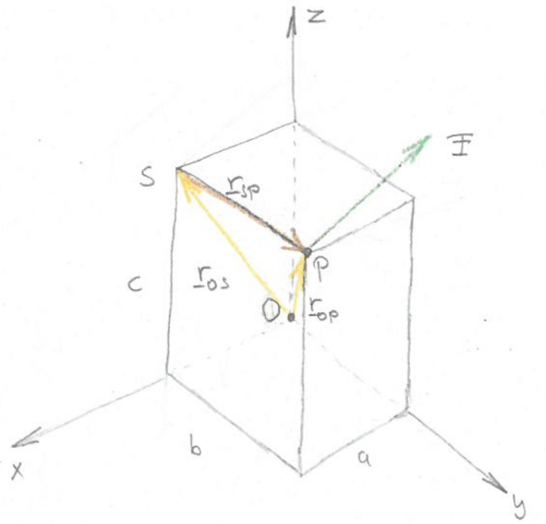
4.1.



Adatok: $a = 20$ [mm] $\quad F = \begin{bmatrix} 10 \\ 50 \\ 70 \end{bmatrix}$ [N]

$b = 40$ [mm]

$c = 60$ [mm]

a, $[\underline{F}, \underline{M}_S]_S$

b, $[\underline{F}, \underline{M}_O]_O$

a, $\underline{r}_{OP} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $\underline{r}_{OS} = \begin{bmatrix} a \\ 0 \\ c \end{bmatrix}$

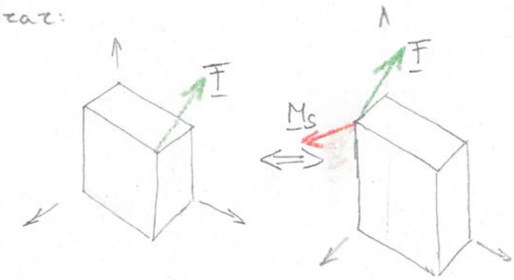
Mivel: $\underline{r}_{OP} = \underline{r}_{OS} + \underline{r}_{SP} \Rightarrow \underline{r}_{SP} = \underline{r}_{OP} - \underline{r}_{OS} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}$

P-ből S-be redukálva:

$$\underline{M}_S = \overbrace{\underline{M}_P}^{\underline{0}} + \underline{r}_{SP} \times \underline{F} = \underline{0} + \begin{bmatrix} 0 \\ 40 \\ 0 \end{bmatrix} \times \begin{bmatrix} 10 \\ 50 \\ 70 \end{bmatrix} = \begin{bmatrix} 2800 \\ 0 \\ -400 \end{bmatrix} \text{ [Nmm]}$$

megj.: irányok,

Azaz:



b, S-ből O-ba redukálva:

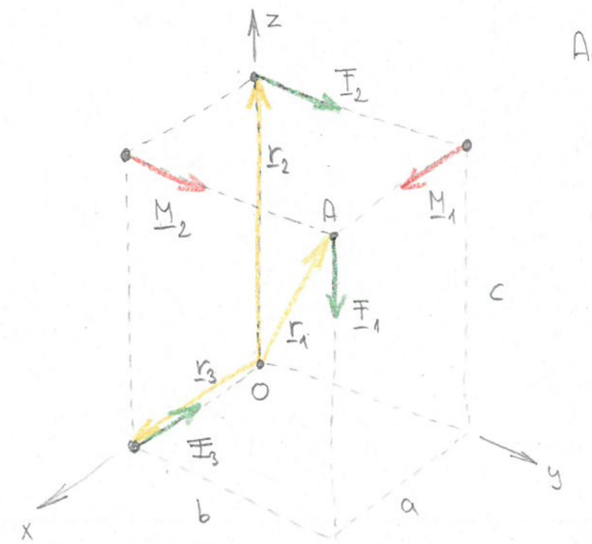
$$\underline{M}_O = \underline{M}_S + \underline{r}_{OS} \times \underline{F} = \begin{bmatrix} 2800 \\ 0 \\ -400 \end{bmatrix} + \underbrace{\begin{bmatrix} 20 \\ 0 \\ 60 \end{bmatrix} \times \begin{bmatrix} 10 \\ 50 \\ 70 \end{bmatrix}}_{\begin{bmatrix} -3000 \\ 600-1400 \\ 1000 \end{bmatrix}} = \begin{bmatrix} -200 \\ -800 \\ 600 \end{bmatrix} \text{ [Nmm]}$$

