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MODELING GEARS WITH COGAL SOFTWARE

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ABSTRACT: Nonlinear racks, noncircular gears and chain rings are special and rare machine parts. The rarity can be explained by the complicated evaluation process of these gears. In the current work, a numerical methods were determined and applied in a computer program called 'CogAL', which makes this process easier and faster for the user. The main concepts of the numerical method and the functionality of our software will be shortly presented thorough some special examples.

Keywords: *gear, cogwheel, noncircular gear, nonlinear rack, cog profile*

INTRODUCTION

In most cases, general gears defined by basic parameter (module, diameter, etc) are manufactured by special machines [1,2]. The complicated surfaces of the gears are automatically generated, thus no accurate model is needed. Gear modelling modules integrated in CAD systems do not reproduce the accurate geometry, as they do not follow the principle of the shear cutting (see Fig. 1). These modules serve mostly just to visualize the gears.

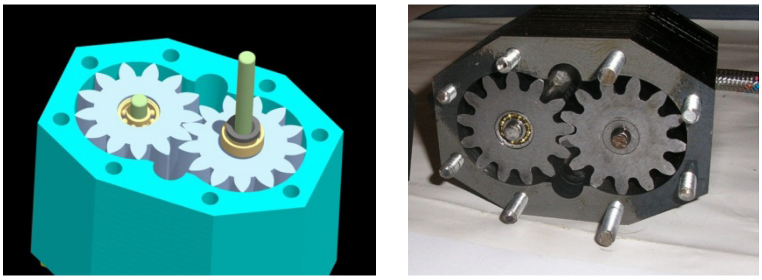

Fig.1. *The illustration of gear pump CAD model and the manufactured gear*

Recently various CNC technologies came to the front, which are suitable also to manufacture gears, like cutting with laser or water, wire-spark cutting, CNC milling, 3D printing [5]. Contrary to the traditional manufacturing methods, these modern technologies need a precise geometry of the cogwheel. Creating these models is usually a slow and difficult process. These machine parts are very rare in the industrial usage, which does not mean that there is no need for them. In many cases, complex mechanisms can be replaced by a single pair of noncircular gear. The reason they are rare, is that their definition is complex and time consuming.
Our main goal was to create a fast and user-friendly software, which can generate a large variety of gear geometry based on the simulation the rolling of the gears. The software, named CogAL, can generate noncircular gears, noncircular chain-rings or nonlinear racks (Fig. 2. [7]) based on the function of transmission.

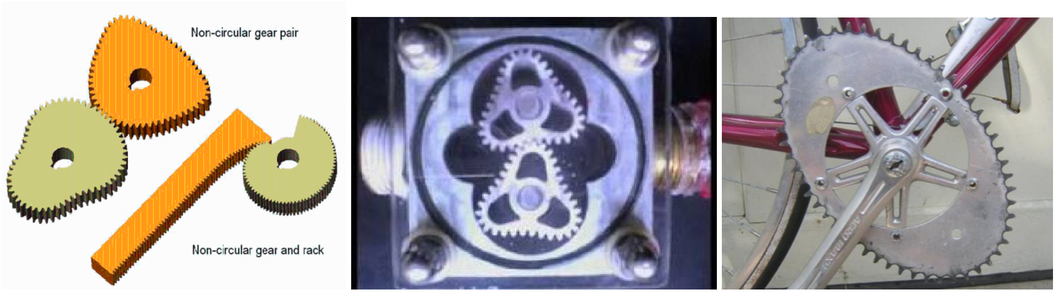

Fig.2. *Non-circular gears, non-circular chain-ring and non-linear rack*

A non-circular gear can be build up as follows. First, the cog profile must be defined, then a circular tool-gear must be evaluated. This can be used to generate the non-circular gear after the main curve is defined based on the function of transmission. CogAL program has the some steps of evaluation.In the next sections, these methods and their implementation in CogAL are presented in details.

COG PROFILES

In CogAl, the frequently used cog profiles are predefined, such as standard, sine, or chain profiles [3]. Custom profiles can also be defined by a set of contour points. See Fig. 3.

