Matek A1 / 10. gyakorlat: Határozatlan integrál
Számítsuk ki az integrálokat!

1. a) $\int x^2 + 3\cos(x) - 2^{2x} \cdot 3^x + \frac{3}{x}\,dx$

   b) $\int \frac{x^2 + 3x^3 - 1}{\sqrt{x}}\,dx$

2. a) $\int \cos(2x-1)\,dx$

   b) $\int \frac{2}{\sqrt[3]{(2x+3)^2}}\,dx$

   c) $\int \frac{3}{2+x^2}\,dx$

   d) $\int \frac{2}{\sqrt{4x^2+12x+10}}\,dx$

   e) $\int \sin^2(x)\,dx$

3. a) $\int \frac{1}{x+3}\,dx$

   b) $\int \tan(x)\,dx$

   c) $\int \frac{2+x}{2+x^2}\,dx$

4. a) $\int \sin(x)\cos^{16}(x)\,dx$

   b) $\int \frac{x^3}{2} \cdot \sqrt[3]{x^4-3}\,dx$

   c) $\int \frac{\ln(x)}{x}\,dx$

5. a) $\int e^{\sin(x)}\cos(x)\,dx$

   b) $\int \frac{\cos(x)}{\sqrt{1+\sin(x)}}\,dx$

6. a) $\int x\sin(x)\,dx$

   b) $\int (x^2+3x)e^{2x+1}\,dx$

   c) $\int (1+x)\sinh(3x)\,dx$

7. a) $\int \ln(x)\,dx$

   b) $\int x\ln(x)\,dx$

   c) $\int (5x^2+3x)\ln(2x)\,dx$

8. a) $\int \arctan(x)\,dx$

   b) $\int \arcsin(x)\,dx$

   c) $\int arsh(x)\,dx$

9. a) $\int \sin(x) \cdot \cosh(x)\,dx$

   b) $\int \sin(2x)\cos(3x)\,dx$

   c) $\int \sin(2x)e^{3x}\,dx$

   d) $\int \frac{\sin(x)}{e^x}\,dx$

10. gyakorlat

1., a) $\int x^2+3\cos x-\underbrace{2^{2x}\cdot 3^x}_{4^x\cdot 3^x=12^x}+\frac{3}{x}dx=\int x^2\,dx+3\int\cos x\,dx-\int 12^x\,dx+3\int\frac{1}{x}\,dx=$

$=\frac{x^3}{3}+3\sin x-\frac{12^x}{\ln 12}+3\ln x+c$

Integrálás: mit deriváltam?
$\int f(x)\,dx=F(x)+c$, ha $F'(x)=f(x)$.

Ellenőrzés deriválással

2. e) $\int \sin^2x\,dx=\int\frac{1-\cos 2x}{2}\,dx=\int\frac{1}{2}-\frac{\cos 2x}{2}\,dx=\frac{1}{2}x-\frac{\sin 2x}{4}+c$

Linearizáló formulák: $\sin^2x$, $\cos^2x$-re
Képletgyűjteményben:

$\cos 2x=\cos^2x-\sin^2x=\underbrace{\cos^2x+\sin^2x}_{1}-2\sin^2x=1-2\sin^2x$

cél: ez tűnjön el} ez kell

$\Downarrow$

$\cos 2x=1-2\sin^2x$

$\sin^2x=\frac{1-\cos 2x}{2}$

1., b) $\int\frac{x^2+3x^3-1}{\sqrt{x}}\,dx=\int x^{\frac{3}{2}}+3x^{\frac{5}{2}}-x^{-\frac{1}{2}}\,dx=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+3\frac{x^{\frac{7}{2}}}{\frac{7}{2}}-\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+c=$

$\uparrow$
$\sqrt{x}=x^{1/2}$

$=\frac{2}{5}\cdot x^{\frac{5}{2}}+\frac{6}{7}\cdot x^{\frac{7}{2}}-2x^{\frac{1}{2}}+c$

10p.

