

# BASIC REGENERATIVE MODELLING OF AXLES ROLLING PROCESS

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## Abstract

In this work, a simple regenerative formulation of the axles rolling process is presented. This work only considers a simple representation of the elastic-plastic deformation subjected to the regeneration that arises during the rolling operation. Simple two degrees of freedom mechanical model is introduced with an ideal constant control force. The deformation force is considered simply according to the literature including elastic and elastic-plastic regions on the empirical deformation characteristics. This induces a very much the same effect as the flyover in cutting processes that causes abrupt vanish of the cutting force, however here the elastic-plastic relay brings nonsmooth behaviour in the rolling process. In order to understand the process, steady rolling is extended to the real axles rolling process with constant feed, where overlapping can occur. The equations are simplified and time domain based simulations are performed including this simple consideration of the elastic-plastic relay.

**Keywords:** rolling, regeneration, nonsmooth, FEM

## 1 Introduction

Axles rolling process is an important cold forming process in the train industry. Almost all cases, train shafts have to be rolled to increase surface hardness by favorable residual compressive stress field, and to eliminate microcracks originated from previous turning processes [1]. This removing of the microcracks can increase the lifetime of train shafts drastically, which is a key importance in avoiding serious train accidents reported by [2]. This cold forming process is a fairly slow process and it requires heavy machinery due to the relatively high press force, that is usually granted by electro-hydraulic control system [3]. With the rotating workpiece (shaft), one can expect similar regenerative effect that appears in cutting processes in [4]. Even though, in this case, the strict geometric accuracy is not that important, however, large amplitude vibrations can cause ripples on the surface, that is very much refused by quality standards. Like any other forming process, rolling is also a quite complicated continuous mechanical process due to the simultaneous operation of elastic and plastic deformations. In the literature, sheet rolling process is quite extensively investigated by [5], and [6]; although it is quite a different process than axles rolling. In sheet rolling process, large portion of the workpiece is deformed, while in axles rolling the elastic foundation of the usually quite thin deformed layer also has its importance. That is why cylinder rolling on elastic-plastic material can come into the front. There are various attempts in the literature to model this operation on finite element basis [7]. These models are quite specific, and due to their large sizes, usually they are not adequate for dynamic modeling. There are much more promising, although much complicated methods to describe the elastic-plastic deformation during cylinder rolling. These models have only a few parameters, however, those use plenty of assumptions related to the plastic zones along slip lines in [8]. We do not attempt to improve the inaccuracies of the formation models of

cylinder rolling, described in the literature in [9]. By accepting their results, we derive the mathematical formalism of regenerative rolling processes that causes high amplitude vibrations. The paper has the following structure. The simple rolling force characteristics is explained after [9] and adjusted to the latter dynamic analysis. A two degrees of freedom (DoF) dynamic model is introduced with a constant control force to have the simplest possible model to describe an axles rolling process. A general model is introduced for steady rolling with no feed using one constant delay, that introduces the concept of plastic penetration function  $\delta_p(t)$  as an additional state of the system. This model is improved to operate with feed introducing multiple penetration functions. The model is transformed into a multi-delayed dynamic model, that is more convenient to perform time domain simulations. Lastly a finite element model (FEM) is shown by predicting the feed dependency of the resulting force generated by the overlapping effect.

