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Unstable Islands in the Stability Chart of Milling Processes Due to the Helix Angle

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Abstract: Dynamics and stability of milling operations with helical fluted tool are investigated in time domain. Stability analysis is performed based on the governing delay-differential equation using the numerical semi-discretization method. It is shown that unstable flip (period doubling) islands arise in the stability lobe diagram due to the helical flutes of the tool. These islands are separated by the lines where the axial depth of cut is equal to the multiples of the helix pitch. The phenomenon is explained using the Floquet theory of periodic delayed systems. The theoretical results are confirmed by experiments.

Keywords: milling, stability lobes, period doubling

1. INTRODUCTION

The commercialization of reliable high-speed machining systems during the past decade has driven the need for thorough dynamical investigations of high-speed cutting processes. One of the most important phenomena that limits the productivity of machining is the development of self-excited vibrations (known also as machine tool chatter) caused by the regeneration of surface waviness during successive cuts. The pioneering work of [Tlusty et al., 1962] and [Tobias, 1965] led to the development of the so-called stability lobe diagram that plots the boundary between stable and unstable depths of cut as a function of spindle speed.

The mathematical model of the regenerative machine tool chatter is a delay-differential equation (DDE) with a corresponding infinite dimensional state space (see, e.g., [Stépán, 1989]). The model equation for turning processes is a DDE with constant coefficients (autonomous DDE) while for milling operations, the repeated entering and exiting teeth of the rotating tool yields a DDE with time periodic coefficients. Stability analysis of periodic delayed systems is extremely difficult, and – apart from some trivial cases – close form stability conditions can not be given. Stability charts for milling processes are usually determined either via numerical simulations (see [Smith and Tlusty, 1991], [Balachandran and Zhao, 2000], [Campomanes and Altintas, 2003]) or via different semi-analytical approximation techniques (see, e.g., [Altintas and Budak, 1995, 1998], [Insperger and Stépán, 2000, 2004a], [Bayly et al., 2003], [Szalai and Stépán, 2003, 2006]).

In turning processes, machine tool chatter is associated with Hopf instability that corresponds to a periodic oscillation of the tool-workpiece system at a well defined chatter frequency. In milling processes, chatter is associated with either secondary Hopf or flip (period doubling) instability. In the case of secondary Hopf instability, quasi-periodic chatter arise, while flip chatter is associated with vibrations at frequency equal to the half of the tooth passing frequency. It should be mentioned that flip instability is directly related to the time-periodic nature of the milling process, it mostly occurs for operations with small radial immersion when the directional cutting force is strongly time-dependent.

Period doubling phenomenon and the corresponding added stability lobes were discovered only in the recent years. [Davies et al., 2000, 2002] modeled small radial immersion milling as an impact-like cutting process and received analytical formulae for the flip stability boundaries. [Insperger and Stépán, 2000] investigated the 1 DoF model of milling and used the Fargue-type approximation of the corresponding periodic DDE. The results were confirmed by several other techniques: [Bayly et al., 2003] used the time finite element method, [Merdol and Altintas, 2004] used the multi-frequency solution developed by [Altintas and Budak, 1995, 1998], [Szalai and Stépán, 2003, 2006] determined the characteristic functions of the system and obtained stability criteria using the argument principle, [Corpus and Endres, 2004] reduced the problem to the flip boundaries where ordinary differential equations (ODEs) describes the system instead of DDEs, [Insperger and Stépán, 2004a] used the semi-discretization method to show the effect of the time-periodicity of the milling process on the stability lobe diagram. Basically, all the abovementioned methods are based on the Floquet theory of DDEs. The existence of flip chatter was also confirmed by experiments in [Davies et al., 2000, 2002], [Bayly et al., 2003], [Mann et al., 2003, 2004], [Gradišek et al, 2005].

Unstable islands in the stability charts were first found by [Szalai and Stépán, 2003, 2006]. They investigated the interrupted turning process that can be considered as a model for small radial immersion milling. Based on a mathematically very accurate analysis, they showed that all the added stability boundaries, except the first one at the highest cutting speed, are closed curves (islands, lenses). Their finding was confirmed for milling processes in [Gradišek et al. 2005] using the numerical semi-discretization method. Still, in spite of the theoretical predictions, clear experimental verification of the unstable islands was not provided yet since the corresponding depth of cut values are usually relatively large.

