## Limit directions of the discontinuity manifolds in extended Filippov systems

Mate Antali<sup>\*</sup> Gabor Stepan<sup>\*</sup> antali@mm.bme.hu stepan@mm.bme.hu

\*Department of Applied Mechanics, Budapest University of Technology and Economics, H-1111 Budapest, Hungary

## Abstract

In classical Filippov systems, there are codimension-1 discontinuity manifolds in the phase space. We consider systems possessing an isolated codimension-2 discontinuity manifold surrounded by smooth behaviour of the vector field. These vector fields are out of the scope of classical Filippov systems and they lead to the definition of *extended Filippov systems*.

In these systems, there are *continuously many* directions which are orthogonal to the discontinuity manifold. By analysing the behaviour of the vector field in the vicinity of the discontinuity manifold, the so-called *limit directions* can be determined. The limit directions show the tangent lines of the trajectories which tend to the discontinuity manifold in forward or backward time. In each point of the discontinuity set, these attracting and repelling limit directions characterize the structure of the dynamics projected onto the orthogonal complement of discontinuity manifold. The limit directions can be used for the definition of the sliding and crossing regions in extended Filippov systems.



FIGURE 1. Simple configurations of the limit directions in the orthogonal space of the discontinuity manifold. Left panel: two attracting limit directions. Right panel: an attracting and a repelling limit direction.

The motivation behind the introduction of extended Filippov systems is the Coulomb friction between rigid bodies in 3D contact problems. In these systems, the limit directions correspond to the direction of the relative velocity at transitions between slipping and rolling of the bodies.

## Acknowledgements

The research leading to these results has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP/2007-2013) / ERC Advanced Grant Agreement n. 340889.