Available online at www.sciencedirect.com

ScienceDirect

Procedia CIRP 00 (2018) 000–000

www.elsevier.com/locate/procedia

8th CIRP Conference on High Performance Cutting (HPC 2018)

Milling stability for slowly varying parameters

Zoltan Dombovari[a,b,*], Jokin Munoa[b], Rachel Kuske[c], Gabor Stepan[a]

[a]*Budapest University of Technology and Economics, Department of Applied Mechanics, Muegyetem rkp. 3, H1111 Budapest, Hungary*
[b]*IK4-IDEKO Dynamics, & Control Department, Arriaga Kalea 2, E20870 Elgoibar, Spain*
[c]*Georgia Institute of Technology, School of Mathematics, 686 Cherry St NW, GA30313 Atlanta, USA*

* Corresponding author. Tel.: +0-000-000-0000; fax: +0-000-000-0000. *E-mail address:* author@institute.xxx

Abstract

In order to predict the quality and the stability properties of milling processes, the relevant dynamics reduced to the cutting edges needs to be known. However, the dynamics varies through the workspace along the tool path during a given machining operation. This is the case for large heavy duty milling operations, where the main source of the relevant dynamics is related to the otherwise slowly varying machine structure rather than to the fairly steady milling tool dynamics. The effect of slowly varying dynamic parameters is presented for milling stability when the cutting process takes place in a region of the work space where the steady-state cutting would change from stable to unstable. After the separation of the slow and fast time scales, the governing non-autonomous delay differential equation is frozen in slow-time in order to determine the time-periodic stationary cutting solution of the milling operation for different parameters. The loss of stability is predicted from the correction to the time-periodic frozen time solution, for which we obtained non-autonomous equation for the accumulated growth over the slow-time. The growth shows loss of stability with a shift on the parameters compared to the static parameter solution.

Peer-review under the responsibility of the International Scientific Committee of the 8th CIRP Conference on High Performance Cutting (HPC 2018).

Keywords: milling; regeneration; chatter; varying dynamics;

1. Introduction

The aim of this work is to point out the effect of slowly changing parameters on milling dynamics. It is well known that machining processes like drilling, turning or milling are subjected to regenerative effect when the past relative motion of the workpiece-tool system influences the present behavior of the operation. By modelling the geometric arrangement of the cutting edges, the corresponding regenerative delays can be identified [16]. Combining with an empirical cutting force characteristics and with the dynamic model of the machine tool structure, the governing equation can be derived [2,5]. This results in delay differential equation (DDE) [12] of autonomous (time-independent) or non-autonomous (time-dependent) kinds for different machining operations. In case of milling, the governing equations are time-periodic due to the non-regular cutter-workpiece-engagement (CWE) and the rotation of the tool [10]. In this time-periodic case, the instability of corresponding time-periodic stationary solution refers to unstable milling operation that leads to chatter [17]. By using the Floquet theory on the linearized variational system [9], stability charts can be constructed usually in the parameter space of spindle speed n and depth of cut a. Between the stable and unstable domains, the stability boundaries correspond to either (secondary) Hopf or period doubling (flip) losses of stability.

The above mentioned methodology is capable to predict chatter for constant parameters; however, in reality, one or more parameters may be slowly varying during the machining operations. For example, large machines are well known to have varying dynamic behavior, thus, slowly moving cutter through their workspace is subjected to slowly varying dynamic properties. In five axis milling, even rigid compact machines operate in slowly changing environment during complex 3D tool motions, while the varying geometry along the tool path also affects the CWE in time.

In mathematical terms, the slowly changing variable introduces a permanent non-cyclic time dependency in the originally time-periodic milling model. This means that the DDE cannot be handled using Floquet theory, or at least, not in a straightforward manner. The slowly changing ordinary differential equation (ODE) models have already revealed the effect of slowly changing parameters on the corresponding stability loss and bifurcation [8,11]. In the case of Hopf bifurcations, by introducing a slow time scale, a shift of the stability boundary can be identified by considering the accumulative effect of the variational dynamics around the slowly changing stationary solution [11].

