Nonlinear dynamics of a dual-point-contact ball

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Summary. A mechanical model of a possible flowmeter is analysed, which contains a ball in a dual-point-contact to a vessel. A nonlinear, nonsmooth dynamical system is used for modelling, which contains several types of dynamics caused by the contacts. We investigate the stationary motions, their stability and the effect of different parameters on the system.

Introduction

Single-point contact is a common-used assumption for modelling mechanical systems, when both rolling and sliding connections are considered between the contacting bodies. The dual-point contact case, when a body has two contact points with either two other bodies or with a single other body, is also usual in engineering practice. For example, dual-point-contact structure can be found in roller bearings [3], and wheelsets of trains can also be modelled as dual-point-contact bodies [4].
A possible principle of constructing a flowmeter can be also based on dual-point rolling. If the flow is swirled and rotating flow is created in a cylindrical measuring vessel, a ball can be driven, which rolls along the edge of the vessel. From counting the rotations of the ball, the flow rate of the fluid could be calculated after calibration. The operation of the device is based on the dynamics of the ball, which is explored during the present work.

Mechanical model

The ball is assumed to be rigid and symmetric, it is described by its radius, mass and mass moment of inertia with respect to its centre of gravity, denoted by r, m and J, respectively. The vessel is modelled as a cylinder, having an inner radius of R. The symmetry axis of the vessel is parallel to the direction of gravitational acceleration. Dual-point contact is assumed, that is the ball touches both the wall and the bottom of the vessel. A simple Coulomb friction model is used at both contact points, with uniform coefficient of friction μ both for the dynamic and kinematic friction cases. We assume that the fluid is rotating in the vessel with a constant angular velocity ω_0. The resistance force from the fluid is modelled with a viscous force proportional to the velocity of the ball, the resistance arising from the rotation of the ball is neglected. Sketch of the system can be seen in Figure 1.

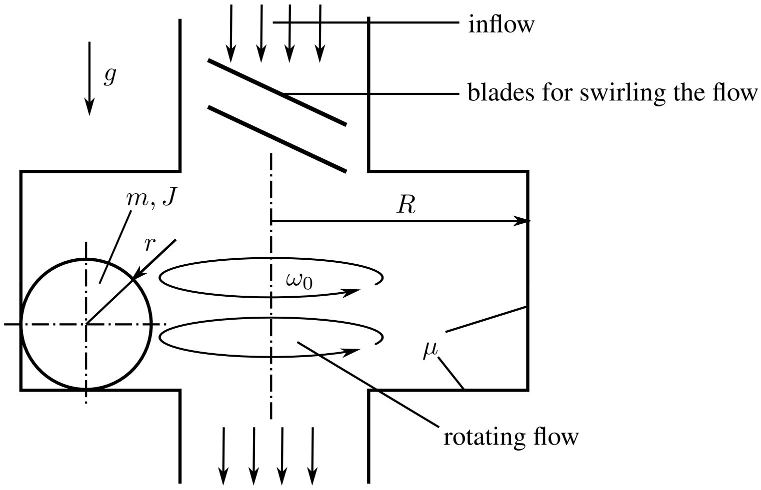

Figure 1: Sketch of the mechanical model of a rolling ball flowmeter. A stationary flow is assumed rotating with an angular velocity ω_0 in the vessel. The dynamics of the ball is analysed, which is running around driven by the flow.

Challenges

The model includes different types of nonlinearities caused by the contact assumptions. Both rolling and sliding motions are allowed at the contact points, and transitions between these motions can cause discontinuity in the accelerations due to the Coulomb friction law. The contact constraints are also unilateral, thus the contact forces cannot be in tension along the direction normal to the surfaces. This property causes a further discontinuity in the system. ([1], [2])
By counting the degrees-of-freedom, it can be seen that the system is dynamically undetermined in case of dual-point-rolling, some components of the contact forces cannot be calculated if the rigid body assumptions are kept. Thus, instead of calculating the forces, we have to find other possibilities to determine the condition of sliding.

Objectives of present work

The main motivation is to determine possible operating parameters of the flowmeter, and to have a deeper insight into the dynamics of two-point-contact systems. The different kinds of stationary motions are explored, and their stability is investigated, taking into consideration of the effects of the parameters. During the bifurcation analysis, we focus on the effect of the friction coefficient μ and the flow velocity ω_0.

Results

The system can be described using four generalised coordinates. By appropriate choice of the coordinates, all of them can be made cyclic, that is, the dynamics of the ball is not effected by the general coordinates themselves, only by the velocities. Therefore, a system of four first-order differential equations is obtained for the velocities.

We have to take into consideration, that sliding or rolling can occur at both contact points, which means four possible kinds of behaviour: dual-point rolling, dual-point sliding and two mixed sliding-rolling cases. In the four-dimensional phase space of the system, the mixed sliding-rolling cases take place on two-dimensional subspaces. These manifolds intersect each other in a one-dimensional subspace, which corresponds to the dual-point rolling case. Finally, the general behaviour in the full four-dimensional space corresponds to the dual-point sliding case.

A stationary motion of the ball can be represented by a fixed point in this dynamical system. The stability of these motions is investigated not only inside the given subspaces of dynamics, but also with respect to transitions between the different kinds of behaviour. For the latter, analysis of discontinuous terms is required because of the Coulomb friction and the unilateral constraint.

We can prove, that the stable stationary dual-point rolling is possible only between certain ω_0 values of the flow. By choosing the coefficient of friction, three rather different situations can occur. The boundary of stable two-point-rolling motion can be seen in Figure 2. At very high values of ω_0, the ball separates from the ground, and a temporal one-point-contact situation is created, the ball sometimes climbs up to the wall of the vessel, and after that, impacts can occur between the ball and the ground. The phase space must be extended to 6 dimensions to analyse the behaviour around these transitions. These unwanted effects create a limit to the application of the theory.

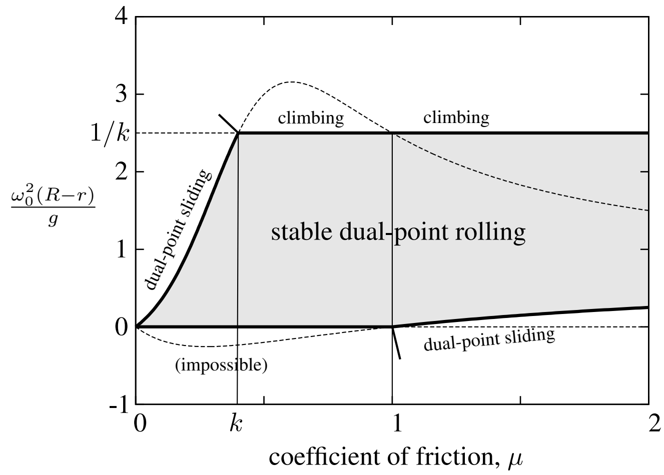

Figure 2: Effect of the parameters on the stability of stationary two-point rolling of the ball. On the boundaries, the type of stability loss is shown in mechanical point of view.

Conclusions

The rolling ball flowmeter can be modelled as a dual-point-contact system. The structure is modelled as a four-dimensional dynamical system, which includes nonlinearities and discontinuities. We have analysed the four structurally different possible motions according to the rolling and sliding state of the contact points. The stationary motions and their stability are determined. We proved that the coefficient of friction has a strong effect on the behaviour of the system, and the effect of other parameters are explored. We can conclude that this type of flowmeter can be used only in a restricted range of flow velocity.

References

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