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Stability case study of an underactuated service robot

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Abstract: Usually the dynamics of robotic systems are described by ordinary differential equations via selecting a minimum set of (independent) generalized coordinates. However, different parameterizations and the use of a non-minimum set of (dependent) generalized coordinates can be advantageous in such cases when the modeled device contains closed kinematic loops and/or it has a complex structure. The use of dependent coordinates, like natural coordinates, leads to differential algebraic equations of motion. On the other hand, the stability analysis and control design of underactuated robots are usually rely on partial feedback linearization based techniques which are exclusively developed for systems modeled by independent coordinates. In this paper we propose a different control algorithm formulated by using dependent coordinates. A case study is presented for the stability analysis of the motion control of a low degree of freedom, digitally controlled, underactuated service robot.

1. Introduction

In case of complex multibody problems or closed loop manipulators, geometric constraint conditions have to be considered during dynamics modeling and simulation. These systems are often described by a dependent set of generalized coordinates subjected to constraints that provide an efficient formalism to generate the equations of motion in the form of differential algebraic equations (DAE).

Control methods for underactuated manipulators, such as partial feedback linearization (PFL) [1] and the computed desired computed torque control (CDCTC) method [2] are mainly developed for systems described by independent coordinates. The PFL can be used to feedback linearize the dynamics corresponding to the active degrees-of-freedom (DoFs), and in case of strong inertial coupling the dynamics corresponding to the passive DoFs of a system. In connection with the categories of active and passive coordinates, reference [2] introduces controlled and uncontrolled coordinates. While the controlled coordinates are prescribed, the trajectories of the uncontrolled coordinates are calculated on-line which makes the error feedback possible for all DoFs. The on-line calculation of these coordinates requires the solution of the equation of motion projected into the space of uncontrolled motion.

Following the same idea, reference [3] presents a computed torque based solution for the position control of a suspended service robot ACROBOTER [4] that is modeled by dependent coordinates. In that study, the desired values of the uncontrolled coordinates and the control input is determined via the direct solution of the DAE equation of motion by applying the backward Euler discretization.

By making use of the concept of servo-constraints (also called as control- or actuator-constraints) [5], here, a computationally more effective computed torque control strategy is proposed based on the method of Lagrangian multipliers combined with a PD controller for enforcing the servo-constraints. This controller provides similar constraint stabilization for the servo-constraints as the Baumgarte stabilization for the geometric constraints.

2. Dynamics modeling and computed torque control

It is assumed that the dynamics of the studied underactuated system is modeled by using dependent coordinates resulting the equation of motion

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\lambda} = \mathbf{Q} + \mathbf{H}\mathbf{u}\,, \tag{1}$$

$$\boldsymbol{\phi}_g = \mathbf{0}\,, \tag{2}$$

where $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n\times n}$ is the mass matrix, $\mathbf{\Phi}_{\mathbf{q}}(\mathbf{q}) \in \mathbb{R}^{m\times n}$ is the Jacobian of geometric constraints $\boldsymbol{\phi}_g(\mathbf{q}, t) \in \mathbb{R}^m$ and $\boldsymbol{\lambda}$ is the vector of the Lagrangian multipliers. Matrix $\mathbf{H}(\mathbf{q}) \in \mathbb{R}^{n\times l}$ is the control input matrix and $\mathbf{u} \in \mathbb{R}^l$ contains the actuating forces and torques. In addition, $\mathbf{Q}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^n$ denotes the remaining generalized forces like the gravity and/or the Coriolis and centrifugal terms.