Flip islands associated with the helix angle of the tool was recently reported in [Zatarain et al., 2006]. They extended the multi-frequency solution of [Budak and Altintas, 1998] for milling cases with helical tool, and received that the flip instability areas (including the first flip lobe) are closed islands separated by the horizontal lines where the depth of cut equals a multiple of the mill helix pitch. Note that the flip islands found by [Zatarain et al., 2006] are directly related to the helix angle of the tool, while the flip islands found by [Szalai and Stépán, 2003] are independent of the helix angle.

In the current paper, the results of [Zatarain et al., 2006] are investigated based on time domain considerations. A mathematical explanation is given for the helix-induced flip islands based on the Floquet theory of DDEs.

2. MILLING MODEL WITH HELIX ANGLE

Machine tool chatter is induced by the relative vibrations between the tool and the workpiece. In general cases, both the workpiece and the tool experience vibrations and the chatter occurs along the most dominant modes of the tool-workpiece structure. Here, we will investigate the 1 DoF system presented in Figure 1. The workpiece is compliant in the y direction that is perpendicular to the feed direction x. The tool is assumed to be rigid. The governing equation of motion reads

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = H(t, a_{\mathrm{p}})(x(t) - x(t-\tau)), \tag{1}$$

where m, c, k are the mass, the damping and the stiffness parameters and $H(t,a_{\mathrm{p}})$ is the directional force coefficient. The regenerative delay is $\tau = 60/(N\Omega)$, where Ω is the spindle speed in rpm and N is the number of the flutes. Due to the helix of the tool, the directional force coefficient can be given in the integral form

$$H(t, a_{\mathrm{p}}) = \sum_{j=1}^{N} \int_0^{a_{\mathrm{p}}} g_j(t,z)(K_{\mathrm{r}} \cos\varphi_j(t,z) - K_{\mathrm{t}} \sin\varphi_j(t,z)) \cos\varphi_j(t,z)\, \mathrm{d}z, \tag{2}$$

where the function $g_j(t,z)$ defines if the j^{th} tooth at axial location z is cutting or not. K_{t} and K_{r} are the specific tangential and radial forces. The axial and radial depths of cut are denoted by a_{p} and a_{e}. Due to the helix angle η, the position of the j^{th} tooth at axial location z is given by

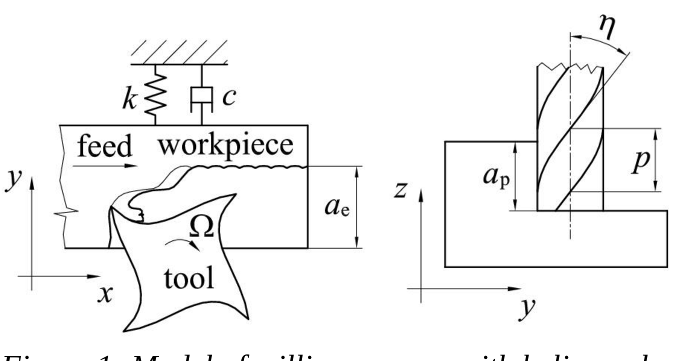

Figure 1; Model of milling process with helix angle.

$$\varphi_j(t,z) = \frac{2\pi\Omega}{60}t + j\frac{2\pi}{N} - z\frac{2\pi}{pN}, \qquad p = \frac{D\pi}{N\tan\eta}, \tag{3}$$

where p is the helix pitch and D is the diameter of the tool. Further details on the model of helical tools in milling processes can be seen in [Balachandran and Zhao, 2000] or in [Insperger and Stépán, 2004b].

Stability of the above milling process is determined by the monodromy operator of Equation (1) that can be represented as an infinite dimensional matrix. If the eigenvalues of this monodromy operator (the so-called characteristic multipliers) are in modulus less than 1, then the system is stable, otherwise the system is unstable and chatter arises. There exist several numerical methods to determine the characteristic multipliers (see in the Introduction). In this paper, the semi-discretization method is used to construct stability charts (see in [Insperger and Stépán, 2004a]).

