Száraz súrlódás és nem-folytonos dinamikai rendszerek

Antali Máté

BME Műszaki Mechanikai Tanszék

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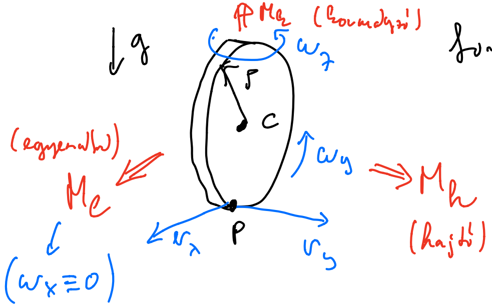
10. Hajtott kerék megcsúszása térben

Az előadás-sorozat elkészülését az MTA/ELKH támogatta a Prémium Posztdoktori Kutatói Programban támogatott PPD2018-014/2018 számú pályázat keretében. Készült 2021-ben.



forgó KR, álló VR

homogén, vékony korong

$$\underline{x} = \begin{bmatrix} v_x \\ v_y \\ \omega_y \\ \omega_z \end{bmatrix} \longrightarrow \dot{\underline{x}} = \underline{F}(\underline{x})$$

(csúszás)

$$m \rightarrow J_y = \frac{1}{2} m\rho^2$$

$$J_z = \frac{1}{4} m\rho^2$$

$$\underline{F}(\underline{x}) = \begin{bmatrix} v_y\omega_z - \frac{2M_h}{m\rho} - 3\mu g \boxed{\frac{v_x}{\sqrt{v_x^2+v_y^2}}} \\ -v_x\omega_z - \rho\omega_y\omega_z - \mu g \boxed{\frac{v_y}{\sqrt{v_x^2+v_y^2}}} \\ \frac{2M_h}{m\rho^2} + \frac{2\mu g}{\rho} \cdot \boxed{\frac{v_x}{\sqrt{v_x^2+v_y^2}}} \\ \frac{4M_k}{m\rho^2} \end{bmatrix}$$

$\Sigma: \; v_x = v_y = 0$

$\rightarrow$ gördül

$\rightarrow$ 2D rész a 4D térben

$$\hat{\underline{x}} \in \Sigma \qquad \hat{\underline{x}} = \begin{bmatrix} 0 \\ 0 \\ \omega_y \\ \omega_z \end{bmatrix} \qquad \underline{\psi}(\varphi) = \begin{bmatrix} \cos\varphi \\ \sin\varphi \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{F}^*(\varphi) = \begin{bmatrix} -\dfrac{2M_h}{m\rho} - 3\mu g \boxed{\cos\varphi} \\ -\rho\omega_y\omega_z - \mu g \boxed{\sin\varphi} \\ \dfrac{2M_h}{m\rho^2} + \dfrac{2\mu g}{\rho} \cdot \boxed{\cos\varphi} \\ \dfrac{4M_g}{m\rho^2} \end{bmatrix}$$

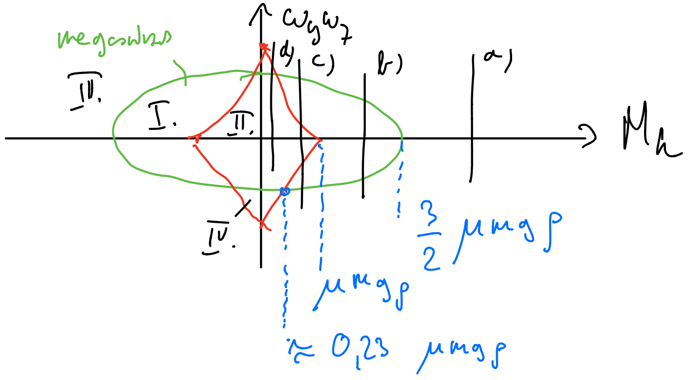
$$\underline{F} = \boxed{\frac{v_x}{\sqrt{v_x^2+v_y^2}}} \cdot \underline{A} + \boxed{\frac{v_y}{\sqrt{v_x^2+v_y^2}}} \underline{B} + \underline{C}$$

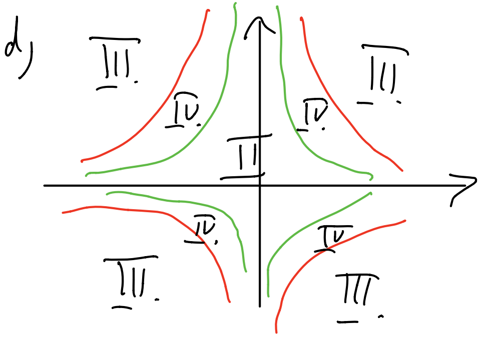
$$\underline{F}^*(\varphi) = \boxed{\cos\varphi} \cdot \underline{A} + \boxed{\sin\varphi} \cdot \underline{B} + \underline{C}$$