Equation (1), (2) form an index 3 DAE, from which the underlying ODE equations can be obtained by differentiating the geometric constraint equations twice and substituting back the solution for $\boldsymbol{\lambda}$ into equation (1). For the sake of simplicity, in the following derivations we assume that the geometric constraints have no components with explicit time dependence. The constraints at the acceleration level has the form

$$\ddot{\boldsymbol{\phi}}_g = \mathbf{\Phi}_{\mathbf{q}}\ddot{\mathbf{q}} + \dot{\mathbf{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}}\,. \tag{3}$$

An important element of the presented approach is the use of the concept of servo-constraints [5] specifying the desired motion of the constrained system as function of the generalized coordinates and time. The servo constraint equations are formulated similar to the geometric ones, but they involve control specification terms that may depend on time explicitly. These constraints are represented at the position and acceleration level by

$$\boldsymbol{\phi}_s(\mathbf{q},t)=\mathbf{0} \quad \text{and} \quad \ddot{\boldsymbol{\phi}}_s=\mathbf{G}_{\mathbf{q}}\ddot{\mathbf{q}}+\dot{\mathbf{G}}_{\mathbf{q}}\dot{\mathbf{q}}+\dot{\mathbf{c}}, \quad \text{where} \quad \mathbf{G}_{\mathbf{q}}=\frac{\partial\boldsymbol{\phi}_s}{\partial\mathbf{q}} \quad \text{and} \quad \mathbf{c}=\frac{\partial\boldsymbol{\phi}_s}{\partial t}\,. \tag{4}$$

Combining the equation of motion (1), the geometric (3) and the servo constraints (4) at the acceleration level, similarly to the method of Lagrange multipliers, the accelerations $\ddot{\mathbf{q}}$, the Lagrange multipliers $\boldsymbol{\lambda}$ and the control input $\mathbf{u}$ can be calculated as the solution of the system

$$\begin{bmatrix} \mathbf{M} & \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}} & -\mathbf{H} \\ \boldsymbol{\Phi}_{\mathbf{q}} & \mathbf{0} & \mathbf{0} \\ \mathbf{G}_{\mathbf{q}} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{Q} \\ -\dot{\boldsymbol{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}} \\ -\dot{\mathbf{G}}_{\mathbf{q}}\dot{\mathbf{q}}-\dot{\mathbf{c}}-\alpha\dot{\boldsymbol{\phi}}_s-\beta\boldsymbol{\phi}_s \end{bmatrix}, \tag{5}$$

where the scalars α and β play a similar role as the Baumgarte parameters [6] in the solution of DAE equations of motion. When there are only geometric constraints the Baumgarte parameters are used to stabilize those constraints. Here, the control parameters α and β realize a PD controller that is intended to enforce the servo constraints which results the desired motion of the system. Assuming that the state of the system is measured and the system matrix of (5) is invertible, the necessary control action can be calculated. The proposed controller will be applied for a novel service robot described in the next section.

3. The ACROBOTER service robot

The ACROBOTER platform is a service robot that crawls in the plane of the ceiling and has a pendulum-like working unit [4]. The different subsystems of this robot are shown in Fig. 1. The system of anchor points is placed on the ceiling in a triangular grid. The climber unit (CU), which is a planar RRT robot, can move by grasping these anchor points. The swinging unit (SU) is connected to the climber unit via a main and three secondary cables. These four cables are fixed in one point by the cable connector (CC). The horizontal motion of the SU is mainly provided by the climber unit, while in vertical direction it can be elevated by the main cable. In addition, the SU has ducted fan actuators that can also contribute to the control of the horizontal motion. The cable connector has no actuators and therefore its position cannot directly be controlled. Consequently the ACROBOTER platform is underactuated.