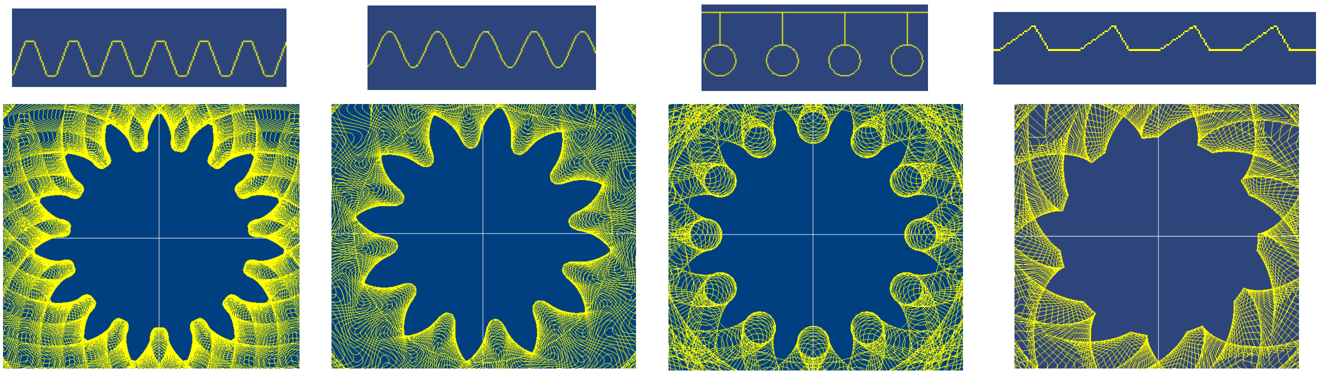

Fig.3. *Standard, sine, chain and custom tooth profiles*

ROLLING A LINE ON A CONVEX CURVE

The circular tool-gear can be generated by rolling a line (the rack) on a circle. Its generalized method is presented here. A steady *x-y* and a rolling ξ-η coordinate system are defined. The f_p convex rolling curve is given in the *x-y* system. The rolling line is defined by the ξ axis on which the predefined cog profiles placed

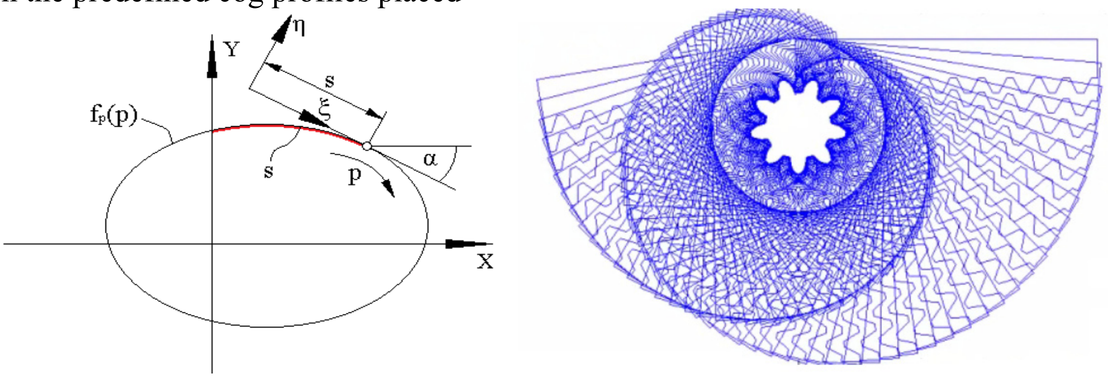

Fig.4. *The steps of the transformation and the resultant set of lines*

The rolling is implemented by using finite numbers of transformations of the ξ-η coordinate system. The transformation can be divided into three steps (see Fig. 4.):

- Offset in the direction of axis ξ with (-*s*), where *s* is the rolled length.
- Rotation with α around the center of ξ-η, where α is the angle of the tangent in the current point.
- Offset of the center of ξ-η into the current point.