Tudjuk, hogy:

$(f(g(x)))'=f'(g(x))\cdot g'(x)$

Emiatt:

$\int f'(g(x))\cdot g'(x)\,dx=f(g(x))+c$

pl.: $\int 2\cdot e^{2x}dx=e^{2x}+c$

Helyettesítés:
$\int f'(\underbrace{g(x)}_{u})\cdot\underbrace{g'(x)\,dx}_{\frac{du}{dx}\cdot dx=du}=\int f'(u)\,du=f(u)+c=f(g(x))+c$

2., Szabály: $\int \overset{f'}{f}(\underbrace{ax+b}_{g(x)})\cdot\underbrace{a}_{g'(x)}dx=\overset{f}{F}(ax+b)+c$, ha $F'=f$

$\int \overset{f'}{f}(ax+b)\,dx=\frac{\overset{f}{F}(ax+b)}{a}+c$

a) $\int\cos(2x-1)\,dx=\frac{1}{2}\int 2\cdot\cos(2x-1)\,dx=\frac{1}{2}\cdot\sin(2x-1)+c$

$\uparrow$
$(\sin(2x-1))'=2\cdot\cos(2x-1)$

$\int\cos(2x-1)\,dx=\frac{\sin(2x-1)}{2}+c$

$\uparrow$
$\int f(ax+b)$

$$\int \cos(\underbrace{2x-1}_{t})\,dx = \int \cos(t)\cdot\frac{1}{2}\,dt = \frac{1}{2}\sin(t)+c = \frac{1}{2}\sin(2x-1)+c$$

$$t = 2x-1 \Rightarrow \frac{dt}{dx} = 2 \Rightarrow dx = \frac{1}{2}\,dt$$

b, $$\int \frac{2}{\sqrt[3]{(2x+3)^2}}\,dx = \int 2\cdot(2x+3)^{-\frac{2}{3}}\,dx \underset{\uparrow \int f(ax+b)}{=} 2\cdot\frac{(2x+3)^{\frac{1}{3}}}{2\cdot\frac{1}{3}}+c =$$

$$= 3\cdot(2x+3)^{\frac{1}{3}}+c = 3\sqrt[3]{2x+3}+c$$

c, $$\int \frac{3}{2+x^2}\,dx = 3\int\frac{1}{2+x^2} = 3\cdot\frac{1}{2}\cdot\int\frac{1}{1+\frac{x^2}{2}}\,dx = \frac{3}{2}\cdot\int\frac{1}{1+(\underbrace{\tfrac{1}{\sqrt{2}}}_{a}x)^2}\,dx =$$

$\rightarrow$ ezek 1-ek legyenek, ez a cél

Mire hasonlít?

$$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$$

$$\underset{\uparrow \int f(ax+b)}{=} \frac{3}{2}\cdot\frac{\operatorname{arctg}(\frac{1}{\sqrt{2}}x)}{\frac{1}{\sqrt{2}}}+c = \frac{3\sqrt{2}}{2}\cdot\operatorname{arctg}\left(\frac{1}{\sqrt{2}}x\right)+c$$

d, $$\int\frac{2}{\sqrt{4x^2+12x+10}}\,dx = 2\cdot\int\frac{1}{\sqrt{1+(2x+3)^2}}\,dx = \cancel{2}\cdot\frac{\operatorname{arsh}(2x+3)}{\cancel{2}}+c$$

Amihez hasonlít:

$$\frac{1}{\sqrt{1+x^2}}$$

Cél: gyök alatti (teljes négyzet) + 1 legyen

$$\underbrace{4x^2}_{(2x)^2}+\underbrace{12x}_{2\cdot 2x\cdot(\underbrace{\dots}_{3})}+10 = (2x)^2+2\cdot 2x\cdot 3+\underbrace{3^2+1}_{10} =$$

$$= (2x+3)^2+1$$

-10p-

3., Szabály:

$$\left(\ln(f(x))\right)' = \frac{1}{f(x)}\cdot f'(x) = \frac{f'(x)}{f(x)}$$

Így:

$$\int\frac{f'(x)}{f(x)}\,dx \underset{\downarrow\, f(x)>0}{=} \ln(f(x))+c$$

$$\int\frac{f'(x)}{f(x)} = \int\frac{-f'(x)}{-f(x)} \underset{\uparrow\, f(x)<0}{=} \ln(-f(x))+c$$

Összerakva a kettőt:

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

a, $\int \frac{1}{x+3}\,dx \overset{*}{=} \begin{cases} \int \frac{1}{x+3}\,dx = \ln(x+3) + c, & \text{ha } x+3>0 \\ \int \frac{-1}{-(x+3)}\,dx = \ln(-(x+3)) + c, & \text{ha } x+3<0 \end{cases}$

$f(x) = x+3$
$f'(x) = 1$

$\overset{*}{=} \ln|x+3| + c$

abszolút értéket is használhatunk, úgy egyszerűbb

b, $\int \operatorname{tg}x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{-\sin x}{\cos x}\,dx = -\ln|\cos x| + c$

$f(x) = \cos x$
$f'(x) = -\sin x$

c, $\int \frac{2+x}{2+x^2}\,dx = \int \frac{2}{2+x^2}\,dx + \int \frac{x}{2+x^2}\,dx =$

$= \frac{2}{3} \cdot \underbrace{\int \frac{3}{2+x^2}\,dx}_{\text{előbb kiszámoltuk}} + \frac{1}{2}\int \frac{\overbrace{2x}^{f'}}{\underbrace{2+x^2}_{f}}\,dx =$

$= \underbrace{\sqrt{2} \cdot \operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right)}_{\text{előbbi eredményből}} + \frac{1}{2} \cdot \ln|2+x^2| + c$

4, Szabály:

$$\Big(\underbrace{(f(x))^{\alpha+1}}_{\text{jelölés: } f^{\alpha+1}(x)}\Big)' = (\alpha+1) \cdot (f(x))^{\alpha} \cdot f'(x)$$

Így:

$$\int f'(x) \cdot f^{\alpha}(x)\,dx = \frac{f^{\alpha+1}(x)}{\alpha+1} + c \quad ; \text{ha } \alpha \neq -1$$

ha $\alpha = -1 \Rightarrow \int \frac{f'}{f}$ típus a, $\int \sin x \cdot \cos^{16} x \, dx = -\int \underbrace{(-\sin x)}_{f'} \underbrace{(\cos x)}_{f}^{16} dx = \frac{-\cos^{17} x}{17} + C$

b, $\int \frac{x^3}{2} \cdot \sqrt[3]{x^4-3} = \int \frac{x^3}{2} \cdot (\underbrace{x^4-3}_{f})^{1/3} dx = \frac{1}{8} \int \underbrace{4x^3}_{f'} \left(\underbrace{x^4-3}_{f} \right)^{1/3} dx =$

$= \frac{1}{8} \cdot \frac{(x^4-3)^{4/3}}{\frac{4}{3}} + C = \frac{3}{32} (x^4-3)^{4/3} + C$

c, $\int \frac{\ln x}{x} dx = \int \underbrace{\frac{1}{x}}_{f'} \cdot (\underbrace{\ln x}_{f})^1 dx = \frac{(\ln x)^2}{2} + C$

5, Szabály:

$\int g'(x) \cdot f(g(x)) \, dx = F(g(x)) + C$

a, $\int \underbrace{e^{\overbrace{\sin x}^{g}}}_{f'(g)} \cdot \underbrace{\cos x}_{g'} dx = e^{\sin x} + C$

Vagy helyettesítéssel:

$\int e^{\sin x} \cdot \cos x \, dx = \int e^t \cdot dt = e^t + C = e^{\sin x} + C$

$t = \sin x \Rightarrow \frac{dt}{dx} = \cos x \Rightarrow dx = \frac{dt}{\cos x}$

b, $\int \frac{\cos x}{\sqrt{1+\sin x}} dx = \int \underbrace{\cos x}_{g'} \cdot \frac{1}{\sqrt{1+\underbrace{\sin x}_{g}}} dx = \int \underbrace{\cos x}_{g'} \cdot \underbrace{(1+\overbrace{\sin x}^{g})^{-\frac{1}{2}}}_{f(g)} dx =$ $\rightarrow f = (1+g)^{-\frac{1}{2}}$

$= \frac{(1+\sin x)^{\frac{1}{2}}}{\frac{1}{2}} + C$

Vagy helyettesítéssel: $t = \sin x \Rightarrow \frac{dt}{dx} = \cos x \Rightarrow dx = \frac{dt}{\cos x}$