3. STABILITY CHARTS

Stability charts for down-milling operations using a 4 fluted 20 mm diameter tool with different helix pitches are presented in Figure 2. The modal parameters of the 1 DoF system are m = 5.364 kg, c = 421.948 N/ms, s = 21.6 kN/mm. The corresponding relative damping is 1.96% and the natural frequency of the system is f_n = 319.375 Hz. The radial depth of cut is a_e = 1 mm and the specific cutting forces are K_t = 804.3 N/mm^2 and K_r = 331 N/mm^2. These parameters correspond to the experimental measurements presented in the next section. The horizontal axes in the stability charts are scaled according to the ratio of the tooth passing frequency and the system's natural frequency that yields a kind of normalized spindle speed.

In Figure 2, it can be seen that as the helix pitch p decreases, unstable islands appear in the stability chart. These islands are separated by the lines where the depth of cut is equal to the multiples of the pitch length (denoted by dotted lines in Figure 2). This phenomenon was first observed recently in [Zatarain et al., 2006]. Here, we will give some mathematical explanation to this phenomenon based on the Floquet theory of DDEs.

In order to obtain a clear picture about the nature of these unstable islands, the qualitative differences between time-periodic and autonomous (time-independent) systems should be considered. A system is time-periodic if its governing differential equation contains time periodic coefficients, like $H(t,a_p)$ in Equation (1). If the governing differential equation does not contain time-dependent coefficients, then the system is autonomous.

Stability of autonomous systems is determined by the so-called characteristic roots: the system is stable if all the characteristic roots are in the left half of the complex plane. For autonomous systems, the following routes of instability may occur:

- Hopf instability: a complex pair of characteristic roots cross the imaginary axis,
- fold instability: a real characteristic root crosses the imaginary axis at the origin.

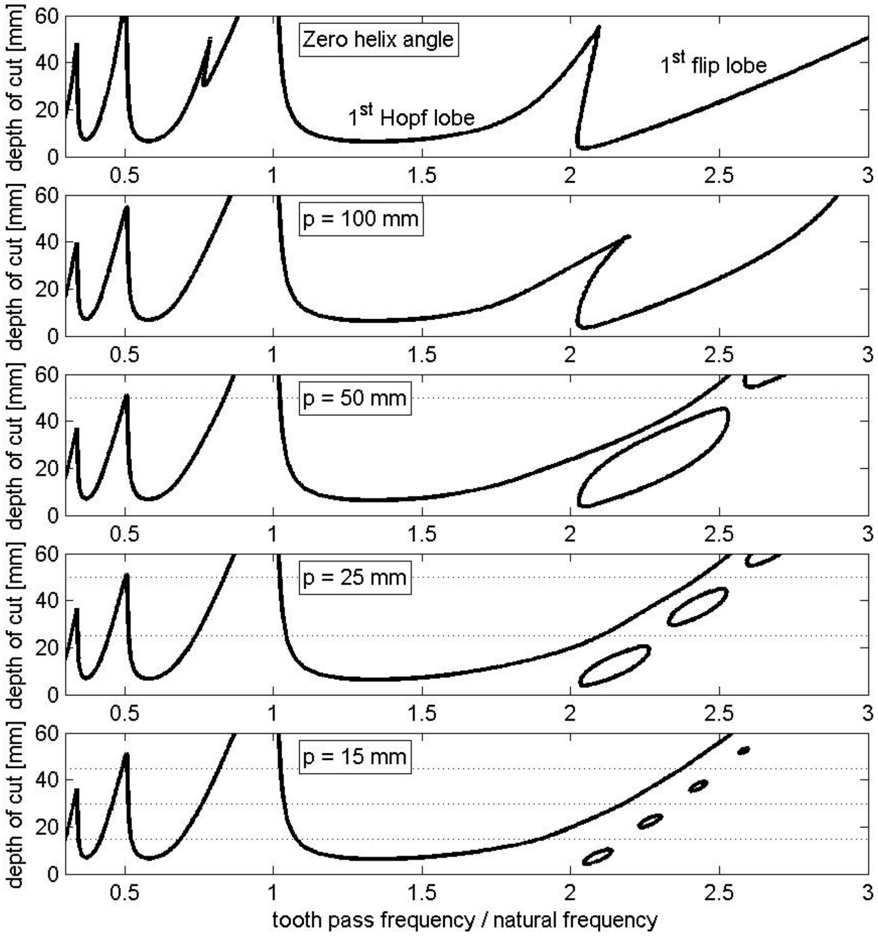

Figure 2; Stability charts for different helix pitches.