VAGY:

P-ből O-ba redukálva:

$$\underline{M}_O = \underbrace{\underline{M}_P}_{\underline{0}} + \underline{r}_{OP} \times \underline{F} = \begin{bmatrix} 20 \\ 40 \\ 60 \end{bmatrix} \times \begin{bmatrix} 10 \\ 50 \\ 70 \end{bmatrix} = \begin{bmatrix} 2800-3000 \\ 600-1400 \\ 1000-400 \end{bmatrix} = \begin{bmatrix} -200 \\ -800 \\ 600 \end{bmatrix} \text{ [Nmm]}$$

4.2.



Adatok: $F_1 = 25$ [N] $\quad M_1 = 90$ [Nm] $\quad a = 2$ [m]

$F_2 = 70$ [N] $\quad M_2 = 110$ [Nm] $\quad b = 3$ [m]

$F_3 = 60$ [N] $\quad c = 4$ [m]

a, $[\underline{F}, \underline{M}_O]_O$

b, $[\underline{F}, \underline{M}_A]_A$

c, $\underline{F} \cdot \underline{M}_A = \underline{F} \cdot \underline{M}_O$?

d, centrális egyenes és $[\underline{F}, \underline{M}_C]_C$

e, O pontban erőrendszer, hogy egyensúly legyen?

a, Az erő- és nyomatékvektorok:

$$\left.\begin{array}{l}
\underline{F}_1 = \begin{bmatrix} 0 \\ 0 \\ -F_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -25 \end{bmatrix} [N] \\[2ex]
\underline{F}_2 = \begin{bmatrix} 0 \\ F_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 70 \\ 0 \end{bmatrix} [N] \\[2ex]
\underline{F}_3 = \begin{bmatrix} -F_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -60 \\ 0 \\ 0 \end{bmatrix} [N]
\end{array}\right\} \underline{F}_i \, , \; i = 1, \dots, 3$$

A támadáspontok helyvektorai:

$$\underline{r}_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} [m] \qquad \underline{r}_2 = \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} [m] \qquad \underline{r}_3 = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} [m]$$

$$\left.\begin{array}{l}
\underline{M}_1 = \begin{bmatrix} M_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 90 \\ 0 \\ 0 \end{bmatrix} [N] \\[2ex]
\underline{M}_2 = \begin{bmatrix} 0 \\ M_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 110 \\ 0 \end{bmatrix} [N]
\end{array}\right\} \underline{M}_j \, , \; j = 1, \dots, 2$$

A statikai vektorkettős az O pontra: $[\underline{F}, \underline{M}_O]_O$

⟹ $\underline{F}$ az eredő erő: $\underline{F} = \sum_{i=1}^{3} \underline{F}_i = \begin{bmatrix} 0 \\ 0 \\ -25 \end{bmatrix} + \begin{bmatrix} 0 \\ 70 \\ 0 \end{bmatrix} + \begin{bmatrix} -60 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -60 \\ 70 \\ -25 \end{bmatrix} [N]$

És $\underline{M}_O$ az erőrendszer nyomatéka az O pontra:

$$\underline{M}_O = \sum_{j=1}^{2} \underline{M}_j + \sum_{i=1}^{3} \underbrace{\underline{r}_i \times \underline{F}_i}_{\underline{M}_{OF_i}} = \underbrace{\begin{bmatrix} 90 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 110 \\ 0 \end{bmatrix}}_{\begin{bmatrix} 90 \\ 110 \\ 0 \end{bmatrix}} + \underbrace{\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -25 \end{bmatrix}}_{\begin{bmatrix} -3 \cdot 25 \\ +2 \cdot 25 \\ 0 \end{bmatrix}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \times \begin{bmatrix} 0 \\ 70 \\ 0 \end{bmatrix}}_{\begin{bmatrix} -4 \cdot 70 \\ 0 \\ 0 \end{bmatrix}} + \underbrace{\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} -60 \\ 0 \\ 0 \end{bmatrix}}_{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}} = \begin{bmatrix} -265 \\ 160 \\ 0 \end{bmatrix} [Nm]$$

