Root Iteration Method for Modal Parameter Identification

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1. Introduction

This paper represents an intermediate state of an automatic modal identification tool that is able to extract modal behavior of a mechanical structure. These parameters are mandatory from vibrational monitoring and model developing perspective.

During the measurements one collects the so-called frequency response functions (FRF's) between various excitation and sensing points. All of these functions are collected into a 3D array which serves as input for the fitting algorithm. The algorithm uses the assumption of linearity for the curve fitting procedure. After the fitting was performed the modal parameters can be determined and stored for evaluation of the vibrational behavior or used for creating hybrid finite element models (FEM) based on the results of experimental modal analysis results.

The dynamic models based on the fitting algorithm are carrying the actual tested vibratory behaviour of the mechanical system. This results in a better simulation environment where the initial state of the system is ensured due to this characterization technique.

2. Fitting algorithm

The transfer function (TF) contains all information about the transient and stationary behavior of a given mechanical system. However, there is no possible way to actually measure TF.

2.1 FRF function in modal analysis

The FRF is a slice at $s=\mathrm{i}\omega$ of the TF. In connection with this relation we identify s and ω being in Laplace and Fourier domain, respectively. Due to the multiple excitation and sensing points the TF can be given in the following, so-called right polynomial fraction form:

$$\mathbf{H}(s)=\mathbf{A}(s)\mathbf{B}^{-1}(s)\,. \qquad (1)$$

The modal parameters are hidden inside the poles, which are the singular points of the Transfer function. In the fractional form this means

$$\det \mathbf{B}(s)=0\,. \qquad (2)$$

The goal is to calculate the modal parameters such as the natural frequency ω_n and damping ratio ζ from s

$$s_k=-\zeta_k\,\omega_{\mathrm{n},k}\pm\mathrm{i}\omega_{\mathrm{n},k}\sqrt{1-\zeta_k^2}\rightarrow\omega_{\mathrm{n},k},\zeta_k\,, \qquad (3)$$

where $k=1,\ldots,N$. There is one thing only the FRF can be measured instead of a TF, but not even all necessary points can be reached during a real industrial test. Thus only a truncated version of the theoretical FRF $\mathbf{H}(\omega)$ can be measured, which forms a non-symmetric matrix as

$$\mathbf{\Phi}(\omega):R\rightarrow C^{m\times n}\,. \qquad (4)$$

Eq. (4) describes the data input for the algorithm and (1) describes the fitting model which was used in the algorithm.

2.2 Mathematical model of the algorithm

According to (1), the FRF can be modeled as a fraction of two matrices as well, here: a non-symmetric $\mathbf{A}$ and symmetric matrix functions $\mathbf{B}$ [1]:

$$\mathbf{H}(\omega)=\mathbf{A}(\omega)\mathbf{B}^{-1}(\omega), \qquad (5)$$

$$\mathbf{A}(\omega):R\rightarrow C^{m\times n},\ \mathbf{B}(\omega):R\rightarrow C^{n\times n},$$

where each of the two matrices are made from Forsythe base polynomials $P_l(\omega)$ [2], which are orthogonal (avoid numerical errors) and Hermitian symmetric (property of a real linear system). To do the fitting the following error term were defined and minimized ($\mathbf{A}(\omega):=\Sigma_l\,\mathbf{A}_l\,P_l(\omega)$, $\mathbf{B}(\omega):=\Sigma_l\,\mathbf{B}_l\,P_l(\omega)$), as

$$\mathbf{E}(\omega)=\mathbf{A}(\omega)-\mathbf{\Phi}(\omega)\mathbf{B}(\omega)\ \text{ with} \qquad (6)$$

$$J(\mathbf{A}_l,\mathbf{B}_l)=\mathrm{tr}\,\mathbf{E}^{\mathsf{H}}(\omega)\,\mathbf{E}(\omega)\rightarrow\min.$$

2.3 Calculation of the modal parameters

The results of (6) are the polynomial coefficients. Using that and the base polynomial terms, so-called comrade matrix [3] can be constructed, which eigenvalues are approximately the s_k's, thus from (3) the modal parameters (ω_n, ζ) are definable.

2.4 Measurement results

Putting all together, s_k's calculated with several fitting polynomial order, from which the modal parameter, in this case the "founded" eigenfrequencies can be calculated. Plotting these eigenfrequencies for every fitting order the stability diagram can be constructed as shown in Fig. 2.

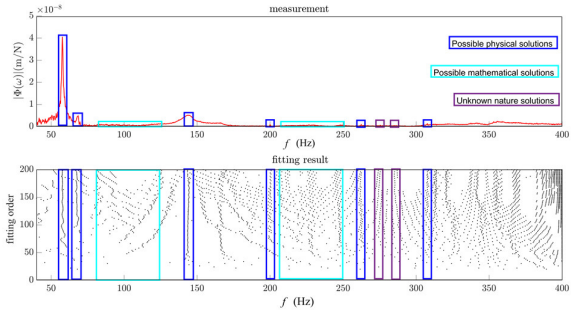

Fig. 2. Stability diagram.

From Fig. 2. it can be seen that for several cases the algorithm was able to identify the proper modal parameters: in this case the eigenfrequencies. But there were certain situation where it did wrong. This was because the fitting process is a root (poles) finding method after all. Some of the roots contain physical meaning i.e.: describes the structure's vibrational mode, others just arise with the mathematical nature of the method.

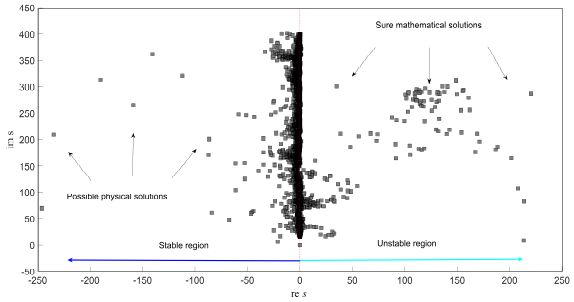

Fig. 3. Mathemathical roots.

Fig. 3. shows that some of the mathematical roots show unstable behavior already at the end of the fitting process. The problem is that there are unstable roots hidden on the stable region of the complex plane. This phenomena brings the question: how can the real, physical solutions be separated from the wrong, mathematical ones.

3. Pole iteration

One possible solution candidate could be an iteration process where all of the poles that was calculated in a certain fitting order will be iterated in another fitting order with the Newton-Raphson method. Here, the assumption was that, any physical roots are stable and remain stable through the iteration process. In the other hand the unstable roots may wander to the unstable region of the complex plane through the iteration, thus a filtering condition can be set up to get rid of these solutions.

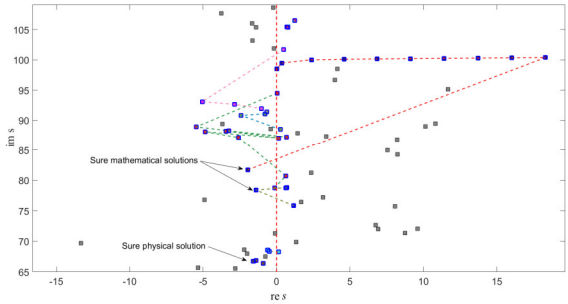

Fig. 4. Pole iteration.

4. Conclusions

The fitting process itself is not enough to create and automatic fitting tool, because the appearance of the mathematical solutions. The pole iteration could filter out those solutions but further tests are need to check whether the unstable behavior appears at every mathematical poles during the iteration process. This behavior could serve as a filter condition to separate the solutions.

5. Acknowledgement

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