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EXPERIMENTAL ANALYSIS OF A FAN DRIVEN UNDERACTUATED PENDULUM

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1. Introduction

Underactuated systems form a special group of controlled dynamical systems, when the number of independent control inputs is less than the degrees of freedom. The common characteristic of the underactuated systems is that the so called zero-dynamics cannot be actuated directly. Let us consider a crane where the position of the upper endpoint of the cable and the cable length are actuated but, the swinging motion of the load can only be controlled indirectly. For the accurate positioning of the load the exact understanding of the dynamics of the crane is needed. Besides, in this work we also consider the natural coordinate based modeling approach of the controlled dynamical system.

2. Theoretical background

In recent decades the most common way of robot regulation became the feedback linearization of the dynamics of the regulated system that results a linear dynamical system. Thereafter arbitrary motion can be prescribed for the robot if we assume that the actuator torques are unbounded. This approach cannot be used if the system is underactuated. To overview the problem let us consider the following equation of motion of a general dynamical system:

$$\ddot{\mathbf{q}} = \mathbf{f}_1(\mathbf{q},\dot{\mathbf{q}},t) + \mathbf{f}_2(\mathbf{q},\dot{\mathbf{q}},t)\mathbf{u}\,, \qquad (1)$$

where $\mathbf{q}$ is the minimum set of generalized coordinates and $\mathbf{u}$ is the control input vector. We can say that the system is fully actuated if the rank of $\mathbf{f}_2(\mathbf{q},\dot{\mathbf{q}},t)$ equals to the degrees of freedom:

$$\mathrm{rank}(\mathbf{f}_2(\mathbf{q},\dot{\mathbf{q}},t)) = \dim(\mathbf{q})\,. \qquad (2)$$

In such case the control input can be chosen as:

$$\mathbf{u} = \mathbf{f}_2^{-1}(\mathbf{q},\dot{\mathbf{q}},t)\left[-\mathbf{f}_1(\mathbf{q},\dot{\mathbf{q}},t) + \tilde{\mathbf{u}}\right]. \qquad (3)$$

This can be substituted into (1) resulting

$$\ddot{\mathbf{q}} = \tilde{\mathbf{u}} \qquad (4)$$

linear system, which means that $\ddot{\mathbf{q}}$ can be prescribed arbitrarily by the synthetic control input $\tilde{\mathbf{u}}$. In the case of underactuated systems this approach cannot be used because $\mathbf{f}_2(\mathbf{q},\dot{\mathbf{q}},t)$ is not invertible. Hence we say that a system is underactuated [1] if

$$\mathrm{rank}(\mathbf{f}_2(\mathbf{q},\dot{\mathbf{q}},t)) < \dim(\mathbf{q})\,. \qquad (5)$$

From the above overview we can conclude that the control of underactuated systems is a challenging task, even so the literature [2-4] provides different techniques for this kind of systems.

Complex structured manipulators are often modelled by natural coordinates [5] especially in real time applications. The equation of motion arises in the form of a differential algebraic equation:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{\Phi}_{\mathbf{x}}^{\mathrm{T}}(\mathbf{x})\boldsymbol{\lambda} = \mathbf{Q}(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{H}(\mathbf{x})\mathbf{u}\,, \qquad (6)$$

$$\boldsymbol{\varphi}(\mathbf{x}) = \mathbf{0}\,, \qquad (7)$$

where $\mathbf{x}$ is the vector of the non minimum set of descriptor coordinates (so called natural coordinates), $\mathbf{M}$ is the mass matrix, $\mathbf{\Phi}_{\mathbf{x}}$ is the Jacobian of the geometric constraint $\boldsymbol{\varphi}$, $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers, $\mathbf{Q}$ is the vector of external forces and $\mathbf{H}$ is the input matrix. The task of the manipulator is formulated by the servo constraint vector [2,3]

$$\boldsymbol{\varphi}_s(\mathbf{x},t) = \mathbf{0}\,, \qquad (8)$$

which has the same mathematical form as the geometric constraint equation (7).

References [2,3] propose a control method for underactuated and natural coordinate based systems that is based on the backward Euler discretization of equations (6-8), which leads to a nonlinear algebraic equations for the control input $\mathbf{u}_i$ in the i th time step and it is solved by Newton-Raphson method.