VAGY: $\underline{M}_{OF_1} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 3 & 4 \\ 0 & 0 & -25 \end{vmatrix} = \underline{i} \cdot \underbrace{\begin{vmatrix} 3 & 4 \\ 0 & -25 \end{vmatrix}}_{-75} - \underline{j} \cdot \underbrace{\begin{vmatrix} 2 & 4 \\ 0 & -25 \end{vmatrix}}_{-50} + \underline{k} \cdot \underbrace{\begin{vmatrix} 2 & 3 \\ 0 & 0 \end{vmatrix}}_{0} = \begin{bmatrix} -75 \\ 50 \\ 0 \end{bmatrix} [Nm]$

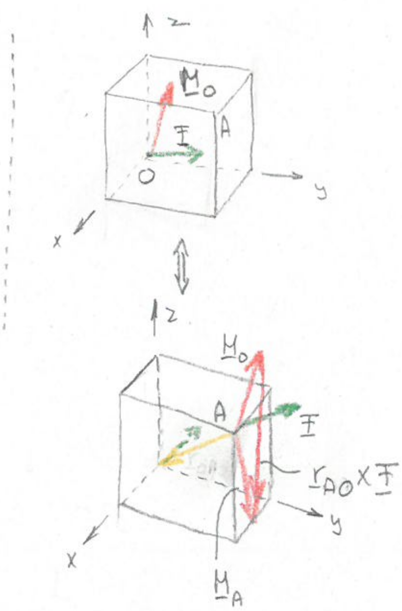
$\underline{M}_{OF_2} = \dots$, $\underline{M}_{OF_3} = \dots$

b, Az erőrendszer redukáltja a A pontra: $[\underline{F}, \underline{M}_A]_A$

⟹ $\underline{F}$ változatlan.

Továbbá:

$$\underline{M}_A = \underline{M}_O + \underbrace{\underline{r}_{AO}}_{-\underline{r}_1} \times \underline{F} = \begin{bmatrix} -265 \\ 160 \\ 0 \end{bmatrix} + \underbrace{\begin{bmatrix} -2 \\ -3 \\ -4 \end{bmatrix} \times \begin{bmatrix} -60 \\ 70 \\ -25 \end{bmatrix}}_{\begin{bmatrix} (-3)\cdot(-25) - (-4)\cdot 70 \\ (-4)\cdot(-60) - (-2)\cdot(-25) \\ (-2)\cdot 70 - (-3)\cdot(-60) \end{bmatrix} = \begin{bmatrix} 355 \\ 190 \\ -320 \end{bmatrix}} = \begin{bmatrix} 90 \\ 350 \\ -320 \end{bmatrix} [Nm]$$



c) $\underline{F}\cdot\underline{M}_A = \begin{bmatrix}-60\\70\\-25\end{bmatrix}\cdot\begin{bmatrix}90\\350\\-320\end{bmatrix} = 27100\ [Nm^2]$

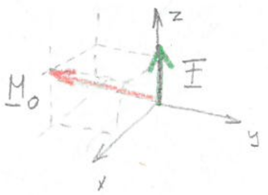
$\underline{F}\cdot\underline{M}_0 = \begin{bmatrix}-60\\70\\-25\end{bmatrix}\cdot\begin{bmatrix}-265\\160\\0\end{bmatrix} = 27100\ [Nm^2]$

$\Big\} = \checkmark$

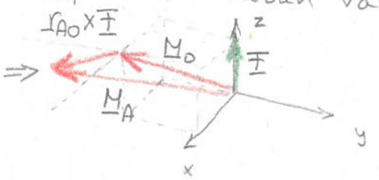
De miért?

$$\underline{F}\cdot\underline{M}_A = \underline{F}\cdot(\underline{M}_0 + \underline{r}_{A0}\times\underline{F}) = \underline{F}\cdot\underline{M}_0 + \underbrace{\underline{F}\cdot(\underbrace{\underline{r}_{A0}\times\underline{F}}_{\perp\underline{F}})}_{0} = \underline{F}\cdot\underline{M}_0$$

$\underline{F}$-re merőleges síkban van $\Rightarrow$ $\underbrace{\underline{M}\text{ merőleges vetülete }\underline{F}\text{-re}}$ nem változik.



