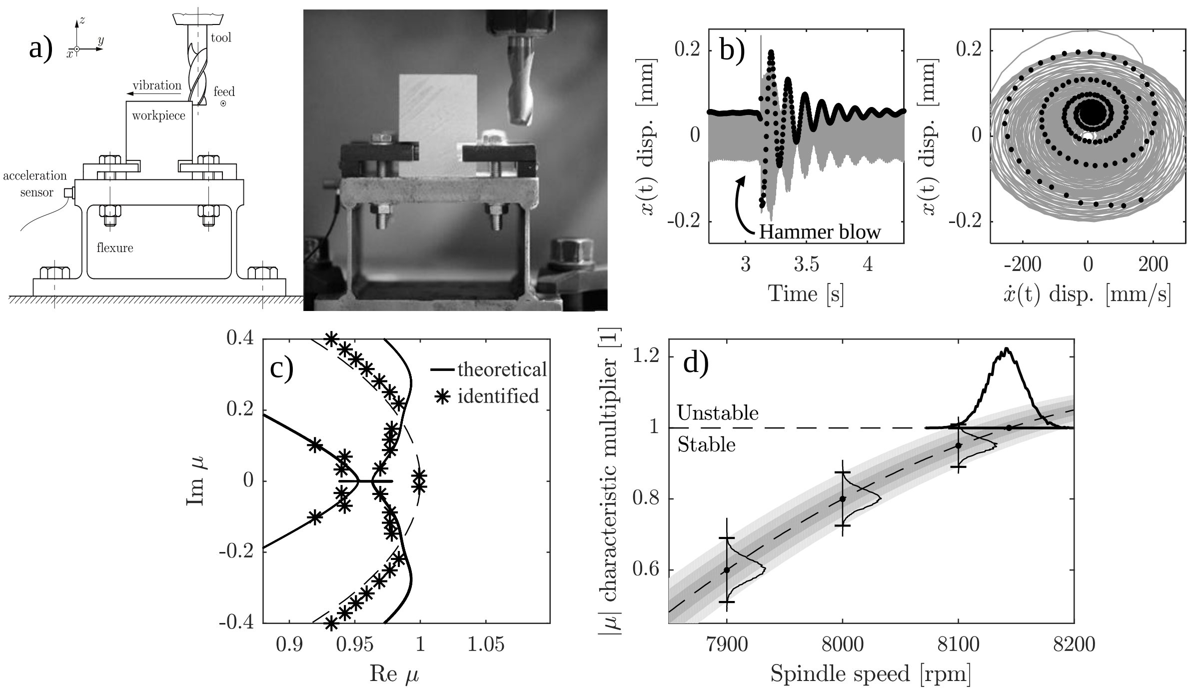

Figure 1 a: The experimental setup for milling with single-degree-of-freedom structure [3]*; b: time domain representation and phase plane of measured signal and its stroboscopic sections before and after hammer excitation; c: identified dominant multipliers for different spindle speeds (denoted by stars) in the complex plane compared to the path of the theoretical ones (continuous line). d: schematic figure representing the extrapolation method of the stability boundary based on stable measurement points.*

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