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Cumulative Surface Location Error for milling processes based on tool-tip Frequency Response Function

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Abstract

In milling processes, the desired machined surface cannot be perfectly achieved even in case of chatter-free machining due to the thermally induced errors, the trajectory following errors and the most significant one: the cutting force induced vibration errors. In case of vibration, the error is represented by the so-called Surface Location Error (SLE), which is the distance between the machined and the required surface position. In case of roughing operations, these errors can have a significant impact on the surface position due to the interaction between the subsequent SLEs. The machined surface depends on the previously resulted SLE through the variation of the radial immersion. In this paper, the series of the consecutive SLEs are investigated in a multi-degree-of-freedom model. The dynamical behaviour of the milling tool is described by frequency response functions. The variation of the SLE values is governed by a discrete map, which may lead to an unpredictable final surface position. The parameter range where this unpredictable final SLE occurs is presented together with the traditional stability chart representing the chatter-free domains of cutting parameters. With the proposed methods, the traditional stability chart can be improved, from which chatter-free and CSLE-stable technological parameters can be selected.

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1. Introduction

In industry, milling is a widely used manufacturing method. This machining process may induce harmful vibrations which are responsible for unacceptable surface quality. These vibrations are classified into two groups [1]. One of them is the self-excited vibration that is due to the loss of stability typically related to the surface regeneration effect [2]. This effect can be modelled with delay-differential equations (DDE) [3]. The other type of vibration is the large amplitude forced vibration near to resonant spindle speeds [4]. These forced vibrations lead to the so-called *Surface Location Error (SLE)*, which is the largest deviation between the machined and the desired surface.

The so-called stability chart [5] presents the chatter-free (stable) parameter domain which is usually illustrated in the plane of the spindle speed and the axial immersion. It can be calculated by means of methods in frequency domain or in

time-domain [6-10]. The stability boundary has pockets between the so-called stability lobes typically around the resonant spindle speeds. The most productive chatter-free parameter regions are located in these pockets, however, large amplitude vibrations occur there due to the resonant excitation of the periodic milling force, which leads to substantial *SLE* [11]. The stability chart is useful during the roughing operations. Due to the fact that *SLE* is relevant at finishing operations [12], its effect is usually neglected in the model of roughing operations. Despite this, the *SLE* can still have significant impact on the surface quality in case of consecutive immersions during roughing, which is used to remove the oversize of a workpiece.

At each immersion, the machined surface differs from the desired one due to the Surface Location Error. This current offset error modifies the next radial immersion and generates different cutting force, which leads to a modified *SLE* at the subsequent immersion. During this process, the *SLE* can be

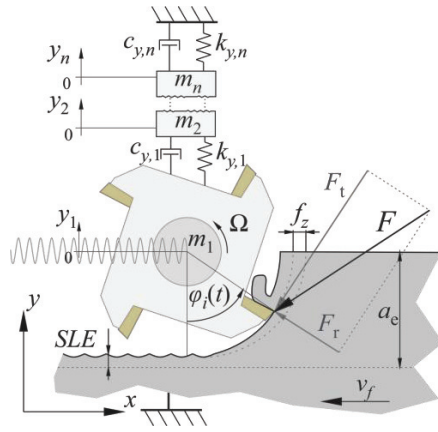


Fig. 1. Mechanical model of the milling process and representation of the Surface Location Error (SLE)

accumulated immersion-by-immersion which converge to a certain value, which is called *Cumulative Surface Location Error (CSLE)* [13, 14].

In this paper the evaluation of the series of *SLE* values is investigated in milling operations by means of a multi-degree-of-freedom model with multiple modes based on measured tool-tip frequency response functions (*FRF*).

2. Forced vibration

In what follows, the main steps of the *SLE* computation are summarised based on [15]. The quality of the machined surface, specifically, the *Surface Location Error (SLE)* is determined numerically in case of a straight fluted tool.