This work intends to apply the slow time scale methodology

Peer-review under the responsibility of the International Scientific Committee of the 8th CIRP Conference on High Performance Cutting (HPC 2018).

in the time-periodic DDE model of milling operations to show how those slowly varying parameters affect the classical chatter predictions. The paper considers a simple one degree of freedom (DOF) model of the milling operation as a demonstrative example. Stability of the slowly changing dynamics is determined by considering the exponential growth of the amplitude deviation from the frozen-time solutions. The single degree of freedom model can describe naively the cantilever-like structural arrangement of a heavy-duty milling machine. The model can represent the slowly changing dynamics during milling processes performed in ram-axis-direction.

2. One DOF milling model

A simple one DOF model of the regenerative milling process is considered here with the following second order time periodic DDE

$$\ddot{q}(t) + 2\xi\omega_{\mathrm{n}}\dot{q}(t) + \omega_{\mathrm{n}}^2 q(t) = aK_{\mathrm{c},t}B(t)(q(t-\tau) - q(t)) + G(t), \quad (1)$$

where ω_{n}, ξ, τ and $K_{\mathrm{c},t}$ stand for the natural angular frequency, damping ratio, regenerative delay and tangential cutting coefficient, respectively. The regenerative delay is originated in the tooth passing frequency $\Omega_Z = Z\Omega$ as $\tau = 2\pi/\Omega_Z$, where $\Omega(\mathrm{rad/s}) = 2\pi n(\mathrm{rpm})/(60(\mathrm{s/min}))$ is the angular velocity of the tool. The milling cutter is assumed to be equally spaced with cutting edges, that is, the principle period equals with the regenerative delay as $T_{\mathrm{p}} = \tau$. The stability diagram is usually depicted in the parameter space spanned by the spindle speed n and axial depth of cut a. As shown in [13] and [20], the time periodicity of (1) appears in

$$B(t) \equiv B(t+T_{\mathrm{p}}) = \mathbf{U}^{\mathsf{T}} \sum_{i=1}^{Z} \frac{g(\varphi_i(t))}{\sin\kappa} \mathbf{T}(\varphi_i(t)) \boldsymbol{\kappa}_{\mathrm{c}} \otimes \mathbf{n}(\varphi_i(t)) \mathbf{U}, \quad (2)$$

and in the periodic excitation with $G(t) \equiv G(t + T_{\mathrm{p}})$

$$G(t) = -a\mathbf{U}^{\mathsf{T}} \sum_{i=1}^{Z} \frac{g(\varphi_i(t))}{\sin\kappa} \mathbf{T}(\varphi_i(t)) \left(\mathbf{K}_{\mathrm{e}} + \mathbf{K}_{\mathrm{c}} \otimes \mathbf{n}(\varphi_i(t)) \begin{bmatrix} f_Z \\ 0 \\ 0 \end{bmatrix} \right). \quad (3)$$

The mass normalized mode shape vector, lead angle and the feed per tooth are denoted by $\mathbf{U}$, κ and f_Z. The edge normal, the edge coefficients and the cutting coefficients are stored in $\mathbf{n}$, $\mathbf{K}_{\mathrm{e}}$ and $\mathbf{K}_{\mathrm{c}} = K_{\mathrm{c},t}\boldsymbol{\kappa}_{\mathrm{c}}$ vectors defined in local edge (t, r, a) system, while the transformation between (t, r, a) and (x, y, z) is realized by $\mathbf{T}(\varphi)$. The edge angular position and the effect of CWE are traced by $\varphi_i(t)$ and $g(\varphi)$ [6]. Linear stability of the time periodic stationary solution $Q(t) = Q(t + T_{\mathrm{p}})$ of (1) can be determined by various methods, like multi-frequency solution [4,15] and semidiscretization [14].