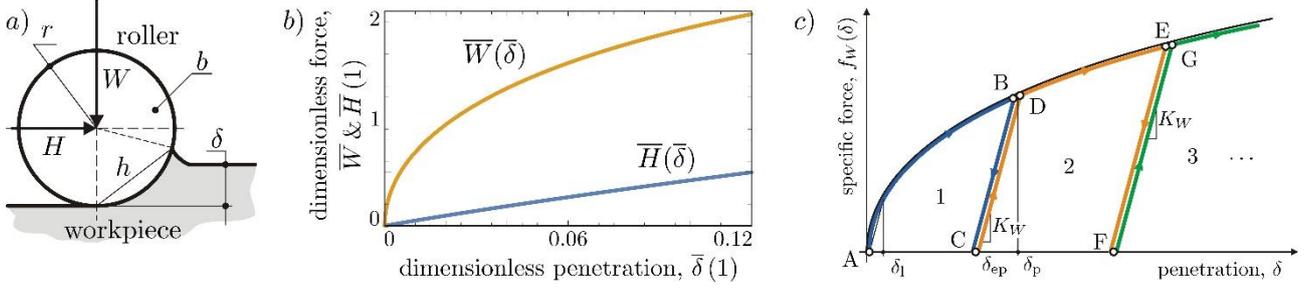


Figure 1: *a)* shows the sketch of the rolling process, while *b)* presents the specific force based on [9]. Panel *c)* shows the simple consideration of elastic-plastic effect during rolling by tracing plastic and elastic-plastic deformations  $\delta_p$  and  $\delta_{ep}$ , respectively.

## 2 Simplified rolling force model

Simple plane strain model is considered here after the work of [9]. In that work an analytical formula is presented for narrow wheel penetration and rolling (see Figure 1*a*). The paper of [9] claims that, if the width  $b$  is relatively small compared to the radius of the wheel  $r$ , a simple power characteristics can be derived between the chord length  $h$  of the material contact and the penetration  $\delta$ . By accepting the derivations in [9] nonlinear specific plastic force characteristics can be given as

$$f_{W,p}(\delta) := \kappa r \chi(\delta) \sqrt{2 \frac{\delta}{r}} \sqrt{1 - \frac{1}{2} \frac{\delta}{r}}, f_{H,p}(\delta) := \kappa r \chi(\delta) \frac{\delta}{r} \text{ and } \chi(\delta) = \left( 2 + \pi - 4 \arcsin \sqrt{\frac{\delta}{2r}} \right). \quad (1)$$

This specific force characteristics are depicted in Figure 1*b*. Obviously, there are plenty of questions about the validity of these elastic-plastic models presented in [9]. Although, one could imagine the overall specific rolling force should vanish for no penetration, and should be a nonlinear function of the roller penetration  $\delta$ , what is currently true for (1). These formulae do not deal with the redistributed material along the side 'edges' of the roller, and do not describe the probably different rolling behaviour of multiple passes.

The specific force formulae (1) follow ones expectation to have much larger vertical force component than the one on the horizontal direction (Figure 1*b*). In the vertical direction rolling behaves more like an indentation, while in the horizontal direction the rubbing of the forward upheaval material is the influencing phenomenon.

In this very simple framework, the following description can be derived based on the elastic-plastic effect dominating the multi-passing rolling process. Post the yield condition the material is plastically deformed, then elastically released. Having the second loading, after the elastic uploading, the force is 'continued' exactly from the same level, from where the first plastic deformation had been released. This is very different than the operation of the cutting force characteristics, where in each cutting passes the same behaviour is happening over and over [10].

This means, actually the material 'remembers' its deformation state, which can be modelled by using a coordinate transformation on the rolling force characteristics as presented in Figure 1c. In detail, the material undergoes plastic deformation from the intact state of surface A to the primary deformed state B. This penetration is measured with the value of plastic deformation  $\delta_p$ . By releasing the surface from rolling pressure it recovers along its elastic stiffness  $K_W$  ( $W$ : vertical direction after [9]) line to C. This released state is measured by the value of elastic-plastic deformation  $\delta_{ep}$ . Note that, from now on, more deformation can only be achieved by using more force. That is, in the next round, if specific force on that surface segment is not increased the segment only undergoes elastic deformation only reaching D along  $f_{W,e}(\delta, \delta_{ep}) = K_W(\delta - \delta_{ep})$ . Additional plastic deformation is only possible from D to E by pressing more the surface segment. In order to ease the latter dynamic modelling, and since the actual rolling force characteristics is unknown, the rolling force is considered elastic until a limit penetration  $\delta_l$  that can be calculated from the following equality  $K_W\delta = f_{W,p}(\delta) \rightarrow \delta_l$ .

Due to this simple description, an inconsistency can be felt with the very definition of the penetration  $\delta$ , which was measured to the intact surface according to [9]. This penetration in the next pass would be much smaller, but this highly deformed elastic-plastic consideration of the rolling is not available yet in the literature. In this work, we point out the dynamic modeling problems, that can be later improved with the real rolling force characteristics considering multiple passes, which probably introduces different  $f_W$  &  $f_H$  characteristics for each number of passes.

