

Sign-constancy of Green's Functions for Two-point Boundary Value Problems with Second Order Impulsive Delayed Differential Equations

Alexander Domoshnitsky¹ and Yulia Mizgireva¹

¹*Department of Mathematics, Ariel University, Israel (e-mail: adom@ariel.ac.il, julia.mizgireva@gmail.com)*

We consider the following second order impulsive differential equation with delays

$$(Lx)(t) \equiv x''(t) + \sum_{j=1}^p a_j(t)x'(t - \tau_j(t)) + \sum_{j=1}^p b_j(t)x(t - \theta_j(t)) = f(t), \quad t \in [0, \omega], \quad (1)$$

$$\begin{aligned} x(t_k) &= \gamma_k x(t_k - 0), \quad x'(t_k) = \delta_k x'(t_k - 0), \quad k = 1, 2, \dots, r, \\ 0 &= t_0 < t_1 < t_2 < \dots < t_r < t_{r+1} = \omega, \end{aligned} \quad (2)$$

$$x(\zeta) = 0, \quad \zeta < 0, \quad (3)$$

For equation (1) we consider the following variants of boundary conditions:

$$x(0) = 0, \quad x(\omega) = 0, \quad (4)$$

$$x'(0) = 0, \quad x(\omega) = 0, \quad (5)$$

$$x(0) = 0, \quad x'(\omega) = 0, \quad (6)$$

$$x'(0) = 0, \quad x'(\omega) = 0. \quad (7)$$

In our talk we obtain sufficient conditions of nonpositivity of Green's functions for impulsive differential equation without an assumption about the sign of $b_j(t)$. All results are formulated in the form of theorems about differential inequalities. Then choosing the test functions, we obtain explicit conditions of nonpositivity of Green's functions.

Keywords: second order impulsive differential equations; boundary value problems; sign-constancy of Green's functions