3. Slowly changing milling model

A real milling process is usually subjected to time-dependent slowly changing parameters. In this manner, we can modify the DDE (1) by considering some slowly changing parameters with respect to the so-called slow time $s := \varepsilon t$, where the general rate of change is denoted by ε. Thus,

$$\begin{aligned} \ddot{y}(t) + 2\xi(s)\omega_{\mathrm{n}}(s)\dot{y}(t) + \omega_{\mathrm{n}}^2(s)y(t) &= \\ a(s)K_{\mathrm{c},t}B(t,s)(y(t-\tau(s)) - y(t)) &+ G(t,s). \end{aligned} \quad (4)$$

Surely, not all parameters are changing at the same rate during a given process. For example, in the general form of (4) one can model a cutting process defined for a ramp-like workpiece [18] with slowly changing $a := a(s)$ and $G(t) := G(t, s)$. Note that $y \equiv q$; it has been introduced only to distinguish the solution of (1) and that of (4). In any case, the governing DDE (4) form of the slowly changing milling operation is no longer exactly time-periodic. It behaves as a general non-autonomous (time-dependent) system with two different time scales described by the (real) fast time t and the (introduced) slow time s.

3.1. Quasi-Stationary Solution

It is straightforward to assume that (4) must have a slowly changing, but in this case not time-periodic, quasi-stationary solution $Y(t) \neq Y(t + T_{\mathrm{p}})$. Perturbation x is introduced in the solution as

$$y(t) := Y(t, s) + x(t). \quad (5)$$

Abusing the notation, the real time and slow time dependencies are dropped for a while. Substituting the assumption (5) into (4), and using $Y^{\tau,\varepsilon\tau} := Y(t-\tau, s-\varepsilon\tau) \approx Y(t-\tau, s) - \varepsilon\tau\frac{\partial}{\partial s}Y(t-\tau, s) =: Y^{\tau,0} - \varepsilon\tau Y_s^{\tau,0}$ one can get the following form if $Y(t, s) \approx Q(t; s) = Q(t + T_{\mathrm{p}}; s)$.

$$\begin{aligned} \ddot{x} + 2\xi\omega_{\mathrm{n}}\dot{x} + \omega_{\mathrm{n}}^2 x &= \\ aK_{\mathrm{c},t}B(x^{\tau,\varepsilon\tau} - x) &- aK_{\mathrm{c},t}\varepsilon\tau B Q_s^{\tau,0}, \end{aligned} \quad (6)$$

where additional slow time derivatives of Q have been incorporated into the definition of Y. Considering the time periodicity in $Q_s^{\tau,0}$, (6) has the same form as (4). For the remainder of the paper we concentrate on the homogeneous form of (6) in order to show the shift of the parameter value for the dynamic stability loss.

3.2. Asymptotic behavior of the quasi-stationary solution

The asymptotic behavior of this non-autonomous slow/fast system can be derived by the WKB method [3] considering the original time periodicity slightly depending on the slow time s. According to Floquet theory [9], the general solution of a linear time periodic system is given by an exponential and a time-periodic term [7]. Similarly, using the WKB method, the following general solution can be assumed for the homogeneous part of the slowly changing equation (6):

$$x(t) := x(t, s) = \mathrm{e}^{\frac{\sigma(s)}{\varepsilon}} A(t, s), \text{ where } A(t, s) = A(t + T_{\mathrm{p}}, s). \quad (7)$$

From (7) the slow time variation of σ captures the asymptotic behavior describing stability: as σ increases from positive to negative values there is rapid exponential growth of x. Substitution of (7) into the homogeneous part of (6) leads to a partial differential equation as

$$\varepsilon\sigma_{ss}A + \sigma_s^2 A + 2\sigma_s\dot{A} + \ddot{A} + 2\xi\omega_{\mathrm{n}}(\sigma_s A + \dot{A}) + \omega_{\mathrm{n}}^2 A = aK_{\mathrm{c},t}B(\mathrm{e}^{-\tau\sigma_s} - 1)A \tag{8}$$