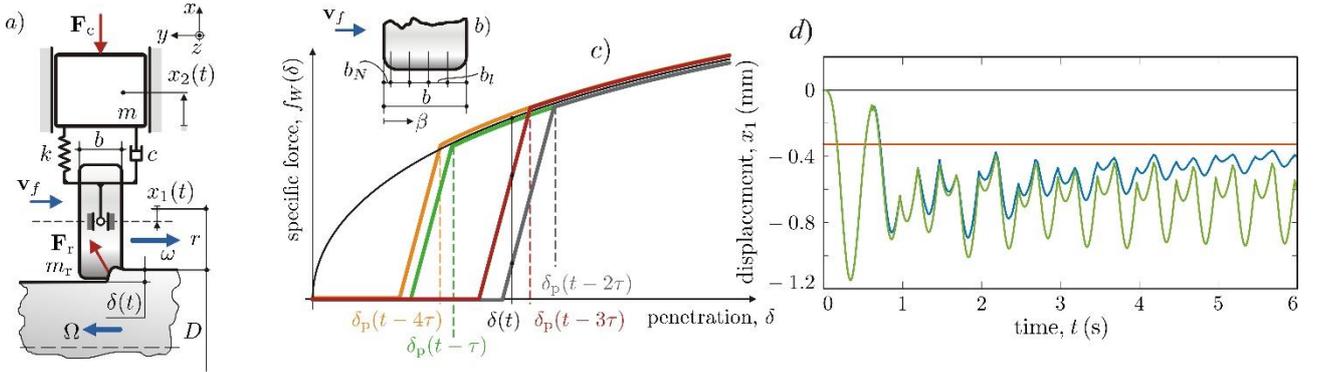


Figure 2: The two DoF rolling model is depicted in a), where feed is  $\mathbf{v}_f$ , the assigned generalized coordinates are  $(x_1, x_2)$ . In b) the parameters used for describing overlapping are presented, while c) shows the elastic-plastic force for different previous deformations. Sample time domain simulations are presented in d), where equilibrium  $\bar{x}_1$  denoted with red line. A stable and a saturated solution corresponding with an unstable equilibrium are depicted by blue and green.

### 3 Dynamics

In this section, we model the behavior of the axles rolling process taking into account the simple rolling force behaviour in Figure 1c. The aim is to model the mechanics with an ideal control providing constant force  $F_c = |\mathbf{F}_c| := \text{const.}$ . It is necessary to know, in the standard, the desired pressing force is specified for a given rolling process not the value of the local plastic deformation. That means, the model lacks all electro-hydraulic components that keep this constant force  $\mathbf{F}_c$  controlled. This actually results in, the model described below would not cover all types of self-excited vibration phenomena that cause poor rolled surface.

Accepting the simple specific rolling force characteristic originated from [9] at (1) and their elastic-plastic behaviour (see Figure 1c), the simplest two DoF mechanical model can be built with rigid roller and elastic-plastic workpiece (WP in Figure 2a). The dynamics of the machine is mimicked by one (dominant, not rigid) mode with a linear spring with stiffness  $k$ , and a linear viscous damper with damping  $b$ . The two DoF motion

is described by two generalized coordinates  $x_1$  and  $x_2$  (see Figure 2c), with which the roller centre ( $x_1$ ) and the modal displacement of the dominant mode ( $x_2$ ) are described. One can build a model, where the roller rolls through the same material over and over, the one which actually performs real rolling process with moderate feed and overlapping (Figure 2a). Lastly, the version with extremely high feed in such, the roller actually goes on a helical spiral without any overlapping. Real process follows case where the plastic deformation is dominant with

$$\delta(t) := -x_1(t). \quad (2)$$

In this sense, the specific rolling force depends on the momentary penetration  $\delta(t)$  and it is integrated along  $\beta$  (see Figure 2b) the contact length of the roller as

$$\mathbf{F}_r(\delta(t)) = [F_W(\delta(t)) \quad F_f(\delta(t)) \quad -F_H(\delta(t))]^T = \int_{\beta} [f_W(\delta(t)) \quad f_f(\delta(t)) \quad -f_H(\delta(t))]^T d\beta \quad (3)$$

In (3)  $f_W$ ,  $f_H$  and  $f_f$  are the vertical, horizontal and feed directional specific force components. Note that, in this simple description the horizontal and feed direction does not effect the dynamics. Thus, during the process, the equilibrium state would be defined, when the  $x$  component of rolling force  $\mathbf{F}_r$  is in balance with the control force  $\mathbf{F}_c$ . Other components are carried ideally by the frictionless guide constrains.