Stability of periodic systems are described by the so-called characteristic multipliers: the system is stable if all the characteristic multipliers are inside the unit circle of the complex plane. For periodic systems, the possible routes of instability are:

- secondary Hopf instability: a complex pair of characteristic roots cross the unit circle,
- fold instability: a real characteristic root crosses the unit circle at 1,
- flip instability: a real characteristic root crosses the unit circle at -1.

While Hopf and fold instabilities are typical for both autonomous and periodic systems, flip instability is typical only for periodic systems.

Since milling processes are periodic systems, flip instability is a typical for them. Practically, flip instability occurs mostly for milling operations with small radial immersion when the directional cutting force is strongly time-dependent.

The analysis of the directional force coefficient $H(t,a_p)$ in Equation (1) gives a clear explanation for the existence of the unstable islands. Due to the helix of the tool, the directional force coefficient changes with the axial depth of cut. If the axial depth of cut is equal to a multiple of the helix pitch, that is $a_\mathrm{p} = rp$, where r is an integer, then the directional force coefficient becomes constant in time:

$$
\begin{aligned}
H(t,rp) &= \sum_{j=1}^{N} \int_0^{rp} g_j(t,z)(K_\mathrm{r}\cos\varphi_j(t,z) - K_t \sin\varphi_j(t,z))\cos\varphi_j(t,z)\,\mathrm{d}z \\
&\overset{(1)}{=} r\sum_{j=1}^{N} \int_0^{p} g_j(t,z)(K_\mathrm{r}\cos\varphi_j(t,z) - K_t \sin\varphi_j(t,z))\cos\varphi_j(t,z)\,\mathrm{d}z \\
&\overset{(2)}{=} r\int_{\varphi_\mathrm{a}}^{\varphi_\mathrm{e}} (K_\mathrm{r}\cos\varphi - K_t\sin\varphi)\cos\varphi\left(-\frac{pN}{2\pi}\right)\mathrm{d}\varphi = r\int_{\varphi_\mathrm{e}}^{\varphi_\mathrm{a}} (K_\mathrm{r}\cos\varphi - K_t\sin\varphi)\cos\varphi\frac{pN}{2\pi}\,\mathrm{d}\varphi \\
&\overset{(3)}{=} r\int_0^{\tau} \underbrace{\sum_{j=1}^{N} g_j(t)(K_\mathrm{r}\cos\varphi_{0,j}(t) - K_t\sin\varphi_{0,j}(t))\cos\varphi_{0,j}(t)}_{H_0(t)} \underbrace{\frac{pN}{2\pi}\frac{2\pi\Omega}{60}}_{p/\tau}\,\mathrm{d}t \\
&= r\,p\underbrace{\frac{1}{\tau}\int_0^{\tau} H_0(t)\,\mathrm{d}t}_{K} = rpK.
\end{aligned}
\qquad (4)
$$

Here, we used that due to the helical tool, r number of flutes are in continuous contact with the workpiece between the enter and exit angles φ_e and φ_a (equality signs 1 and 2), and due to Equation (3), $\mathrm{d}z = -\frac{pN}{2\pi}\mathrm{d}\varphi$ and $\mathrm{d}\varphi = \frac{2\pi\Omega}{60}dt$ (equality signs 2 and 3). In Equation (4), $H_0(t)$ is the directional cutting force coefficient corresponding to a straight fluted tool ($\eta = 0$) with $\varphi_{0,j}(t) = 2\pi\Omega t/60 + j\,2\pi/N$ as the angular position of the j^{th} flute. K is the integral average of $H_0(t)$ that corresponds to the zeroth-order approximation of the multi-frequency solution of [Budak and Altintas, 1998].

