Bistability in nonlinear elastic robotic arms subject to delayed feedback control

Giuseppe Habib*, Asier Barrios‡, Zoltan Dombovari*
*MTA-BME Lendület Machine Tool Vibrations Research Group, Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, Hungary
‡Dynamics and Control Department, Ideko, Elgoibar, Spain

Summary. Stability and bifurcation analysis of a non-rigid robotic arm controlled in a time delayed feedback loop is addressed in this work. The study aims at revealing the dynamical mechanisms leading to the appearance of limit cycle oscillations existing in the stable region of the trivial solution of the system, which are related to the combined dynamics of the robot control and its structural nonlinearities. A numerical study of the bifurcations occurring at the loss of stability enables the development of strategies to eliminate this undesired bistable phenomenon by the implementation of special additional nonlinearities in the control force.

Introduction

Robots are increasingly adopted in modern manufacturing facilities, thanks to their versatility and relatively low cost [1]. Milling operation is one of the operations robots are intended to be used for, where complicated tool trajectories can be realized in a large workspace with a relatively low cost. The relative vibration between workpiece and tool are a troublesome phenomenon in milling that mainly caused by so-called regenerative vibration. The main solution to avoid them is to increase stiffness and damping and try to disturb delayes introduced by the regenerative effect [2]. Increasing stiffness is hardly achievable in robots, since robotic arms are naturally slender and not especially stiff [3]. This makes them particularly prone to vibrations. The main method to mitigate these vibrations consists in implementing an active controller working in a feedback loop. In most of cases, this controller reads in input the acceleration of the end effector (EE, see Fig. 1a) and sends a proportional signal to the robot controller in order to counteract and suppress the vibrations. This signal is added to the position controller of the robot, required to make the robotic arm follow the prescribed path during machining.

Although this procedure is rather straightforward to be implemented, there are several aspects, which might undermine the system stability if not properly accounted for. (i) Robotic arms are naturally slender and they cannot be assumed rigid, especially if they are subject to strong forces, as in the case of machining nonlinearities can rise stronger. (ii) Moreover, since actuators are placed at the joints of the arm, the system is underactuated. Depending on the position of the sensors, either near the motor or near the EE, the system can be considered as collocated or non-collocated, which have relevant consequences on the system stability [4, 5]. (iii) Robot configuration changes continuously during operation and the drive components of the robot generates non-negligible nonlinearities; as we will illustrate in this study, these nonlinearities might have important consequences on the system robustness. (iv) Robot's controller is unavoidably subject to time delay in the feedback loop; although this is often negligible, if large control gains are required to counteract strong forces, time-delay can still generate instabilities.

This study is motivated by the appearance of unexpected vibrations in a real industrial robotic arm for milling operations. This robot is equipped with a built in most probably nearly PD controller for its correct positioning and with an additional controller proportional to the end effector acceleration, to counteract machine tool vibrations (Fig 1. a). Although the control parameters of the system were set such that the system was stable, in some occasion the robotic arm started oscillating with assumingly tiny but enough external forcing, which suggests that it was in bistable conditions. The objective of this study is to set up a seed work for defining a general simple model for this system in order to understand the origin of the bistability and define methods to avoid it.

Mathematical model

The mathematical model adopted is a two-degrees-of-freedom (DoF) mechanical system (Fig 1. a), consisting in two lumped masses, connected by a linear damper c, and a nonlinear spring k_d. One can program a reference trajectory via x_t, which is followed ideally by the robot control signal x_r with a robot control delay τ_r. The control force then is applied to one of the masses m_1 through a linear spring with k, which representing the combined additional dominant DoF what is actually experienced in measurement in a certain bandwidth. This arrangement is the simplest possible model to mimic the behaviour of a robot that is subjected to some sort of nonlinearities most probably originated from their joints. The equation of motion has the following form:

$$
\begin{aligned}
&m_1\ddot{x}_1 + c\,(\dot{x}_1 - \dot{x}_2) + k_\mathrm{nl}(\Delta x)\,(x_1 - x_2) + k_1 x_1 = k_1 x_\mathrm{r},\\
&m_2\ddot{x}_2 + c\,(\dot{x}_2 - \dot{x}_1) + k_\mathrm{nl}(\Delta x)\,(x_2 - x_1) = 0,
\end{aligned}
\tag{1}
$$

where $k_\mathrm{nl}(\Delta x) = k_2 + \kappa\Delta x^2$ ($\Delta x := x_2 - x_1$), m_1 and m_2 are the two lumped masses, c is the damping coefficient, while the robot mechanical constrained action x_r is realised through the linear spring k. The constrained action is modelled to be behind the reference trajectory with τ_r resulting in $x_\mathrm{r}(t) := x_\mathrm{t}(t - \tau_\mathrm{f})$. The reference trajectory for the robot is compiled by a desired motion x_d and the feedback originated from the EE acceleration through a controller with τ_f delay. This creates a final constrained motion $x_\mathrm{r}(t) = x_\mathrm{d}(t - \tau_\mathrm{r}) + K\ddot{x}_2(t - \tau)$, where $\tau = \tau_\mathrm{f} + \tau_\mathrm{r}$.