by using the assumption $\mathrm{e}^{\frac{\sigma(s-\varepsilon\tau)}{\varepsilon}} \approx \mathrm{e}^{\frac{\sigma(s)}{\varepsilon}}\mathrm{e}^{-\tau\sigma_s(s)}$. In accordance with (7), one can obtain $\dot{A} = A_t + \varepsilon A_s$, $\ddot{A} = A_{tt} + 2\varepsilon A_{ts} + \varepsilon^2 A_{ss}$, $A^{\tau,\varepsilon\tau} = A^{\tau,0} - \tau\varepsilon A_s^{\tau,0}$. However, keeping ε sufficiently small and consequently having small changes in A w. r. t. slow time s, one can assume $A_s = A_{st} = A_{ss} = 0$. Thus, the Fourier expansion is applied on the now exactly time-periodic $A(t) := \sum_{l=-\infty}^{\infty} A_l \mathrm{e}^{\mathrm{i}l\Omega_Z t}$ and $B(t) := \sum_{l=-\infty}^{\infty} B_l \mathrm{e}^{\mathrm{i}l\Omega_Z t}$. The multi-frequency approach [4] or Hill type of infinite expansion of the slowly changing milling dynamics can be given after substituting A and B into (8) as

$$\begin{aligned}((\varepsilon\sigma_{ss} + \sigma_s^2 + 2\xi\omega_{\mathrm{n}}\sigma_s + \omega_{\mathrm{n}}^2)\mathbf{I} + \\ [2\mathrm{i}l\Omega_Z\sigma_s - l^2\Omega_Z^2 + 2\xi\mathrm{i}l\Omega_Z]_{l=-\infty}^{\infty})\mathbf{A} = \\ aK_{\mathrm{c},t}(\mathrm{e}^{-\tau\sigma_s}[\mathrm{e}^{-\mathrm{i}l\Omega_Z\tau}]_{l=-\infty}^{\infty} - \mathbf{I})\mathbf{BA},\end{aligned} \tag{9}$$

where $\mathbf{A} = \mathrm{col}_{l=-\infty}^{\infty} A_l$ and $\mathbf{B} = [B_{k-l}]_{k,l=-\infty}^{\infty}$. For the sake of simplicity, zeroth order (average) consideration can be derived by picking only the averages of $\mathbf{A}$ and $\mathbf{B}$ as in [1], resulting in

$$\varepsilon\sigma_{ss} + \sigma_s^2 + 2\xi\omega_{\mathrm{n}}\sigma_s + \omega_{\mathrm{n}}^2 - aK_{\mathrm{c},t}(\mathrm{e}^{-\tau\sigma_s} - 1)B_0 = 0. \tag{10}$$

3.3. Naive Stability Criteria

The general solution for the slow-time system in (7) suggests that, in case of negative real part σ, the perturbation introduced in (5) dies out, while positive real part σ induces rapid explosion w. r. t. s. This can be traced by the real part of the cumulated value of σ_s over slow time in the form

$$\mathrm{re}\int_0^s \sigma_s(\zeta)\mathrm{d}\zeta. \tag{11}$$

If condition (11) crosses zero, the stability property of the slowly changing quasi-stationary solution $Y(t, s)$ switches (7) as shown in [3].

To have σ_s over s, the first order nonlinear ODE representation (10) has to be integrated by using an initial condition when $s = 0$ assuming $\varepsilon = 0$ as

$$\sigma_s(s) = \lambda(s) \quad \Rightarrow \quad \sigma_s(0) = \lambda, \tag{12}$$

where $\lambda(s)$ is the frozen time characteristic exponent of (1) for fixed s.

Table 1. Parameters of full immersion milling process used in this paper [7].

Z	ω_{n} (Hz)	ξ (Hz)	k (N/µm)	feed direction
4	94	0.66	58.38	$[1\,0\,0]^{\mathsf{T}}$
$K_{\mathrm{c},t}$ (MPa)	$K_{\mathrm{c},r}$ (MPa)	$K_{\mathrm{c},a}$ (MPa)	κ (deg)	mode direction
1459	259	0	90	$[0\,1\,0]^{\mathsf{T}}$

Condition (11) suggests that the accumulated stability is overtaken by the accumulated instability, which results in the shift on the onset of the unstable motion very much depending on the initial values of the slowly changing variables [3]. We note here that additional inhomogeneous terms in (6) can be included to provide additional corrections to $\sigma(s)$ and the resulting shift of unstable motion.