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3. Experimental results

For the testing of the control method proposed in [2,3] we extended a fan driven pendulum with an inverted pendulum fixed by flexible joint (see Fig.1 left). For the 2 DoF sysem there is only 1 actuator so the investigated system is underactuated. We used the Cartesian coordinates to describe the system (see Fig.1 right), and the geometric constraints provided for the constant rod lengths. The experimental setup is shown in Fig.2.

In the experiment the desired trajectory of the actuated pendulum were given. The control input for the fan was calculated with the consideration of the dynamics of the inverted pendulum. Fig.3 shows that the simulated and the measured trajectory are in good correspondence; however both have a deviation from the desired trajectory because the control was open loop.

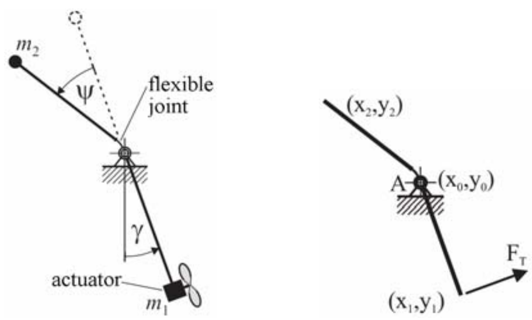

Fig. 1: mechanical model of the experimental equipment (left); natural coordinates (right)

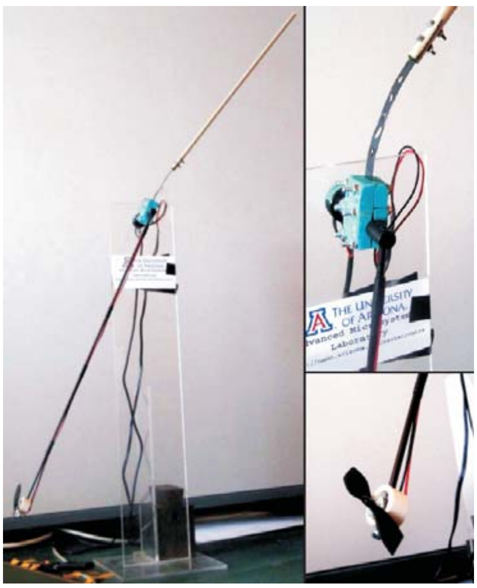

Fig. 2: Experimental equipment

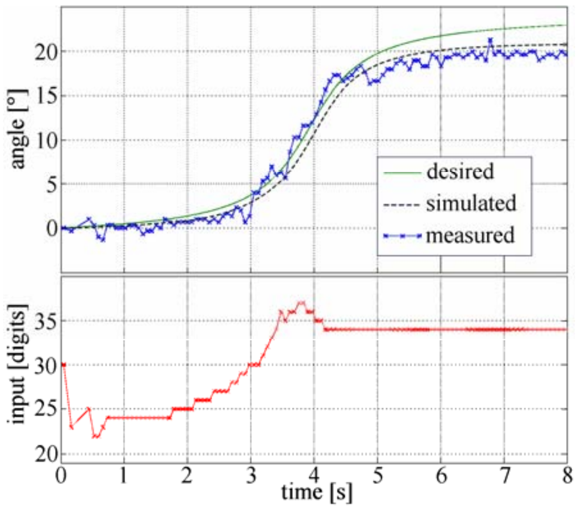

Fig. 3: Simulation and experimental results

4. Acknowledgements

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5. References

[1] J. T.-Y. Wen, Control of nonholonomic systems. Control Handbook, W. S. Levine, Ed. Boca Raton, FL: CRC and IEEE, 1996, pp. 1359-1368.

[2] W. Blayer, K. Kolodziejczyk. Modeling of underactuated mechanical systems in partly specified motion, Journal of Theoretical and Applied Mechanics, 46(2): 383-394, 2008.

[3] L. L. Kovács., A. Zelei, et al. Motion Control of an Under-Actuated Service Robot Using Natural Coordinates, Proc. of ROMANSY 18 - Robot Design, Dynamics and Control, 331-338, Udine, Italy, 2010.

[4] I. M. M. Lammerts, Adaptive computed reference computed torque control, PHD thesis, Eindhoven University of Technology, 1993.

[5] J.G. de Jalón, E. Bayo, Kinematic and dynamic simulation of multibody systems: the real-time challenge, Springer-Verlag, New York, NY, 1994.