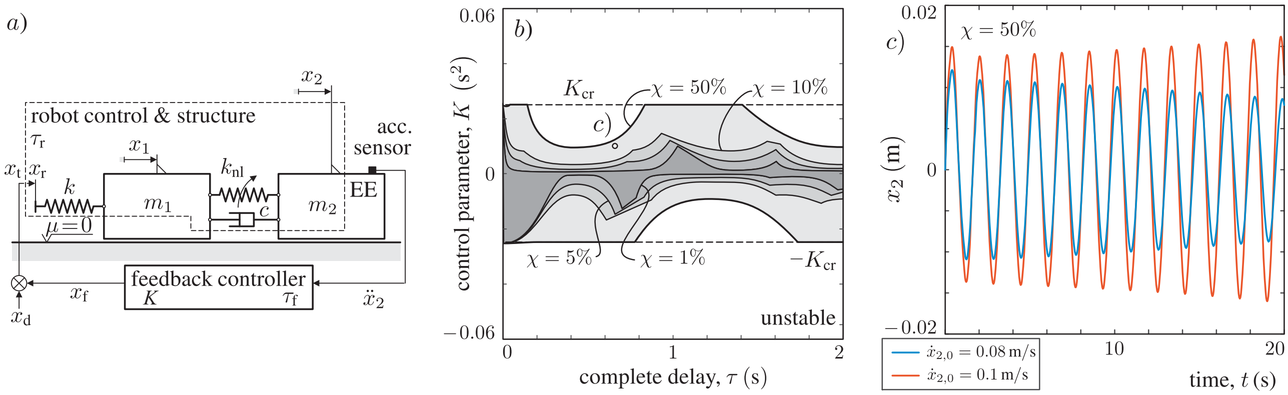

Figure 1: a) shows the sketch of the control ; b) stability chart in the (τ, K) space for various for $\omega_1 = \omega_2 = 2\pi$ rad/s, $\chi_1 = \chi_2 =: \chi = (50,\ 10,\ 5,\ 1)\,\%$, $K_{\rm cr} = 0.0253\,{\rm s}^2$, c) time evolutions for different initial conditions when $\mu_1 = \mu_2 = -1000000\,({\rm s\,m})^{-2}$, $K = 0.01\,{\rm s}^2$, $\tau = 0.6594\,{\rm s}$.

In this work we assume constant desired position, that is, $x_{\rm d}(t) := x_{\rm d}$, which results equilibria at $(\overline{x}_1, \overline{x}_2) = (x_{\rm d}, x_{\rm d})$. By introducing perturbation $x_1 := \overline{x}_1 + u_1$ and $x_2 := \overline{x}_2 + u_2$ the following system can describe the system
Via a standard non-dimensionalization procedure, equations of motion are reduced to

$$\begin{aligned}
&\ddot{u}_1 + 2\chi_1\omega_2\left(\dot{u}_1 - \dot{u}_2\right) + \omega_2^2\left(u_1 - u_2\right) + \mu_1(u_1 - u_2)^3 + \omega_1^2 u_1 = \omega_1^2 K \ddot{u}_{2\tau},\\
&\ddot{u}_2 + 2\chi_2\omega_2\left(\dot{u}_2 - \dot{u}_1\right) + \omega_2^2\left(u_2 - u_1\right) + \mu_2(u_2 - u_1)^3 = 0,
\end{aligned} \tag{2}$$

where $\omega_1^2 := k_1/m_1$, $\omega_2^2 := k_2/m_2$, $\chi_1 := c/(2m_1\omega_2)$, $\chi_2 := c/(2m_2\omega_2)$, $\mu_1 := \kappa/m_1$ and $\mu_2 := \kappa/m_2$, while $\ddot{u}_{2\tau} := \ddot{u}_2(t-\tau)$. By linearising (2) by setting $\chi_1 = \chi_2 := 0$ the linear stability of the corresponding neutral equation can be investigated (see Fig. 1b). It is really important to emphasize that the sufficient condition for neutral equation to have the neutral coefficient (here $\omega_1^2 K$) less than one in its magnitude, that is, $K_{\rm cr} = \omega_1^{-2}$. It can be seen the stability limit is constructed either the above mentioned condition or a repeated pattern originated from the delayed sense of the equation.

Stability and bistable behavior

Stability calculation showed the intricate stability limit of the simplest possible dynamic model of robotic arm subjected to acceleration feedback control. We presented that the linear behaviour is govern by the neutral sense of the system, when the largest order of state coordinate is delayed. The calculated linear stability limit showed the sufficient stability behaviour $K_{\rm cr}$ for neutral type differential equations. It has been presented the system is even more unstable by substracted common repeating lobe structure. Due to the cubic nonlinearity appearing in the combined dynamics of the robot control and structure bistability can occur, what we have presented by time domain simulations. The bistable behaviour is caused by the subcriticality of the corresponding bifurcation on the linear stability limit. Successively, additional nonlinearities can be purposely introduced in the control force algorithm in order to control the characteristic of the bifurcations and enforce supercritically, therefore eliminating the bistable behavior.

References

[1] Hagele M. (2016) Robots Conquer the World [Turning Point]. *IEEE Robot Autom Mag* **23**:120-118.

[2] Munoa J., Beudaert X., Dombovari Z., Altintas Y., Budak E., Brecher C., Stepan G. (2016) Chatter suppression techniques in metal cutting. *CIRP Annals* **65**(2):785-808.

[3] Mousavi S., Gagnol V., Bouzgarrou B.C., Ray P. (2018) Stability Optimization in Robotic Milling through the Control of Functional Redundancies. *Robot Com-Int Manuf* **50**:181-192.

[4] De Luca A., Schroder D., Thummel M. (2007) An Acceleration-Based State Observer for Robot Manipulators with Elastic Joints. *Proc IEEE Int Conf Robot Autom*:3817-3823.

[5] Habib G., Rega G., Stepan G. (2015) Stability Analysis of a Two-degree-of-freedom Mechanical System Subject to Proportional-Derivative Digital Position Control. *J Vib Control* **21**:1539-1555.

[6] Stépán, G. (1989) Retarded Dynamical Systems: Stability and Characteristic Functions. Longman Scientific & Technical, London.