The result of the transformations is a set of lines (Fig. 4.). The inside contour gives the geometry of the gear. The details of the contour finding algorithm are presented later [6].

ROLLING A CIRCLE ON AN CURVE

The determined tool-gear can be used to generate the *concave* gear. The radius of this circle cannot be larger than the smallest radius of the f_p rolling curve. The concept of the previous section are used, however, the transformation of the ξ-η system consist of two main steps (Fig. 5.):

- Rotation with ($\varphi=\alpha+\phi$) around the center of ξ-η, where α is the angle of the tangent in the current point and ϕ is the angle of the rolling rotation of the tool-gear.
- Offset of the center of ξ-η into P_m current point.

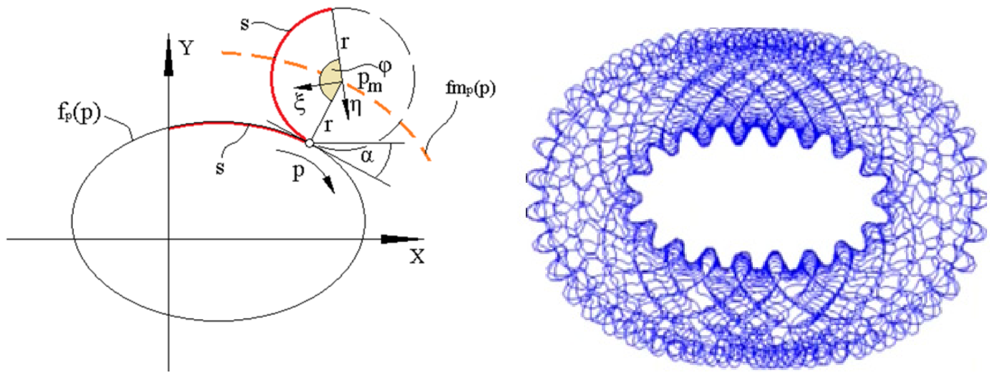

***Fig.5.** Steps of the rolling of the tool-gear on the rolling curve and the resultant set of lines*

With these two methods we can place general cog-profile even on a concave curves which defined by a rolling curve. The generation of the rolling curves for a given function of transmission is presented in the next section [6].

ROLLING CURVES

The rolling curves of a pair of noncircular gears which have fixed axis distance can be defined by the *a* axis distance, which is the sum of the working radii $a=r_1+r_2$ and function of transmission, which is given by

$$i = \frac{\omega_1}{\omega_2} = \frac{\partial\varphi_1}{\partial\varphi_2}, \tag{1}$$

where ω_i is the angular velocity, φ_i is the angle position of the *i*th gears. In **Fig.6.** Fig. 6a, kinematic chain of the rolling of two non-circular gears is presented. The contact point is placed between the axes. The gears have the same velocity vector in this point, so:

$$r_1\omega_1 = r_2\omega_2. \tag{2}$$

Hence, the transmission can be defined by the working radii and/or the axis distance:

$$i = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{a-r_1}{r_1}. \tag{3}$$

Rearranging Eq (3), the first rolling curve in polar coordinate system can be define as

