Statika - 2. gyakorlat

Vektoralgebra pluszfeladatok

Plusz 1 Ha az $\underline{a}+3\underline{b}$ vektor merőleges a $7\underline{a}-5\underline{b}$ vektorra, valamint az $\underline{a}-4\underline{b}$ vektor pedig merőleges a $7\underline{a}-2\underline{b}$ vektorra, akkor mekkora $\underline{a}$ és $\underline{b}$ szögének koszinusza?

$$(\underline{a}+3\underline{b})(7\underline{a}-5\underline{b})=0 \rightarrow 7\underline{a}^2+21\underline{a}\cdot\underline{b}-5\underline{a}\cdot\underline{b}-15\underline{b}^2=0$$

$$(\underline{a}-4\underline{b})(7\underline{a}-2\underline{b})=0 \rightarrow 7\underline{a}^2-28\underline{a}\underline{b}-2\underline{a}\underline{b}+8\underline{b}^2=0$$

$$\left.\begin{aligned}16\underline{a}\underline{b} &= -7\underline{a}^2+15\underline{b}^2\\ 30\underline{a}\underline{b} &= 7\underline{a}^2+8\underline{b}^2\end{aligned}\right\} \quad \begin{aligned}&\rightarrow \underline{a}\underline{b}=\frac{1}{16}(-7\underline{a}^2+15\underline{b}^2)\\ &\rightarrow \underline{a}\underline{b}=\frac{1}{30}(7\underline{a}^2+8\underline{b}^2)\end{aligned}$$

$$30(-7\underline{a}^2+15\underline{b}^2)=16(7\underline{a}^2+8\underline{b}^2)$$

$$-210\underline{a}^2+450\underline{b}^2=128\underline{b}^2+112\underline{a}^2$$

$$\underline{\underline{322\underline{b}^2=322\underline{a}^2}}$$

$\hookrightarrow$ I. $\underline{a}=\underline{b}=\underline{0}$

$\hookrightarrow$ II. $\underline{a}^2=\underline{b}^2 \rightarrow |\underline{a}|=|\underline{b}|$

- $\underline{a}\cdot\underline{b}=\underbrace{|\underline{a}||\underline{b}|}_{\underline{a}^2}\cos\alpha = \underline{\underline{\underline{a}^2\cos\alpha}}$ $\qquad \underline{\underline{\cos\alpha=\frac{1}{2}}}$

- $\underline{a}\cdot\underline{b}=\frac{1}{16}(-7\underline{a}^2+15\underline{b}^2)=\frac{1}{16}(-7+15)\underline{a}^2=\underline{\underline{\frac{1}{2}}}\,\underline{a}^2$

Plusz 2

Hozzuk egy szebb alakra!

$$(3\underline{a}-\underline{b})\times(\underline{b}-3\underline{a}) = 3(\underline{a}\times\underline{b}) \underbrace{-\underline{b}\times\underline{b}}_{=0} + \underbrace{9\underline{a}\times\underline{a}}_{=0} - 3\underline{b}\times\underline{a}$$

$$= 3\underline{a}\times\underline{b} - 3\underline{b}\times\underline{a} = 3\underline{a}\times\underline{b} + 3\underline{a}\times\underline{b} = \underline{\underline{6\underline{a}\times\underline{b}}}$$

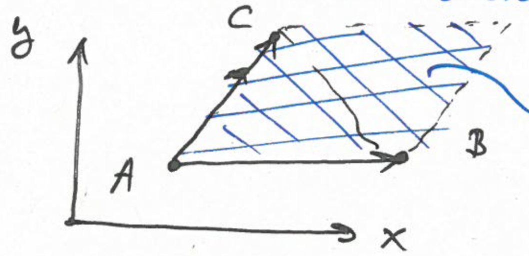
Plusz 3

Mekkora az ABC háromszög területe?

$A(1,0,2)$

$B(-1,4,7)$

$C(5,-2,1)$



$$T = |\underline{r}_{AB}\times\underline{r}_{AC}|$$

$$\underline{r}_{AB} = \underline{r}_B - \underline{r}_A = \begin{bmatrix}-1\\4\\7\end{bmatrix} - \begin{bmatrix}1\\0\\2\end{bmatrix} = \underline{\underline{\begin{bmatrix}-2\\4\\5\end{bmatrix}}}$$

$$\underline{r}_{AC} = \underline{r}_C - \underline{r}_A = \begin{bmatrix}5\\-2\\1\end{bmatrix} - \begin{bmatrix}1\\0\\2\end{bmatrix} = \underline{\underline{\begin{bmatrix}4\\-2\\-1\end{bmatrix}}}$$

$$\underline{r}_{AB}\times\underline{r}_{AC} = \begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ -2 & 4 & 5\\ 4 & -2 & -1\end{vmatrix} = \begin{bmatrix}6\\-18\\-12\end{bmatrix}$$

$$|\underline{r}_{AB}\times\underline{r}_{AC}| = \sqrt{6^2+(-18)^2+(-12)^2} = 20{,}45 = T$$

$$T_\Delta = \frac{T}{2} = \underline{\underline{11{,}225}}$$

$\boxed{\text{Plusz 4.}}$ Igaz-e, ha $\underline{a} \times \underline{c} = \underline{b} \times \underline{c}$ akkor $\underline{a} = \underline{b}$?

$\underline{a} \times \underline{c} - \underline{b} \times \underline{c} = \underline{0}$

$(\underline{a} - \underline{b}) \times \underline{c} = \underline{0}$

↳ Ez akkor zérus, ha

1) $(\underline{a} - \underline{b}) = \underline{0} \Rightarrow \underline{a} = \underline{b}$ ✓ ezt állítottuk!

DE
2) $\underline{c} = \underline{0}$
3) $(\underline{a} - \underline{b}) \parallel \underline{c} \Rightarrow (\underline{a} - \underline{b}) = \lambda \cdot \underline{c}$

Tehát nem igaz, mert más eset is lehet!