In the mechanical model, the workpiece is considered as a rigid body and the cutting tool is modeled as shown in Fig. 1. This assumption is valid for long thin peripheral milling tools. The *SLE* is influenced by the vibration perpendicular to the machined surface only, hence, the modes are projected to the *y* direction. The stable (chatter-free) forced vibration of the tool centre can be described by a non-homogeneous ordinary differential equation in the following form:

$$M\ddot{\mathbf{y}}(t) + C\dot{\mathbf{y}}(t) + K\mathbf{y}(t) = \mathbf{Q}(t) \tag{1}$$

where the generalized coordinates are arranged in the column vector $\mathbf{y}(t) = [y_1(t), y_2(t), \dots, y_n(t)]^T$, the dynamical parameters of the tool are defined in the modal matrices \mathbf{M} , \mathbf{C} and \mathbf{K} , and the resultant cutting-force acting on the tool-tip is included in the general force $\mathbf{Q}(t) = [F_y(t), 0, \dots, 0]^T$. In the applied model, the linear force model [5] is used to describe the cutting force, where the magnitude of the resultant force is linearly proportional to the current chip thickness and chip width [15]:

$$F_y(t) = \sum_i^N a_p f_z \sin(\varphi_i(t)) g_i(\varphi_i(t)) (K_r \cos(\varphi_i(t)) - K_t \sin(\varphi_i(t))) \tag{2}$$

where N is the number of teeth, a_p is the axial immersion, f_z is the feed per tooth, $\varphi_i(t)$ is the angular position of the i^{th} cutting edge, $g_i(\varphi_i(t))$ is the screen function which indicates that the i^{th} edge is in contact with the material ($g = 1$) or not ($g = 0$). K_r and K_t are the radial and tangential cutting force coefficients, respectively. The current angular position of the i^{th} cutting edge is

$$\varphi_i(t) = \Omega t + \frac{2\pi \cdot (i-1)}{N} \tag{3}$$

where Ω is the angular spindle speed in rad/s. The intermittent cutting force leads to periodic force excitation. The resulting forced vibration is copied to the surface during the cutting processes and creates surface errors.

The periodic motion of the tool is determined after the Fourier transformation of the equation of motion (Eq. (1)):

$$\Psi(\omega) = (-\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K})^{-1} \cdot \Phi(\omega) \tag{4}$$

where $\Phi(\omega)$ and $\Psi(\omega)$ are the Fourier transforms of the general force $\mathbf{Q}(t)$ and displacement vector $\mathbf{y}(t)$, respectively.

During the numerical implementation, $\Phi(\omega)$ is approximated by the Fast Fourier Transformation (*FFT*) of $\mathbf{Q}(t)$ defined by Eq. (2), and $\mathbf{y}(t)$ is calculated by means of the inverse Fast Fourier Transformation (*iFFT*) of the resultant $\Psi(\omega)$ from Eq. (4). For the *SLE* computation, only the forced periodic vibration $y_1(t)$ of the tool-tip is needed in time domain. This scalar component can be defined based on the Measured tool-tip *FRF* ($H_{11}(\omega)$):

$$y_1(t) = iFFT\{H_{11}(\omega) \cdot \Phi_1(\omega)\} \tag{5}$$

3. Surface Location Error

The surface contour is defined by the motion of the cutting edges and their forced vibrations, synchronized with the tooth passing frequency which lead to a constant offset error. The *Surface Location Error (SLE)* is the maximum distance between the milled and desired surface (see Fig. 1.) defined by:

$$Up - mill.: SLE = \min_i \left(-\frac{D}{2} \cos \varphi_i(t) + y_1(t) \right) + \frac{D}{2} \tag{6}$$

$$Down - mill.: SLE = -\left(\max_i \left(-\frac{D}{2} \cos \varphi_i(t) + y_1(t) \right) - \frac{D}{2} \right) \tag{7}$$

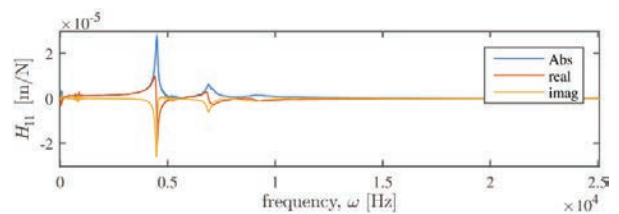


Fig. 2. Measured FRFs at the tool-tip

for the cases of up-milling and down-milling, respectively, where D is the diameter of the milling tool. In this study, the transient vibrations are not analysed, that is, we assume that the transient vibration settles quickly. Note that in case of high feed rates, this assumption cannot be used.

