


Applied Mathematical Sciences

T. Insperger
G. Stépán

Semi-Discretization for Time-Delay Systems

Stability and Engineering Applications

 Springer

T. Insperger • G. Stépán

Semi-Discretization for Time-Delay Systems

Stability and Engineering Applications

 Springer

Tamás Insperger
Department of Applied Mechanics
Budapest University of Technology
and Economics
Műegyetem rkp. 5
1521 Budapest
Hungary
inspi@mm.bme.hu

Gábor Stépán
Department of Applied Mechanics
Budapest University of Technology
and Economics
Műegyetem rkp. 5
1521 Budapest
Hungary
stepan@mm.bme.hu

ISSN 0066-5452
ISBN 978-1-4614-0334-0 e-ISBN 978-1-4614-0335-7
DOI 10.1007/978-1-4614-0335-7
Springer New York Dordrecht Heidelberg London

Library of Congress Control Number: 2011932981

Mathematical Subject Classification (2010): 34K06, 34K13, 34K20, 34K28, 34K35, 37N15, 37N35

© Springer Science+Business Media, LLC 2011

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC, 233 Spring Street, New York, NY 10013, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

To our parents and to our Ágies

Preface

Time delay always arises in engineering models, where the rates of change of state variables depend both on present and on past state variables of the system. Control processes with feedback delay, regenerative machine tool chatter, wheel shimmy models including the elastic contact between the tire and the road, car-following traffic models with the reaction time of the drivers, human motion control with reflex delay, can be mentioned as examples. The analysis of these systems requires the characterization of their local behavior around a desired position or a desired (possibly periodic) path. Such properties can be described by stability charts that present the stability of the linearized system in the plane of the system parameters. These stability charts provide a useful tool for engineers, since they present an overview on the effects of system parameters on the local dynamics of the system.

The main differences between systems with and without time delay is that time delay produces an infinite-dimensional dynamics as opposed to the finite-dimensional dynamics of delay-free systems. For simple time-delay systems, stability charts can be derived analytically. However for complex systems, for instance, when the time-delay effect is coupled with parametric excitation, only numerical techniques can be used.

The scope of this book is to present a numerical technique, called the semi-discretization method, for the stability analysis of linear time-periodic time-delay systems, which is also an essential tool in the study of periodic motions of nonlinear time-delay systems. Semi-discretization is a well-known technique used, for example, in the finite element analysis of solid bodies, or in computational fluid mechanics, where the corresponding partial differential equations are discretized along the spatial coordinates only, while the time coordinates are unchanged. In case of time-delay systems, semi-discretization results in the discretization of delayed terms only, while the actual-time-domain terms are not discretized. In this way, the infinite-dimensional system is approximated by a finite-dimensional one.

The structure of the book is as follows. Chapter 1 gives some introduction to linear time-delay systems. Chapter 2 deals with the construction of the stability charts for some fundamental delay-differential equations. The semi-discretization method is presented in Chapter 3 including higher-order methods, rate convergence esti-

mates, and numerical issues. The semi-discretization method is applied in Chapter 4 to some Newtonian examples with different delay types, such as single point delay, multiple delays, distributed delay, and time-periodic delay. Finally, Chapter 5 presents real-world mechanical engineering applications. Turning and milling processes are considered with varying spindle speed, resulting in time-periodic time delays. Then, the so-called act-and-wait control concept is introduced, and it is analyzed through applications to the stick-balancing problem and to a force-control process with feedback delay. It is shown that the inclusion of waiting periods in the control rule may have a stabilizing effect. This provides the surprising conclusion that doing nothing and rather waiting for the response of a previous action might be a superior control strategy for systems with feedback delay. Finally, the stick-balancing model with reflex delay is investigated in the case of parametric forcing at the stick's base. The book concludes with an appendix that contains Matlab codes for the semi-discretization of the examples presented in Chapter 4.

The book is designed for graduate and PhD students as well as for researchers working in the fields of mechanical, electrical, and chemical engineering, control theory, biomechanics, population dynamics, neurophysiology, even climate research in which time-delay models occur.