$$r_1(\varphi_1) = \frac{a}{i(\varphi_1)+1} \tag{4}$$

and the rolling curve of the second gear can be defined as

$$r_2(\varphi_2(\varphi_1)) = r_1(\varphi_1)i(\varphi_1). \tag{5}$$

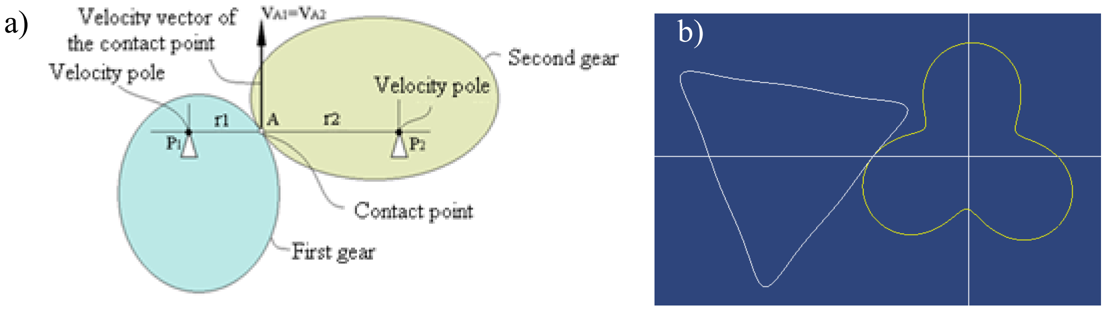

Fig.6. *a) Sketch for the evaluation of the curves b) a pairs of rolling curves*

CONTOUR FINDING

Based on the previous sections, a set of lines can be defined for a general concave non-linear gear. To get the final geometry, we need to find the inside contour of these lines. Many methods have developed, implemented and tested, and finally an image based method shows the best and fastest results [4]. The essence of this method is to draw a very high resolution raster image of the lines using the graphics card. From this image we can obtain the contour, pixel by pixel very rapidly by using the GPU. Some filtering methods are also applied on the final geometry. The resolution of the applied image can be adjusted to the given surface tolerance of the gear surface.

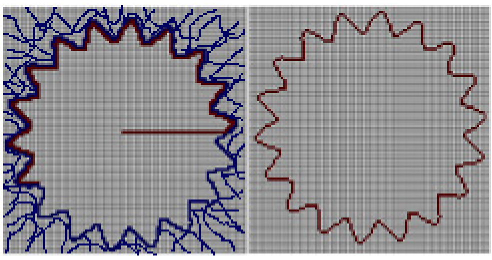

Fig.7. *Image based contour finding presented in a very low resolution*

THE COGAL APPLICATION

For ease of the more convenient use, the previously reviewed computations were implemented in C# environment. The name of the software: CogAL, derived from "***Co***gwheel ***g***enerator ***al***gorithm". Our goal was to create an easy-to-use application for circular and non-circular gears and non-linear racks. In the next subsections, the four separate functions of the program will be covered.

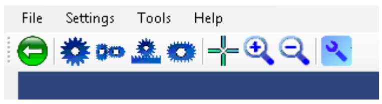

Fig.8. *The menubar of the main functions*

Generating a circular gear

This module can generate circular gears with multiple types of profiles. The standard profile can be defined with its main parameters (like module, contact angle, etc.). Aside from the standard trapezoid profile, it is possible to define sine, chain and custom profiles.

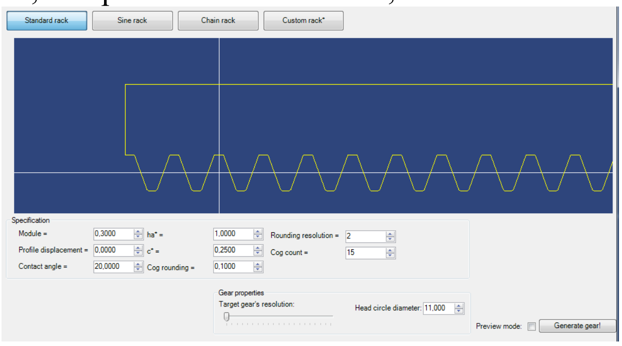

Fig.9. *CogAl's circular gear generator module*

Generating non-circular gear pair, with fixed axis distance

To generate this kind of gear pairs, three steps have to be taken. The first step is to define the rolling curves with their axis distance, and the angle positions pairs (see Fig. 10a). The angle positions can be defined by a function or by a CSV table.

In the next step, the profile has to be specified similarly to the *circular gear* section. When both the profile and the lines are available, a preview image of the final geometry will be generated (Fig. 10b).