### 3.1 Equations of Motions

The equations of motion can be derived by e.g. Lagrange equation approach II by assuming the rolling force, which only depends on the momentary penetration  $\delta(t)$  and does not on the penetration speed  $\dot{\delta}(t)$ . In this manner a simple equations of motion can be derived as

$$\begin{aligned} m_r \ddot{x}_1(t) + c \dot{x}_1(t) - c \dot{x}_2(t) + kx_1(t) - kx_2(t) &= F_W(\delta(t)), \\ m \ddot{x}_2(t) - c \dot{x}_1(t) + c \dot{x}_2(t) - kx_1(t) + kx_2(t) &= -F_c. \end{aligned} \quad (4)$$

This two DoF model has a rigid body mode and a finite mode with  $\omega_{n,1} = 0$  and  $\omega_{n,2} = \sqrt{k \frac{m+m_r}{mm_r}}$ . Applying a real (not constant) control force  $\mathbf{F}_c$ , which would be state dependent, the first natural frequency would be consolidated diverting from a rigid mode.

### 3.2 Simple axles rolling with overlapping

The overlapping is caused by the constant feed  $f = \mathbf{v}_f \tau$ . Consequently, in each revolution of the workpiece, there is always a primary region, where mainly plastic deformation, while in the subsequent regions elastic-plastic deformation are working. The constant feed  $f$  defines the width of the regions, simply  $b_l = f$  for each region  $l = 1, \dots, N - 1$ , where  $N = \lceil b/f \rceil$ . The last region has the width as  $b_N = b - \lfloor (b - \epsilon)/f \rfloor f$  ( $\epsilon$  is a sufficiently small number). Since, the regions on the surface moving forward, due to the constant feed multiple regeneration of the surface can occur. This local plastic deformation can be defined in a recursive formula, as

$$\delta_{p,l}(t) = \max_{i=0}^{l-1} \delta(t - i\tau), \text{ for } l = 1, 2, \dots, N. \quad (5)$$

Consequently, all possible plastic deformations during the process are originated back to one of the delayed state, since (2). In Figure 2c) the same train of thought is introduced as in (5) by considering all possible plastic and elastic-plastic states. Hence, always the minimum possible force is going to be realized connected possible plastic states (see  $\delta_p(t - 2\tau)$  and  $\delta_p(t - 4\tau)$ ).

For example, the current state  $\delta(t)$  in Figure 2c) causes tiny elastic specific force part related to the previous remained plastic deformation  $\delta_p(t - 2\tau)$ . All other previous plastic deformations relate to a larger force, but those deformations are no longer active since (5). Consequently, on the  $l$ th portion the following force arises

$$F_{W,l}(\delta(t)) = b_l \min_{i=0}^{l-1} f_{W,l-i}(\delta(t), \delta_{p,l-i-1}(t - (i+1)\tau)), \text{ where}$$

$$f_{W,l}(\delta(t), \delta_{p,l-1}(t - \tau)) = \begin{cases} 0, & \text{if } \delta(t) \leq \delta_{ep,l-1}(t - \tau), \\ f_{W,e}(\delta(t), \delta_{p,l-1}(t - \tau)), & \text{if } \delta_{ep,l-1}(t - \tau) < \delta(t) \\ & \text{and } \delta(t) \leq \delta_{p,l-1}(t - \tau), \\ f_{W,p}(\delta(t)), & \text{otherwise.} \end{cases} \quad (6)$$