$\Rightarrow$



Hiszen:

$\underline{F}\cdot\underline{M} = |\underline{F}|\cdot\underbrace{|\underline{M}|\cdot\cos\alpha}_{\text{merőleges vet.}}$

d) A centrális egyenes pontjain:

$$[\underline{F},\underline{M}_c]_c \Rightarrow \underbrace{\underline{F}\parallel\underline{M}_c \text{ vagy } \underline{M}_c=\underline{0}}_{\text{1. felt.}}$$

$$\underline{M}_c = \underline{M}_0 + \underline{r}_{c0}\times\underline{F} \quad \Rightarrow \underline{r}_{c0} = ? \qquad / \underline{F}\times(\ldots)$$

$$\underbrace{\underline{F}\times\underline{M}_c}_{\underline{0},\text{ 1. felt. miatt}} = \underline{F}\times\underline{M}_0 + \underbrace{\underline{F}\times(\underline{r}_{c0}\times\underline{F})}_{\underline{r}_{c0}\cdot(\underline{F}\cdot\underline{F}) - \underline{F}\cdot(\underline{F}\cdot\underline{r}_{c0})}$$

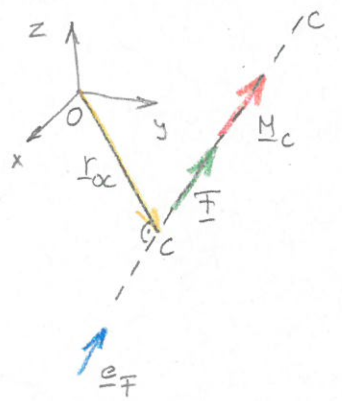
$\underbrace{(\underline{F}\cdot\underline{r}_{c0})}_{:=\underline{0}}$, ha $\underline{r}_{c0}$-t úgy választjuk, hogy $\underline{F}\perp\underline{r}_{c0}$ $\Rightarrow \underline{0}$

Így:

$$\underline{0} = \underline{F}\times\underline{M}_0 + \underbrace{\overbrace{\underline{F}^2}^{|\underline{F}|}}_{\text{skalár}}\cdot\underline{r}_{c0} \quad\Rightarrow\quad \underline{r}_{0c} = \frac{\underline{F}\times\underline{M}_0}{|\underline{F}|^2}$$

A centrális egyenes egyenlete:

$$\underline{r}(\lambda) = \underline{r}_{0c} + \overbrace{\lambda}^{\text{me.: [m]}}\cdot\underbrace{\underline{e}_F}_{\underline{F}/|\underline{F}|} \quad (\underline{e}_F\text{: egységvektor})$$



$|\underline{F}|^2 = \ldots = 9125\ [N^2]$

$\underline{e}_F = \dfrac{\underline{F}}{|\underline{F}|} = \begin{bmatrix}-0,6281\\0,7328\\-0,2617\end{bmatrix}\ [-]$

$\underline{r}_{0c} = \dfrac{1}{9125}\cdot\begin{bmatrix}-60\\70\\-25\end{bmatrix}\times\begin{bmatrix}-265\\160\\0\end{bmatrix} = \begin{bmatrix}0,4384\\0,7260\\0,9808\end{bmatrix}\ [m]$

Azaz: $\underline{r}(\lambda) = \begin{bmatrix}0,4384\\0,7260\\0,9808\end{bmatrix} + \lambda\cdot\begin{bmatrix}-0,6281\\0,7328\\-0,2617\end{bmatrix}$

A statikai vektorkettős pedig: $[\underline{F},\underline{M}_c]_c$

$$\underline{M}_c = \underline{M}_0 + \underbrace{\underline{r}_{c0}}_{-\underline{r}_{0c}}\times\underline{F} = \underline{M}_0 - \underline{r}_{0c}\times\underline{F} = \ldots = \begin{bmatrix}-178,192\\207,890\\-74,247\end{bmatrix}$$