In the subsequent section, the influence of the dimensionless radial immersion on SLE is analysed. In order to do this, define the function f_{SLE} that describes the dependence of the *Surface Location Error* on the dimensionless radial immersion:

$$SLE = f_{SLE}(a_c / D). \tag{8}$$

4. Cumulative Surface Location Error

Consider the influence of the actual SLE_i on the dimensionless radial immersion of the subsequent cutting, which leads to a modified SLE_{i+1} (see in Fig. 3.). During the roughing process, one can set the desired dimensionless radial immersion $a_0 = a_{e,0}/D$, where $a_{e,0}$ is the pre-set radial immersion designed for the tool path. The radial immersion is modified by the actual SLE_i , which results the subsequent dimensionless radial immersion $a_{i+1} = a_0 + SLE_i/D$ and the subsequent error

$$SLE_{i+1} = f_{SLE}\left(a_0 + SLE_i \frac{1}{D}\right). \tag{9}$$

This discrete map can be calculated by means of the dynamical milling model described in Section 3. Eq (9) determines how an SLE develops into another SLE over immersion-by-immersion. These series of the $SLEs$ may converge to a fix point, called *Cumulative Surface Location Error (CSLE)*:

$$CSLE = \lim_{i \rightarrow \infty} SLE_i \tag{10}$$

and it can be computed as a root of the following equation:

$$f_{SLE}\left(a_0 + CSLE \frac{1}{D}\right) - CSLE = 0. \tag{11}$$

The solutions of this non-linear equation are obtained by two different methods. One of them is a numeric iteration of the map Eq. (9) for different initial SLE values. The other method is based on the numerically calculated roots of Eq. (11) by means of the so-called Multi-Dimensional Bisection Method [17]. Note, that this mapping can have stable and unstable fix points ($CSLE$), moreover, it can also have periodic or chaotic solutions, which means that the SLE varies after each immersion.

5. Case study

The above-described $CSLE$ computation method is applied to a case study. During the calculation, the measured tool-tip FRF (see Fig. 2.) of a long peripheral milling tool and the technological parameters presented Table 1 are used. The resulted SLE is presented in Fig. 4a as a function of the dimensionless radial immersion for constant spindle speed 7100 rpm. The dimensionless range of the horizontal axis in

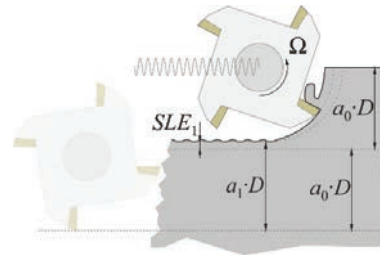


Fig. 3. Sketch of the variation of the radial immersion as a result of the previous SLE

Fig. 4 is $a_0 \in [-1, 0]$ and $a_0 \in [0, 1]$ representing down-milling and up-milling, respectively. Note, that $a = \pm 1$ refers to full immersion.

The calculated stable $CSLE$ values (thick line) and unstable ones (dotted line) are shown in Fig. 4b together with the corresponding stable period-doubling solutions of the SLE series (thick closed curves around the unstable $CSLE$ values). The blue regions denote areas, where period-2 solutions of SLE branches enclose an unstable $CSLE$ solution. In practice, this phenomenon appears as an alternating SLE series.

It is also shown, that multiple (bistable) solutions co-exist in the red colored regions. Here, one unstable and two stable $CSLE$ occur. In this case, based on the initial condition, the SLE series will converge to one of the stable values.

In both *surface error stability phenomena*, the final surface error cannot be predicted at the end of the roughing process. Thus, the oversize of the workpiece will be uncertain before the finishing process, therefore the corresponding technological parameters are unfavourable. These colored (blue or red) domains always contain at least one unstable $CSLE$ solution, which is enough to identify them and their boundaries.

The influence of the spindle speed on the *surface error stability phenomenon* is also analysed. The boundaries of the alternating (blue) and bistable (red) $CSLE$ regions are presented together with the traditional stability chart (black) in the plane of the spindle speed and dimensionless radial immersion in

Table 1. Parameters of the case study.