Substituting consistently (5) into (6), summing the force portions, and finally considering the same two DoF dynamics introduced in (4) the following convenient form can be formulated

$$m_r \ddot{x}_1(t) + c \dot{x}_1(t) - c \dot{x}_2(t) + kx_1(t) - kx_2(t) = \sum_{l=1}^N b_l \min_{i=0}^{l-1} f_{W,l-i}(-x_1(t), \max_{j=0}^{l-i-2} (-x_1(t - (i+1+j)\tau))), \quad (7)$$

$$m \ddot{x}_2(t) - c \dot{x}_1(t) + c \dot{x}_2(t) - kx_1(t) + kx_2(t) = -F_c$$

The form (7) does not contain any additional state variable related to the surface. Although, it is only convenient for rough time domain simulations, where 'min' and 'max' functions are evaluated momentarily fairly easily. In accurate time domain simulation a possible switch on the 'min' and 'max' functions has to be located with event detection. However, one can argue that many different effect actually influences this simple clear mathematical framework, and it is fair to say, the rough time domain simulation should be actually closer to the reality with its heuristic approach.

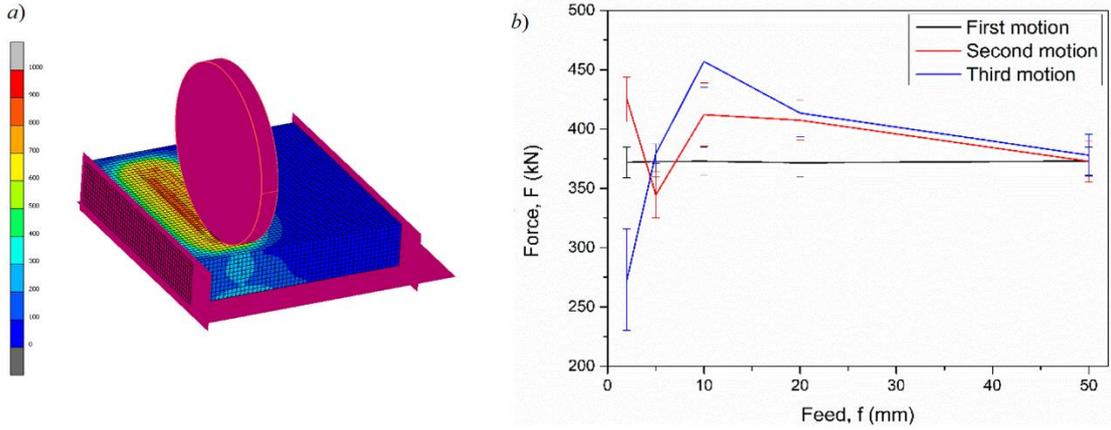


Figure 3: *a)* Equivalent von Mises stress at the FEM model after overlapping rolling simulation, *b)* reaction force acting on the roller as a function of the feed.

## 4 Finite element modelling

In order to improve the modelling of the specific cutting FEM is developed for cold rolling process. We perform single, multi and overlapped passes to achieve a general model where the effect of the parameter changes can be simulated. The final aim is to have simple analytical representation of the rolling force to be included in the dynamic description providing better results given by the simple description originated from [9]. In Figure 3 a sample simulation is shown where the feed ( $\mathbf{v}_f$ ) was varied from the case when the roller goes exactly in its path to the one where it actually produces a helix on the workpiece. One can realize the obvious difference between the FEM results (Figure 3b) and the analytical description (6, 7) considering constant initial penetration  $\delta = -x_1$ .

## 5 Conclusions

The dynamic model of the axles rolling process was derived in this work. The goal of this research to reveal the dynamic modeling issues in this cold forming process. By introducing a simple elastic-plastic force characteristics the delayed differential equation (DDE) form of the steady axles rolling process is introduced. The algebraic condition for the momentary plastic deformation is simplified by introducing multiple constant delays. Consequently, the current plastic deformation is actually originated from all other previous states of the roller itself. Time domain simulation is shown that the process itself can loose its stability without considering the force control in the machine. Finite element model of the axles rolling process was presented that links the simple model with the reality for example pointing out the feed dependency in the overall force, which is clearly not modelled correctly in the simple analytical description.

## 6 Acknowledgement

The research was supported by the Hungarian National Research, Development and Innovation Office (NKFI FK 124361) and ASTRACOMP Project (EXP-00102217) from the Innoglobal program of the Spanish Ministry of Economy, Industry and Competitiveness.

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