4. Case Study

In this section we provide an example, in which the behavior of the dynamic bifurcation analysis is relevant. It is important to emphasize that this is a completely artificial example. Real case studies may show variable significance of the demonstrated effect. In the literature there are various measurement examples, when the test required a pre-manufactured workpiece with a gentle slope [7,18,19]. These measurements typically are aimed to present stability limits or so-called nonlinear hysteresis phenomenon, the direct consequence of subcritical Hopf bifurcation of the stationary milling process. The slowly changing parameter in (4) the axial depth of cut is defined as

$$a(s) = a_{\min} + \varepsilon t, \tag{13}$$

with $a_{\min} = 1$ mm and $a_{\max} = 10$ mm. The presented example with 90 deg lead angle κ and zero helix angle η (see Tab. 1). In Fig. 1*ab*) the system loses its stability after the constant parameter limit a_{c} at the dynamic one a_{din} according to the criterion (11). The loss of stability can be realized as the time domain solution in Fig. 1*c*) escapes the quasi-stationary solution simply replaced with $Q(t; s)$. One can realize the linear dependency of the axial depth of cut a on the solution in Fig. 1*c*). It can be also realized from Fig. 1*c*) that the actual onset value where the solution escapes from the stationary solution is a bit ahead of the predicted position. This suggests deeper dependency of the rate of change ε on the dynamics which needs further, more detailed study of the problem.

5. Conclusion

There are several industrial problems, including ones related to cutting technologies, which may involve dynamic processes on (very) different time scales. In the present work, we study milling operations that clearly have fast time-periodic dynamics. In the meantime, there exists a slow rate of change of some system parameters originating in the slowly varying structural dynamics as the tool moves in the working space of the milling machine. In this paper, the so-called dynamic bifurcation phenomenon has been introduced for the analysis of milling stability. A new generalized governing equation was derived, with

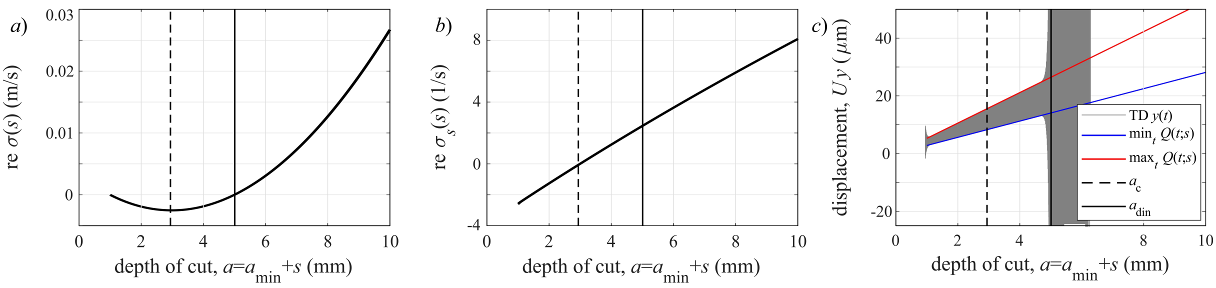

Fig. 1. The real part of the critical eigenvalue $\sigma(s)$ *a*) and its derivative $\sigma_s(s)$ *b*) solving (9) w. r. t. s under slowly changing axial depth of cut $a(s)$. In panel *c*) time domain simulation $y(t)$ and the frozen time stationary solution $Q(t;s)$ are shown. Here, $n = 1869$ rpm and $\varepsilon = 10^{-4}$ m/s.

which the stability of the slowly changing milling dynamics can be predicted. Two simplified case studies are presented with slowly varying behavior. The non-trivial, sometimes counter-intuitive theoretical predictions based on the analysis of the new governing equations were confirmed by time domain simulation results, although some parameter domains still need further and deeper study.

The results may have industrial relevance when the milling cutter moving in the workspace has varying reduced dynamics. The results are somewhat counter intuitive. On the one hand, the accumulated stability, in theory, does not or weakly depends on the rate of change, which suggests that shifting of the stability appears even for extremely small rates of change. On the other hand, the shift (e.g., a_{din} and a_{c}) carries the initial parameter value, since this has direct effect on the accumulated stability behavior.

It is important to emphasize that the above presented results apply only in cases when the cutting operation starts from stable stationary cutting; at this point, the results do not explain the transition backward from chatter to stable stationary cutting, which requires different modelling techniques that are applicable also for quasi-periodic oscillations.