Parameter	Value	Parameter	Value
Feed per tooth	f_t 0.15 [mm]	Axial immersion	a_p 6 [mm]
Number of teeth	N 3 [-]	Radial force coeff.	K_r $237 \cdot 10^6$ [N/m ²]
Tool diameter	D 8 [mm]	Tang. force coeff.	K_t $644 \cdot 10^6$ [N/m ²]

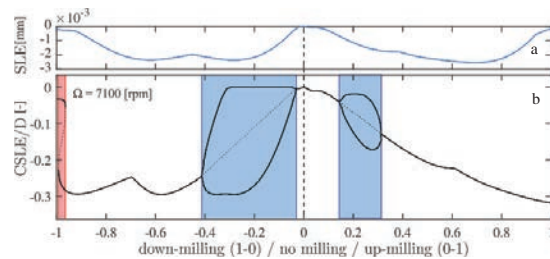


Fig. 4. Calculated SLE and $CSLE$ for up-milling and down-milling. Blue regions represent period-2 solutions; red areas represent bistable regions

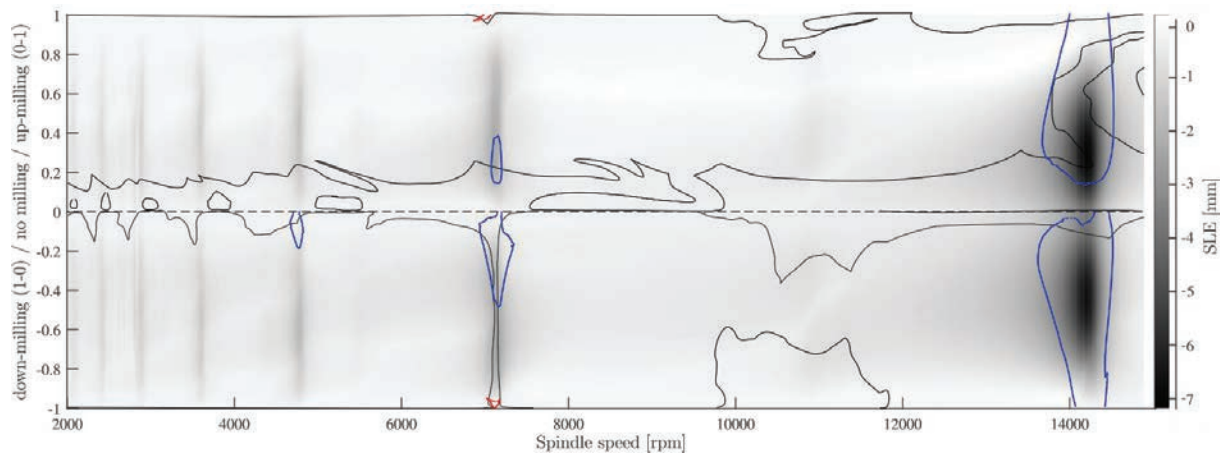


Figure 5. Stability charts; Red contour represents the period-2 solution, blue represents the bistable region, black represents the traditional stability chart (which comes from the regenerative effect), gray scale shows the calculated *SLE*

Figure 5. Note that the stability chart does not show the traditional lobe structure since the vertical axis is not the axial immersion. Still, the gray scale representing the *SLE* values after a single immersion refers to the location of the pockets (dark areas) between the lobes. Note, that the source of extreme high *SLE* values is the resonant excitation of the slightly damped mechanical system. In practice, these high values are limited by non-modelled phenomenon. The stability chart was calculated by the Extended Multi Frequency Solution [8].

Figure 5 shows that the dark favourable parameter domains between the stability lobes, where large material removal rates (*MRR*) can be achieved, the *surface error stability* problem does appear. While the surface roughness may be acceptable in these chatter-free high *MRR* domains, the oversize of the material may not be precisely predicted before the finishing process.

4. Conclusion

In the present study, we show a new type of stability loss related to a surface quality parameter, which is called *surface error stability* problem. This can occur with respect to the *Surface Location Error* parameter during roughing processes. This *CSLE-stability* problem leads to an unpredictable final *Surface Location Error*, which can affect the finishing operation substantially. With the proposed methods, the traditional stability chart can be improved, from which chatter-free and *CSLE-stable* technological parameters can be selected. Unfortunately, the *CSLE-unstable* domains usually appear around the resonant spindle speeds, where high material removal rate could be achieved otherwise.

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