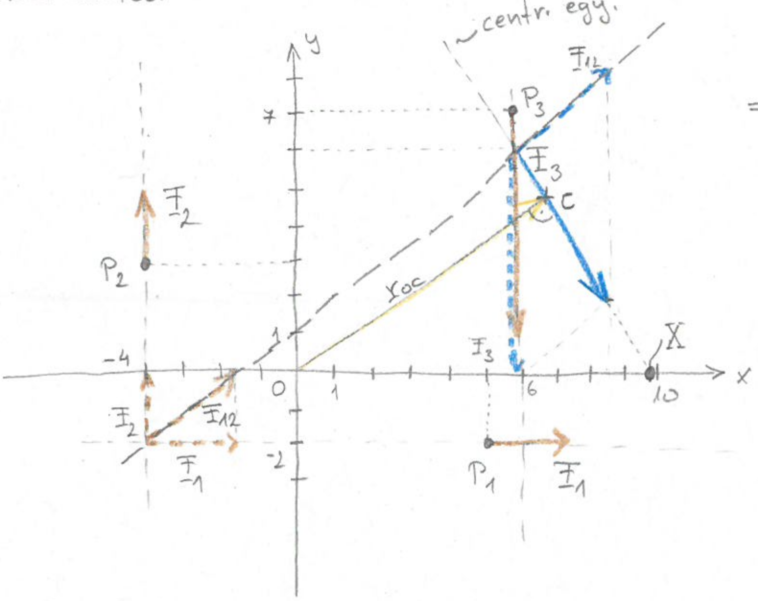
e.) A statikai vektorkettős az O pontban: $[\underline{F}, \underline{M}_O]_O$

Ennek egyensúlyozásához kell: $[\underline{F}^*, \underline{M}_O^*]_O = [-\underline{F}, -\underline{M}_O]_O$

Így: $\underline{F}^* = -\underline{F}$ és $\underline{M}_O^* = -\underline{M}_O$

4.3. $F_1 = 5$ [kN], $F_2 = 4$ [kN], $F_3 = 12$ [kN]

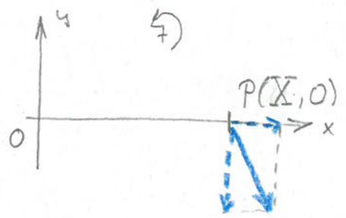
Szerkesztés:



$\Rightarrow \quad \underline{F}_e = \begin{bmatrix} 5 \\ -8 \\ 0 \end{bmatrix}$ [kN],

Számítás: ∘ $\underline{F}_e = \underline{F}_1 + \underline{F}_2 + \underline{F}_3 = \begin{bmatrix} 5 \\ -8 \\ 0 \end{bmatrix}$ [kN]

∘ Hatásvonal helye:



$$M_O = F_1 \cdot k_1 + F_2 \cdot k_2 + F_3 \cdot k_3 = 5 \cdot 2 - \overbrace{4 \cdot 4}^{16} - \overbrace{12 \cdot 6}^{72} = -78 \text{ [Nm]}$$

$$M_O = -|F_{ey}| \cdot X$$

$$X = \frac{-M_O}{|F_{ey}|} = -\frac{-78}{|-8|} = 9{,}75 \text{ [m]}$$

YAGY:
$$\underline{r}_{oc} = \frac{\underline{F} \times \underline{M}_O}{|\underline{F}|^2} = \frac{1}{5^2 + 8^2} \cdot \begin{bmatrix} 5 \\ -8 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -78 \end{bmatrix} = \frac{1}{89} \cdot \begin{bmatrix} 624 \\ 390 \\ 0 \end{bmatrix} \cong \begin{bmatrix} 7{,}01 \\ 4{,}38 \\ 0 \end{bmatrix} \text{ [m]}$$

Ezzel a x tengely metszet:

$\underline{r}(\lambda) = \underline{r}_{oc} + \lambda \cdot \underline{F}$, és most $\underline{r}(\lambda) = \begin{bmatrix} X \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} X \\ 0 \\ 0 \end{bmatrix} = \frac{1}{89} \cdot \begin{bmatrix} 624 \\ 390 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 5 \\ -8 \\ 0 \end{bmatrix}$$

$$\frac{390}{89} - 8\lambda = 0 \Rightarrow \lambda = \frac{390}{8 \cdot 89}$$

$$X = \frac{624}{89} + \frac{390}{8 \cdot 89} \cdot 5 = 9{,}75 \checkmark$$