6. Acknowledgement

The research was partially funded by the European Research Council under the European Unions Seventh Framework Programme (FP7/2007-2013) ERC Advanced grant agreement No. 340889 and by NSERC Discovery Grant 22R23106 and NSERC Discovery Accelerator Supplement Grant 477876-2015.

References

[1] Y. Altintas and E. Budak. Analytical prediction of stability lobes in milling. *CIRP Annals - Manufacturing Technology*, 44(1):357–362, 1995.

[2] Y. Altintas and Z.M. Kilic. Generalized dynamic model of metal cutting operations. *CIRP Annals - Manufacturing Technology*, 62(1):47–50, 2013.

[3] S.M. Bear, T. Erneux, and J. Rinzel. The slow passage through a hopf bifurcation: Delay, memory effects, and resonance. *SIAM Journal on Applied Mathematics*, 49(1):55–71, 1998.

[4] E. Budak and Y. Altintas. Analytical prediction of chatter stability in milling—part i: General formulation. *Journal of Dynamic Systems, Measurement, and Control*, 120(1):22–30, 1998.

[5] Z. Dombovari, J. Munoa, and G. Stepan. General milling stability model for cylindrical tools. *Procedia CIRP*, 2012.

[6] Zoltan Dombovari, Yusuf Altintas, and Gabor Stepan. The effect of serration on mechanics and stability of milling cutters. *International Journal of Machine Tools and Manufacture*, 50(6):511 – 520, 2010.

[7] Zoltan Dombovari, Alex Iglesias, Mikel Zatarain, and Tamas Insperger. Prediction of multiple dominant chatter frequencies in milling processes. *International Journal of Machine Tools & Manufacture*, 51:457–464, 2011.

[8] P. Erneux, T. Mandel. Imperfect bifurcation with a slowly-varying control parameter. *SIAM Appl. Math.*, 46:1–16, 1986.

[9] M. Farkas. *Periodic Motions*. Springer-Verlag, Berlin and New York, 1994.

[10] W. B. Ferry and Y. Altintas. Virtual five-axis flank milling of jet engine impellers—part i: Mechanics of five-axis flank milling. *Journal of Manufacturing Science and Engineering*, 130(1):011005, 2008.

[11] R. Haberman. Slowly-varying jump and transition phenomena associated with algebraic bifurcation problems. *SIAM J. Appl. Math.*, 37:69–105, 1986.

[12] J.K. Hale. *Theory of functional differential equations*. Springer-Verlag, New York, 1977.

[13] A. Iglesias, J. Munoa, J. Ciurana, Z. Dombovari, and G. Stepan. Analytical expressions for chatter analysis in milling operations with one dominant mode. *Journal of Sound and Vibration*, 375:403–421, 2016.

[14] T. Insperger and G. Stepan. *Semi-Discretization for Time-Delay Systems: Stability and Engineering Applications*. Springer, New York, 2011.

[15] S. D. Merdol and Y. Altintas. Multi frequency solution of chatter stability for low immersion milling. *Journal of Manufacturing Science and Engineering*, 126(3):459–466, 2004.

[16] J. Munoa, X. Beudaert, Z. Dombovari, Y. Altintas, E. Budak, C. Brecher, and G. Stepan. Chatter suppression techniques in metal cutting. *CIRP Annals - Manufacturing Technology*, 65(2):785–808, 2016.

[17] A. Otto, S. Rauh, M. Kolounch, and G. Radons. Extension of tlustys law for the identification of chatter stability lobes in multi-dimensional cutting processes. *International Journal of Machine Tools and Manufacture*, 82–83:50–58, 2014.

[18] H.M. Shi and S.A. Tobias. Theory of finite-amplitude machine-tool instability. *International Journal of Machine Tools and Manufacture*, 24(1):45–69, 1984.

[19] G. Stepan, Z. Dombovari, and J. Muñoa. Identification of cutting force characteristics based on chatter experiments. *CIRP Annals-Manufacturing Technology*, 1:113–116, 2011.

[20] M. Zatarain and Z. Dombovari. Stability analysis of milling with irregular pitch tools by the implicit subspace iteration method. *International Journal of Dynamics and Control*, 2:26–34, 2014.