Assuming that the position of the CU and the length of the secondary cables are fixed the ACROBOTER robot can be modeled as a double pendulum. For the presented stability case study we consider only the corresponding simplified planar model of the system shown right in Fig. 1. This model has one (ducted-fan) actuator considered as an external force

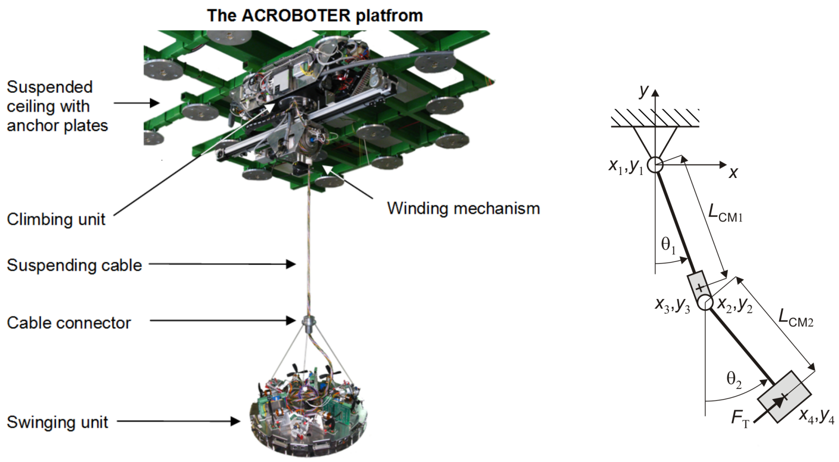

Figure 1. The ACROBOTER robot and its planar model

(F_{T}) acting at the center of mass of the second link of the double pendulum. Using the so-called natural coordinates $\mathbf{q} = [x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4]^{\mathrm{T}}$ and based on reference [6], the equation of motion of the system can be given in the DAE form (1) and (2).

4. Stability of the planar ACROBOTER model

The stability of digitally controlled mechanical systems can be investigated by calculating the eigenvalues of the discrete mapping constructed from the piecewise solution of the equation of motion. This solution is known analytically if the system is linear, thus we have to linearize the system around the investigated configurations. During the stability investigation it is assumed that the control forces calculated at the n^{th} time instant are based on the $(n-1)^{\mathrm{th}}$ measured values which are held by a zero-order-hold (ZOH) until the $(n+1)^{\mathrm{th}}$ sampling instant. The stability investigation is based on equation (1) where the input force $\mathbf{u}$ is calculated via the solution of equation (5). In order to avoid the extra eigenvalues that correspond to the realization of the geometric constraints the DAE system (1) and (2) is transformed into the minimum set of equations using the transformation $\dot{\mathbf{q}} = \mathbf{R}\dot{\mathbf{p}}$ with $\mathbf{p} = [\theta_1, \theta_2]^{\mathrm{T}} \in \mathbb{R}^{n-m}$. The transformed equation has the form

$$\mathbf{R}^{\mathrm{T}}\mathbf{M}\mathbf{R}\ddot{\mathbf{p}} + \mathbf{R}^{\mathrm{T}}\mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{p}} = \mathbf{R}^{\mathrm{T}}\mathbf{Q} + \mathbf{R}^{\mathrm{T}}\mathbf{H}\mathbf{u}. \tag{6}$$

Equation (6) can be linearized around an arbitrary configuration and after that the equation of the controlled system can be written in the general state space form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{x}(t_{n-1}), \quad t \in [t_n, t_{n+1}]. \tag{7}$$

By using the state variables at the end of the n^{th} sampling interval the solution can be calculated as

$$\mathbf{x}(t_{n+1}) = e^{\mathbf{A}\Delta t}\mathbf{x}(t_n) + (e^{\mathbf{A}\Delta t} + \mathbf{I})\mathbf{A}^{-1}\mathbf{B}\mathbf{x}(t_{n-1}), \tag{8}$$