$\int \frac{\cos x}{\sqrt{1+\sin x}} dx = \int \frac{1}{\sqrt{1+t}} dt = \int (1+t)^{-\frac{1}{2}} dt = \frac{(1+t)^{1/2}}{1/2} + C = 2 \cdot (1+\sin x)^{\frac{1}{2}} + C$

6, Parciális integrálás

$$\int f' \cdot g = f \cdot g - \int f \cdot g'$$

Alkalmazás: $\underbrace{\int \text{polinom}}_{\text{őt deriváljuk} \Downarrow \text{eltűnik}} \cdot \underbrace{\begin{matrix}\sin\\ \cos\\ ch\\ sh\\ \exp\end{matrix}}_{\text{könnyű integrálni}} \Rightarrow$ pl: $\int x^2 \cdot \sin x \, dx$

$\int (x+1) \cdot e^{3x} \, dx$

$\int (x^3+3) \cdot sh x \, dx$

a,
$$\int \underbrace{x}_{f} \cdot \underbrace{\sin x}_{g'} dx = \underbrace{x}_{f} \cdot \underbrace{(-\cos x)}_{g} - \int \underbrace{1}_{f'} \cdot \underbrace{(-\cos x)}_{g} \, dx = -x \cdot \cos x + \int \cos x \, dx =$$

$$= -x \cdot \cos x + \sin x + C$$

b,
$$\int \underbrace{(x^2+3x)}_{f} \cdot \underbrace{e^{2x+1}}_{g'} \, dx = \underbrace{(x^2+3x)}_{f} \cdot \underbrace{\frac{e^{2x+1}}{2}}_{g} - \int \underbrace{\overbrace{(2x+3)}^{h}}_{f'} \cdot \underbrace{\overbrace{\frac{e^{2x+1}}{2}}^{j'}}_{g} \, dx \stackrel{*}{=}$$

$$\Downarrow \quad g = \frac{e^{2x+1}}{2}$$

$$\stackrel{*}{=} (x^2+3x) \cdot \frac{e^{2x+1}}{2} - \left(\underbrace{(2x+3)}_{h} \cdot \underbrace{\frac{e^{2x+1}}{4}}_{j} - \int \underbrace{2}_{h'} \cdot \underbrace{\frac{e^{2x+1}}{4_2}}_{j} \, dx \right) =$$

$$j = \frac{e^{2x+1}}{4}$$

$$= (x^2+3x) \cdot \frac{e^{2x+1}}{2} - (2x+3) \cdot \frac{e^{2x+1}}{4} + \frac{e^{2x+1}}{4} + C$$

c,
$$\int \underbrace{sh(3x)}_{f'} \cdot \underbrace{(1+x)}_{g} \, dx = \underbrace{\frac{ch(3x)}{3}}_{f} \cdot \underbrace{(1+x)}_{g} - \int \underbrace{\frac{ch(3x)}{3}}_{f} \cdot \underbrace{1}_{g'} \, dx =$$

$f = \frac{ch(3x)}{3}$

$$= \frac{ch(3x)}{3} \cdot (1+x) - \frac{sh(3x)}{9} + C$$

7, Parciális integrálás:

$$\int \underbrace{\text{polinom}}_{\substack{\text{könnyű}\\ \text{integrálni}}} \cdot \underbrace{\ln}_{\substack{\text{deriválva}\\ \text{egyszerűbb lesz}}}$$

a, $$\int \ln x \, dx = \int \underbrace{1}_{f'} \cdot \underbrace{\ln x}_{g} \, dx = \underbrace{x}_{f} \cdot \underbrace{\ln x}_{g} - \int \overbrace{\underbrace{x}_{f} \cdot \underbrace{\frac{1}{x}}_{g'}}^{1} dx = x \cdot \ln x - x + C$$

b, $$\int \underbrace{x}_{f'} \cdot \underbrace{\ln x}_{g} \, dx = \underbrace{\frac{x^2}{2}}_{f} \cdot \underbrace{\ln x}_{g} - \int \underbrace{\frac{x^2}{2}}_{f} \cdot \underbrace{\frac{1}{x}}_{g'} dx = \frac{x^2}{2} \cdot \ln x - \frac{x^2}{4} + C$$

c, $$\int \underbrace{(5x^2+3x)}_{f'} \cdot \underbrace{\ln(2x)}_{g} \, dx = \underbrace{\left(\frac{5x^3}{3} + \frac{3x^2}{2}\right)}_{f} \cdot \underbrace{\ln(2x)}_{g} - \int \underbrace{\left(\frac{5x^3}{3} + \frac{3x^2}{2}\right)}_{f'} \cdot \underbrace{\frac{1}{2x} \cdot 2}_{g'} dx =$$

$$= \left(\frac{5x^3}{3} + \frac{3x^2}{2}\right) \cdot \ln(2x) - \left(\frac{5x^3}{9} + \frac{3x^2}{4}\right) + C$$