In the final step the program will find the contour of the preview shape. The final geometry will be generated, and available for further usage, like exporting into standard CAD system format or a real-time rolling test of the final gears geometries can be performed to check the result (Fig 11).

a)

b)

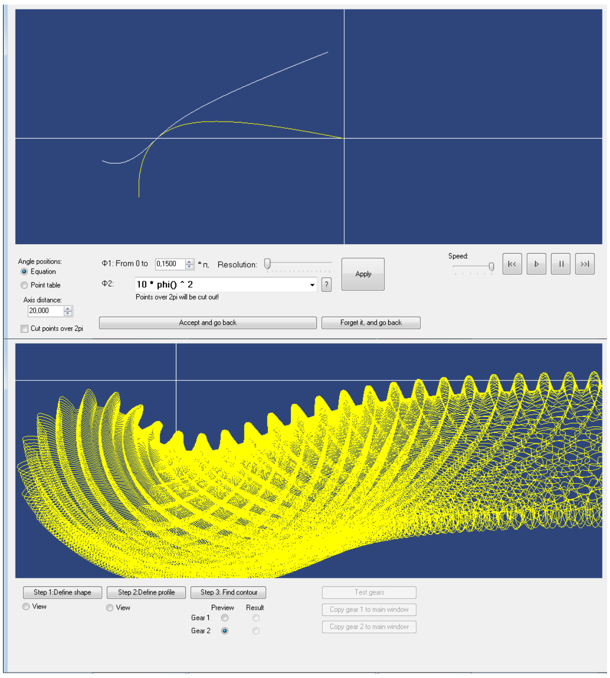

Fig.10. *a) Generated the rolling curves in CogAl b) Preview of the final geometry*

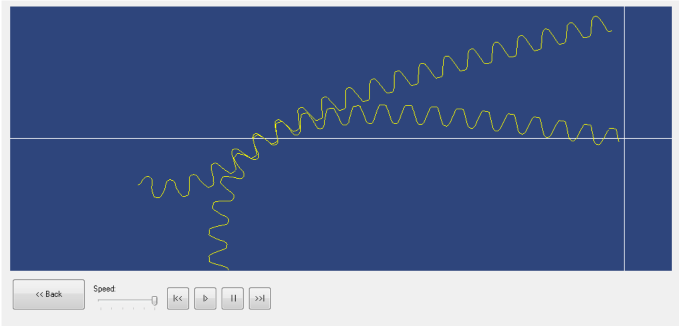

Fig.11. *CogAl shows the final geometry of the non-circular gear pair*

Generating non-circular gear-rack pair

The use of this module is similar in many aspects to the previously presented module. The main difference is in the first step, where the position of the rack is specified as a function of the angular position of the cog-wheel. In this case the transmission function has length units and there is no need to give an axis distance (as it theoretically would be infinite in each case).

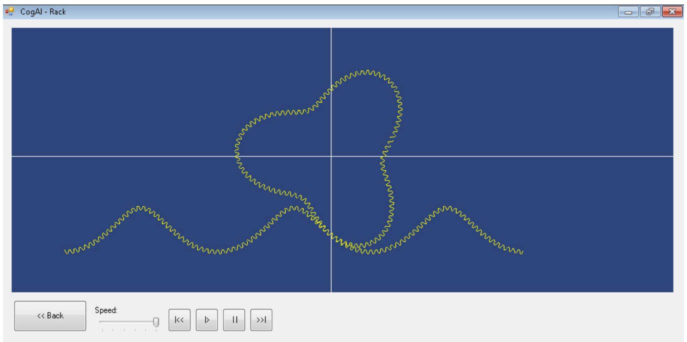

Fig.12. *CogAl shows the final geometry of the non-linear gear-rack pair*

Generating a single, non-circular gear with arbitrary rolling curve

This module can generate a non-circular gear with any kind of rolling curve, with standard, sine, chain or custom tooth profiles. The rolling curve can be defined with two parametric functions in Cartesian or polar coordinate systems, or by loading a CSV file that contains the coordinates.

