

Pseudospectral methods for the bifurcation analysis of nonlinear delay equations

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In mathematical models for biological systems, delays often enter the model as integrals over the past time. Populations with age or size structure, for instance, can be described with a renewal equation for the population birth rate, possibly coupled with a delay differential equation for the environmental variable. Moreover, the different life stages of individuals often lead to history-dependent delays of threshold type.

No software is capable of studying numerically the bifurcation properties of this kind of nonlinear systems. To address this gap, we propose the pseudospectral discretization technique as a way to approximate a general nonlinear delay equation with a low-dimensional system of ordinary differential equations, whose properties can be studied with existing software. The technique can be applied to systems coupling both integral and delay differential equations, and involving bounded or unbounded delays [1, 2, 3].

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