8, Parciális integrálás:

$\int \underbrace{\text{arctg}}_{\text{tudjuk őket deriválni!}}, \int \text{arcsin}, \dots \quad \int \text{arsh}, \dots$

a, $$\int \text{arctg}x \, dx = \int \underbrace{1}_{f'} \cdot \underbrace{\text{arctg}x}_{g} \, dx = \underbrace{x}_{f} \cdot \underbrace{\text{arctg}x}_{g} - \int \underbrace{x}_{f} \cdot \underbrace{\frac{1}{1+x^2}}_{g'} dx =$$

$$= x \cdot \text{arctg}x - \int \frac{1}{2} \frac{\overbrace{2x}^{f'}}{\underbrace{1+x^2}_{f}} dx = x \cdot \text{arctg}x - \frac{1}{2} \cdot \ln|1+x^2| + C$$

b, $$\int \arcsin x \, dx = \int \underbrace{1}_{f'} \cdot \underbrace{\arcsin x}_{g} \, dx = \underbrace{x}_{f} \cdot \underbrace{\arcsin x}_{g} - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx =$$

$$= x \cdot \arcsin x + \frac{1}{2} \int \underbrace{(-2x)}_{f'} \cdot \underbrace{(1-x^2)^{-\frac{1}{2}}}_{f^{\alpha}} dx = x \cdot \arcsin x + \frac{1}{2} \cdot \frac{(1-x^2)^{1/2}}{1/2} + C =$$

$$= x \cdot \arcsin x + \sqrt{1-x^2} + C$$

c, $\int \operatorname{arsh}x \, dx = \int \underbrace{1}_{f'} \cdot \underbrace{\operatorname{arsh}x}_{g} \, dx = \underbrace{x}_{f} \cdot \underbrace{\operatorname{arsh}x}_{g} - \int \underbrace{x}_{f} \cdot \underbrace{\frac{1}{\sqrt{1+x^2}}}_{g'} \, dx =$

$= x \cdot \operatorname{arsh}x - \frac{1}{2} \int \underbrace{2x}_{f'} \cdot \underbrace{(1+x^2)^{-1/2}}_{f^\alpha} \, dx = x \cdot \operatorname{arsh}x - \frac{1}{2} \cdot \frac{(1+x^2)^{1/2}}{1/2} + c =$

$= x \cdot \operatorname{arsh}x - \sqrt{1+x^2} + c$

9, Parciális integrálás:

$\int \begin{matrix} \sin \\ \cos \\ \exp \\ \operatorname{ch} \\ \operatorname{sh} \end{matrix} \cdot \begin{matrix} \sin \\ \cos \\ \exp \\ \operatorname{ch} \\ \operatorname{sh} \end{matrix}$ akármelyik, akármelyikkel lehet szorozva

a, $\underbrace{\int \underbrace{\sin x}_{f'} \cdot \underbrace{\operatorname{ch}x}_{g} \, dx}_{I} = \underbrace{-\cos x}_{f} \cdot \underbrace{\operatorname{ch}x}_{g} - \int \underbrace{-\cos x}_{f} \cdot \underbrace{\operatorname{sh}x}_{g'} \, dx =$

↳ Lehetne fordítva is a szerep kiosztás!