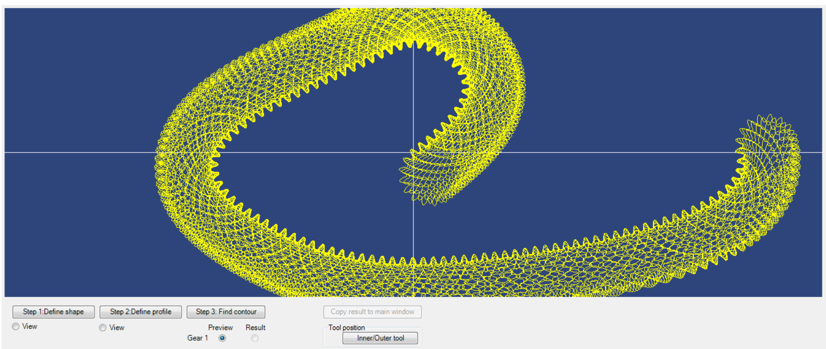

Fig.13. *Generating a single gear with arbitrary curve by CogAl, using the curve of the gears shape*

EXAMPLES

In this section some test applications are shown where the capability of the software and the validity of the results are shown.

In the first test, two gears with identical rolling curves were generated, which creates periodical pulsation in the transmission function. The functions of the angular positions were determined by sinusoidal functions (Fig. 14a).
The second example shows a non-circular gear pair which compensates the angular velocity pulsation of a 45° cardan joint (Fig. 14b).
In the least example, a two-bar open kinematic chain is created, where the linear motion of the end point is guaranteed by non-circular gears, which crates the proper transmission function between angular positions of the bars (Fig. 14c).

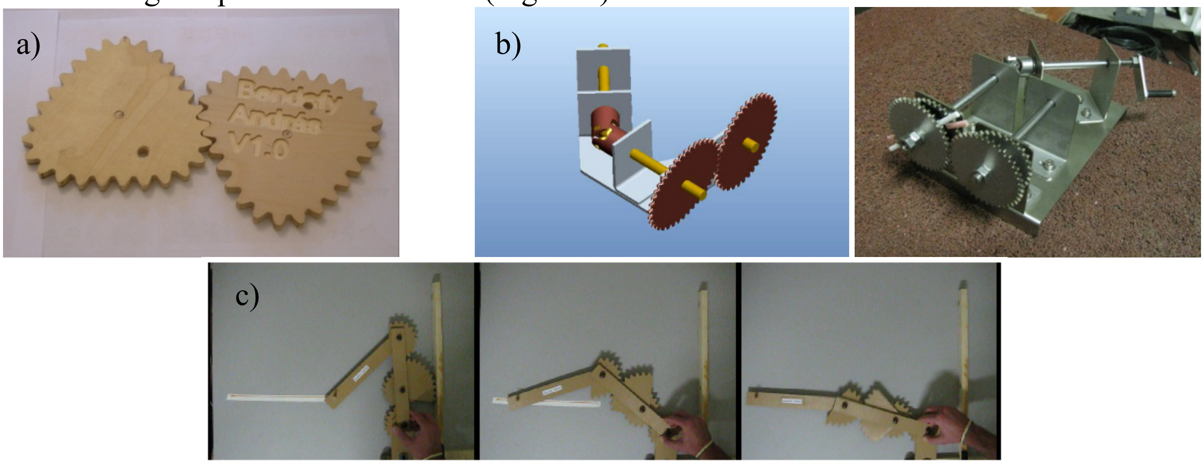

Fig.14. *a) Gears for harmonically varying transmission. b) Eliminating the angular velocity pulsation of the cardan joint. c) Open kinematic chain with linear motion of the free end.*

CONCLUSION

In this work the CogAL software was created which is able to generate general concave non-circular gears, based on a few parameters. The implemented numerical methods were analyzed and optimized to create a fast and reliable geometrical result of the generated cogwheel. A user friendly interface is created where only the main parameters are required and the additional computation is evaluated in the background. The efficiency of the software is represented thorough some engineering examples.
The program is still under development, but it is aimed that later it should be applicable individually, even integrated into CAD software.

ACKNOWLEDGEMENT

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