where Δt is the sampling time of the digital controller. Based on equation (8) the mapping $\mathbf{z}_{n+1} = \mathbf{H}\mathbf{z}_n$ can be composed, where $\mathbf{z}_n = [\mathbf{x}(t_{n-1}), \dot{\mathbf{x}}(t_{n-1}), \mathbf{x}(t_n), \dot{\mathbf{x}}(t_n)]$ is the discrete state vector. The convergence of this discrete mapping is equivalent to the asymptotic stability of the desired motion of the control system. To ensure stability, by considering the mapping as a multi dimensional geometric series, the eigenvalues of $\mathbf{H}$ have to be located within the unit circle of the complex plane. The stability calculations are based on the planar model of ACROBOTER shown in Fig. 1. Based on the analytical formulae provided above, the stability charts are calculated numerically over the grid of the parameters α, β. The third parameter of these charts is associated with the configuration of the system. The ACROBOTER robot is kinematically redundant since the ratio of the lengths of the main and secondary cables can vary while the position of the SU remains unchanged. In the present model the main cable plus the CC are modeled by the first link (L_1), and the effective length associated with the secondary cables plus the height of the SU are modeled by the second link (L_2) of the double pendulum model. The ratio (L_2/L_1) of these lengths characterizes the investigated configuration of the system. The resulting stability charts are shown in Fig. 2. The parameters used in the stability analysis correspond to the real parameters of the ACROBOTER platform. The masses of the links of the pendulum are $m_1 = 0.1$[kg] and $m_2 = 5$[kg], respectively, while the corresponding mass moment of inertias with respect to the centers of masses are $J_{CM1} = 0.01$[kgm^2] and $J_{CM2} = 0.1$[kgm^2]. The centers of gravity along each links are defined by $L_{CM1} = L_1 - 0.02$[m] and $L_{CM2} = L_2 - 0.1$[m], where the typical cable lengths are included in the ranges $L_1 \in (0.1,\ 3\,\text{m})$ and $L_2 \in (0.2\,\text{m},\ 1\,\text{m})$.

The sampling time is given by $\Delta t = 0.01[s]$. Because of the nonlinear behavior of the system the stability properties depend on the configuration. The stability charts are showing the stable domain of control parameters in the hanging down position, i.e., $\theta_1 \approx 0$ and $\theta_2 \approx 0$.

5. Conclusion

In this paper a computed torque method was proposed for underactuated mechanical systems modeled by non-minimum set of generalized coordinates. The proposed controller was applied for the planar model of the ACROBOTER service robot. The stability of this system was analyzed in the parameter space of the proportional and differential control gains (β and α) and the configuration parameter (L_2/L_1) which mainly characterized by the ratio of the

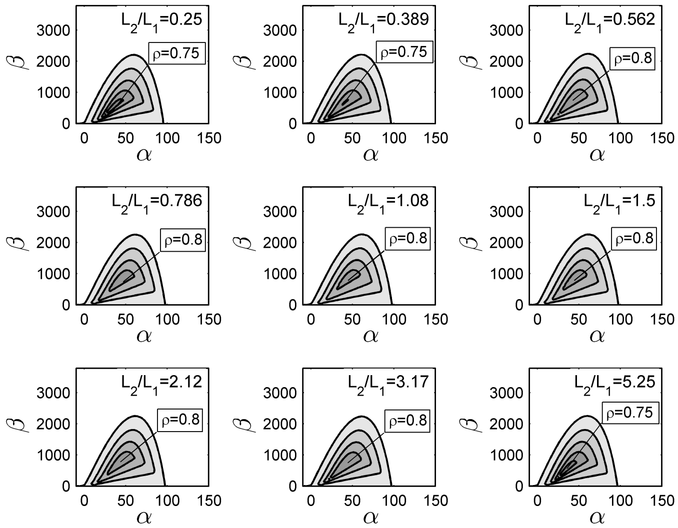

Figure 2. Stability charts

lengths of the main and secondary cables. The stability charts shows that this cable length ratio has no significant effect on the boundaries of the stable domain of control parameters. On the other hand, the stability charts with the lowest minimum spectral radii (see $\rho = 0.75$ in Fig. 2) belong to those cases where the cable length ratio is low or high. These extreme cases corresponds to those configurations in which the ACROBOTER system has a structure which is close to a single pendulum.

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