$= -\cos x \cdot \operatorname{ch}x + \int \underbrace{\cos x}_{f'} \cdot \underbrace{\operatorname{sh}x}_{g} \, dx = -\cos x \cdot \operatorname{ch}x + \underbrace{\sin x}_{f} \cdot \underbrace{\operatorname{sh}x}_{g} - \overbrace{\int \underbrace{\sin x}_{f} \underbrace{\operatorname{ch}x}_{g'} \, dx}^{I}$

A szereposztás ugyanaz legyen, mint az előbb!
(Aki nem hiszi, a próbálja ki)

$I = -\cos x \cdot \operatorname{ch}x + \sin x \operatorname{sh}x - I$

$I = \frac{\sin x \operatorname{sh}x - \cos x \operatorname{ch}x}{2}$

Másik megoldás: trig. azonosságokkal

b, $\underbrace{\int \underbrace{\sin 2x}_{f'} \cdot \underbrace{\cos 3x}_{g} \, dx}_{I} = \underbrace{\frac{-\cos 2x}{2}}_{f} \cdot \underbrace{\cos 3x}_{g} - \int \underbrace{\frac{-\cos 2x}{2}}_{f} \cdot \underbrace{3(-\sin 3x)}_{g'} \, dx =$

$= \frac{-\cos 2x \cdot \cos 3x}{2} - \frac{3}{2} \cdot \int \underbrace{\cos 2x}_{f'} \cdot \underbrace{\sin 3x}_{g} \, dx =$

$$= \frac{-\cos 2x \cos 3x}{2} - \frac{3}{2}\underbrace{\frac{\sin 2x}{2}}_{f}\cdot\underbrace{\sin 3x}_{g} + \frac{3}{2}\int \underbrace{\frac{\sin 2x}{2}}_{f}\cdot\underbrace{3\cdot\cos 3x}_{g'}\,dx =$$

$$= \frac{-2\cos 2x\cos 3x - 3\sin 2x \sin 3x}{4} + \frac{3}{4}I \quad = I$$

$$I = -2\cos 2x\cos 3x - 3\sin 2x\sin 3x \quad + c$$

c) $$\int \underbrace{\sin 2x}_{f'}\cdot\underbrace{e^{3x}}_{g}\,dx = \underbrace{\frac{-\cos 2x}{2}}_{f}\cdot\underbrace{e^{3x}}_{g} - \int \underbrace{\frac{-\cos 2x}{2}}_{f}\cdot\underbrace{3\cdot e^{3x}}_{g'}\,dx =$$

$I =$

$$= \frac{-\cos 2x\cdot e^{3x}}{2} + \frac{3}{2}\int \underbrace{\cos 2x}_{f'}\cdot\underbrace{e^{3x}}_{g}\,dx =$$

$$= \frac{-\cos 2x\cdot e^{3x}}{2} + \frac{3}{2}\cdot\underbrace{\frac{\sin 2x}{2}}_{f}\cdot\underbrace{e^{3x}}_{g} - \frac{3}{2}\cdot\int \underbrace{\frac{\sin 2x}{2}}_{f}\cdot\underbrace{3e^{3x}}_{g'}\,dx =$$

$$= \frac{-2\cos 2x\cdot e^{3x} + 3\sin 2x\cdot e^{3x}}{4} - \frac{9}{4}\cdot\underbrace{\int \sin 2x\cdot e^{3x}\,dx}_{I} \quad = I$$

$$I = \frac{1}{13}\cdot e^{3x}\cdot(-2\cos 2x + 3\sin 2x)$$

d) $$\int \frac{\sin x}{e^x}\,dx = \int \underbrace{\sin x}_{f}\cdot\underbrace{e^{-x}}_{g'}\,dx = \underbrace{\sin x}_{f}\cdot\underbrace{(-e^{-x})}_{g} - \int \underbrace{\cos x}_{f'}\cdot\underbrace{(-e^{-x})}_{g}\,dx =$$

$I =$

$$= -\sin x\cdot e^{-x} + \int \underbrace{\cos x}_{f}\cdot\underbrace{e^{-x}}_{g'}\,dx = -\sin x\cdot e^{-x} + \underbrace{\cos x}_{f}\cdot\underbrace{(-e^{-x})}_{g} - \int \underbrace{-\sin x}_{f'}\cdot\underbrace{(-e^{-x})}_{g}\,dx =$$

$$= -\sin x\cdot e^{-x} - \cos x\cdot e^{-x} - \underbrace{\int \frac{\sin x}{e^x}\,dx}_{I} \quad = I$$

$$I = -e^{-x}\cdot\frac{\sin x + \cos x}{2}$$