So, if $a_\mathrm{p} = rp$ then the helix averages the variations of the directional force coefficient according to Equation (4) and the equation of motion becomes

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = rpK(x(t) - x(t-\tau)). \qquad (5)$$

This equation is an autonomous DDE (i.e., it does not contain time-dependent coefficients), therefore it never experiences flip instability. Consequently, flip stability boundaries never intersect the lines $a_\mathrm{p} = rp$ in the stability charts. If flip stability boundaries exist for other axial depths of cut (when $a_\mathrm{p} \neq rp$) then they should form bounded islands between the lines $a_\mathrm{p} = rp$.

As a numerical verification, Figure 3 presents the stability chart associated with helix pitch $p = 25$ mm and the directional force coefficients for different depth of cut values as a function of time. It can be seen that if the axial depth of cut equals to a multiple of the helix pitch then the directional force coefficient is constant in time, as it was shown in Equation (4).

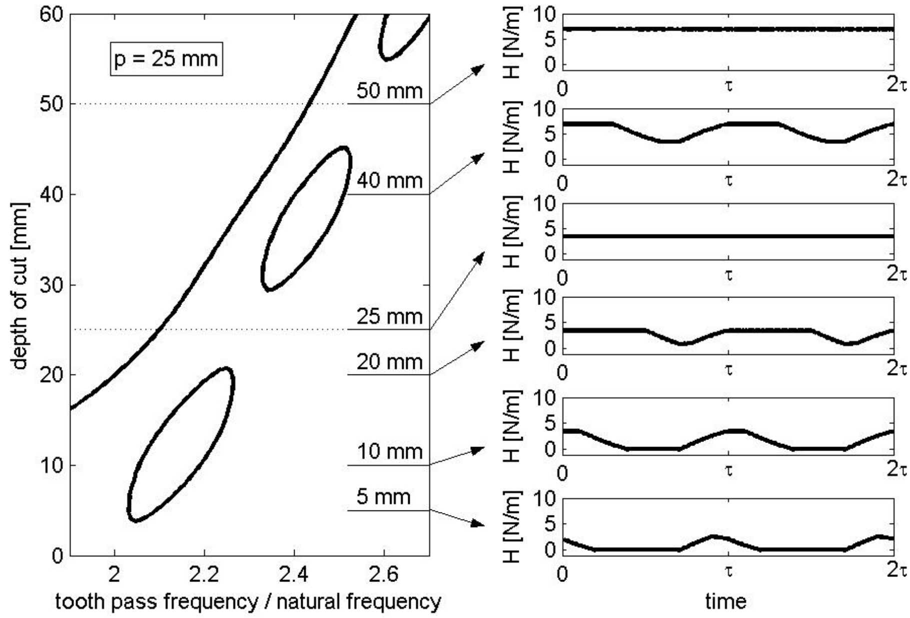

Figure 3; Connection between the flip islands and the directional force coefficient.

Note that the above flip islands are different from the ones reported in [Szalai and Stépán, 2003]. The flip islands in Figure 2 and 3 are directly related to the helix of the tool, and from now on, we will refer them as helix-induced islands. The flip islands found by [Szalai and Stépán, 2003] are related to the periodic nature of the machining process independently of the helix of the tool, therefore, we will refer them as parametrically induced islands. [Szalai and Stépán, 2003] investigated an interrupted machining model with piecewise zero and constant directional cutting coefficient without modelling the helix angle. Based on a mathematically accurate analysis, they showed that all the flip stability boundaries are closed curves (lenses or islands) except the first one above normalized spindle speed 2. Here, we saw that the helix-induced stability islands may occur for all flip boundaries including the first one, too.

Note that parametrically induced flip islands arise for the current milling model with straight fluted tool, too. If the radial depth of cut is set to a_e = 0.5 mm, then a clear (parametrically induced) flip island arises at about normalized spindle speed 0.7 (see Figure 4.)