The book is based on the authors' research work over the last 10 years, but many colleagues have contributed to different parts. Hereby, the authors thank and acknowledge the useful discussions with and comments of Mikel Zatarain, Jokin Muñoa, Grégoire Peigné, and Sébastien Seguy regarding the computational efficiency of the semi-discretization for different milling applications. The helpful consultations and joint works related to the mathematical issues of the method with Janos Turi, Ferenc Hartung, and Barnabás Garay are greatly appreciated. The comments and novel ideas provided by our young colleagues Zoltán Dombóvári and Dániel Bachrathy are gratefully recognized. Finally, the inspiring long-term cooperation and mutual comparative studies for machining operations with Philip V. Bayly, Brian P. Mann, and Firas A. Khasawneh are gratefully acknowledged.

The appearance of this book and the related research work during recent years were supported by the Hungarian National Science Foundation (OTKA) under grant no. K72911 and K68910 and the János Bolyai Research Scholarship of the Hungarian Academy of Sciences. This work is linked to the scientific program of the "Development of quality-oriented and harmonized R+D+I strategy and functional model at BME" project. This project is supported by the New Hungary Development Plan (Project ID: TÁMOP-4.2.1/B-09/1/KMR-2010-0002). This book is related to activities performed within the DYNXPERTS project, funded by the European Commission FP7 Factories of the Future with Grant Number 260073.

Budapest,
April 2011

Tamás Insperger
Gábor Stépán

Contents

1	Introducing Delay in Linear Time-Periodic Systems	1
1.1	Linear Autonomous ODEs	2
1.2	Linear Periodic ODEs	4
1.3	Linear Autonomous DDEs	6
1.4	Linear Time-Periodic DDEs	9
2	Stability Charts for Fundamental Delay-Differential Equations	13
2.1	First-Order Scalar Equations	13
2.1.1	The Hayes Equation	13
2.1.2	The Cushing Equation	16
2.2	Delayed Oscillators	19
2.3	Stabilization with Feedback Delay	22
2.4	Delayed Mathieu Equation	25
2.4.1	Special Cases	26
2.4.2	D-curves	27
2.4.3	The Number of Unstable Characteristic Multipliers	32
2.4.4	Damped Case	34
3	Semi-discretization	39
3.1	Basic Ideas	39
3.2	General Concept	46
3.3	Toward Higher-Order Methods	52
3.3.1	Higher-Order Approximations	53
3.3.2	Rate of Convergence Estimates	60
3.4	Numerical Issues	62
3.4.1	Effective Matrix Multiplication	62
3.4.2	Construction of the Monodromy Matrix in One Step	66
3.4.3	Eigenvalue Computation	69
3.4.4	An Alternative Semi-Discretization Method	69

4	Newtonian Examples	73
4.1	Parametrically Excited Delayed Oscillators	73
4.2	Multiple Delays	79
4.3	Distributed Delays	84
4.4	Time-Periodic Delays	90
5	Engineering Applications	93
5.1	Varying Spindle Speed Turning	93
5.1.1	Mechanical Model	94
5.1.2	Constant Spindle Speed	97
5.1.3	Varying Spindle Speed	99
5.2	Stability of Milling Processes	103
5.2.1	Single-Degree-of-Freedom Model	104
5.2.2	Milling with Helical Tools	114
5.2.3	Varying Spindle Speed Milling	118
5.2.4	Two-Degrees-of-Freedom Model	122
5.3	Periodic Control Systems with Feedback Delay	126
5.3.1	The Act-and-Wait Control Concept	128
5.3.2	Stick-Balancing with Reflex Delay	131
5.3.3	Force Control	136
5.4	Parametric Excitation of Balancing	142
A	Appendix	151
A.1	Solution of Linear Inhomogeneous ODEs	151
A.2	Routh–Hurwitz Criterion	154
A.3	Matlab Code for the Delayed Mathieu Equation	155
A.4	Matlab Code for Multiple Delays	156
A.5	Matlab Code for Distributed Delays	158
A.6	Matlab Code for Time-Periodic Delays	160
	References	163