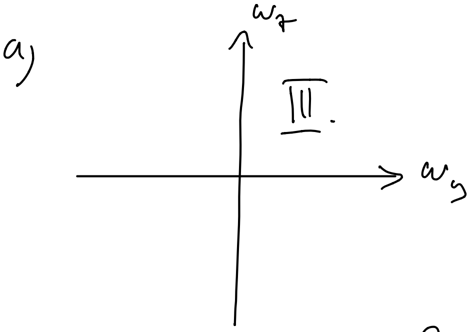
$$\underline{A} = \begin{bmatrix} -3\mu g \\ \boxed{0} \\ 2\mu g/\rho \\ 0 \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} \boxed{0} \\ -\mu g \\ 0 \\ 0 \end{bmatrix} \qquad \underline{C} = \begin{bmatrix} -2M_h/m\rho \\ -\rho \boxed{\omega_y\omega_z} \\ 2M_h/m\rho^2 \\ 4M_h/m\rho^2 \end{bmatrix}$$

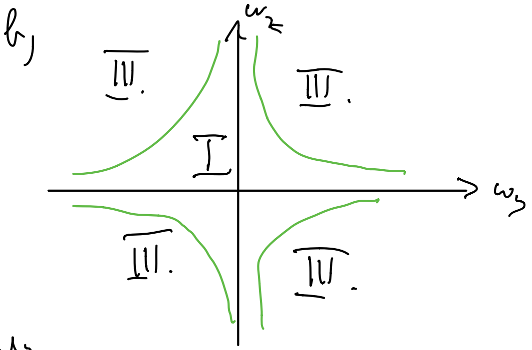
$$A_1 = -3\mu g, \quad B_2 = -\mu g, \quad A_2 = B_1 = 0, \quad C_1 = \frac{-2M_h}{m\rho}, \quad C_2 = -\rho\omega_y\omega_z$$

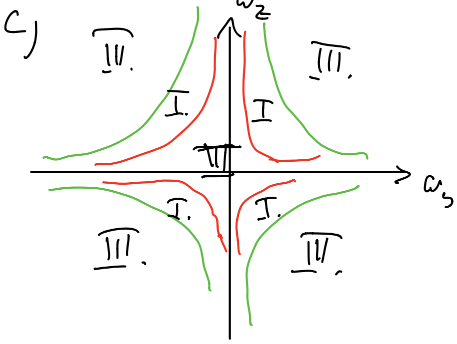
- megoldható! $\left(\dfrac{2M_h}{\rho m}\right)^2 + \left(3\rho\omega_y\omega_z\right)^2 = \left(3\mu g\right)^2$
- 3 határ-ig: $\left(\dfrac{2M_h}{\rho m}\right)^{2/3} + \left(\rho\omega_y\omega_z\right)^{2/3} = \left(2\mu g\right)^{2/3}$











2 kontakt-ion, nincs gördül } nincs

Tapadó dinamika (gördülés):

$$\underline{F}^0 = a\cdot\underline{A} + b\,\underline{B} + \underline{C}$$

$$\langle \underline{F}^0, \underbrace{\underline{\psi}(0)}_{\begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix}} \rangle = 0 \qquad \langle \underline{F}^0, \underbrace{\underline{\psi}(\pi/2)}_{\begin{bmatrix}0\\1\\0\\\vdots\\0\end{bmatrix}} \rangle = 0$$

$$F_1^0 = a\cdot(-3\mu g) + b\cdot 0 - \frac{2M_h}{m\rho} = 0$$

$$F_2^0 = a\cdot 0 + b\cdot(-\mu g) - \rho w_y w_z = 0$$

$$\rightarrow \quad a = -\frac{2M_h}{3\mu m g\rho} \qquad b = -\frac{\rho w_y w_z}{\mu g}$$

$$\rightarrow \underline{F}^0 = \begin{bmatrix} 0 \\ 0 \\ \frac{2}{3}\frac{M_h}{m\rho^2} \\ \frac{4M_h}{m\rho^2} \end{bmatrix}$$

$\downarrow$

gördülés dinamikai egyenletei!!!