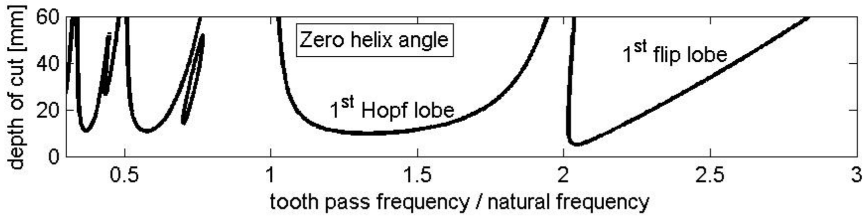

Figure 4; Stability chart for a straight fluted tool with a_e=0.5 mm .

4. EXPERIMENTAL VERIFICATION

A 4 fluted 20 mm diameter tool with helix pitch p = 27.2 mm (η = 30°) was used to down-mill aluminium 7075T6 samples at radial depth of cut a_p = 1 mm. The workpiece was mounted on a compliant fixture in order to receive a system with relatively small natural frequency (f_n = 319.375 Hz). This way, the first flip boundaries are attained at spindle speeds around 10 krpm. All the mechanical and technological parameters were given in the previous section.

Since, for a 4 fluted tool, the flip (period doubling) frequencies interfere with the second harmonics of the tooth passing frequency, clear identification of flip chatter and chatter free cases based on the vibration spectra is difficult. Therefore, the experimental stability charts is demonstrated by the contour lines of the vibration level measured during the tests. The spindle speed – depth of cut pairs where the tests were taken are denoted by crosses. The sizes of the crosses change according to the corresponding vibration level (see the contour lines). This way, experimental stability chart can be presented without making a clear limit value of the vibration level. As it can be seen, the experimental results fully validate the theoretical predictions. The existence of a (helix-induced) flip island at the area of the first flip lobe can clearly be seen.

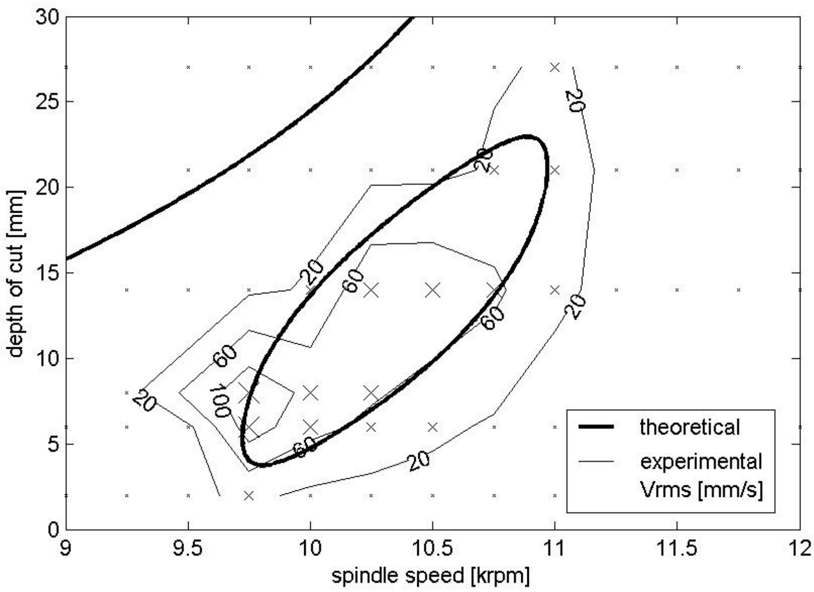

Figure 4;Theoretical and experimental stability charts for p=27.2 mm.

5. CONCLUSION

Flip islands in the stability diagram of milling processes associated with the tool helix was recently reported in [Zatarain et al., 2006]. In this paper, these islands were investigated based on time domain considerations. It was shown that if the axial depth of cut is equal to the multiples of the helix pitch, then the directional force coefficient becomes constant in time, and flip chatter cannot arise. This explains that the stability islands are separated by the lines where the axial depth of cut is equal to the multiples of the helix pitch. It was shown that the helix-induced stability islands differ from the parametrically induced islands reported in [Szalai and Stépán, 2003, 2006]. While the parametrically induced islands occur at all flip boundaries except the first one, the helix-induced islands arise at the first flip boundary, too. The theoretical results were confirmed by experiments.

ACKNOWLEDGEMENT

This work was supported in part by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences, and by the Hungarian National Science Foundation under grants no. OTKA T043368 and F047318.

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