# The influence of parametric excitation on floating bodies

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In the present study, we investigate the possibility of stabilizing a floating body that is unstable without parametric excitation. We prescribe a non-stationary geometric constraint, that is, the centre of gravity is moved periodically in vertical direction. This periodic movement causes the parametric excitation described by a non-autonomous differential equation. By using the stability chart of the Mathieu equation, we can find sets of parameters where the stabilization of the floating body is realizable. This can help in the stabilization of a canoeist, but it can also cause stable ships to capsize. Numerical simulations and an experiment were also accomplished to confirm that the stability chart derived for the case of the floating body is realistic.

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## 1 Introduction

The stability of floating bodies, especially boats and ships can be affected by the vertical periodic motion of the centre of gravity. For example, canoes and kayaks are pretty labile, using mechanical terminology, we call them statically unstable. Nevertheless, the boat basically moves in lengthwise direction, and it also oscillates in vertical direction due to the rowing motion of the athlete. In this work, we examine whether the parametric excitation caused by the vertical periodic motion of the boat helps in the balancing of these row-vessels. First, we briefly summarize the theory, then we apply it for the model of the vertically oscillating floating body.

## 2 Background for parametrically excited systems

Parametric excitation means that some of the parameters of the system change as a periodic, quasi-periodic or stochastic function of time. The simplest form of a parametrically excited system is given by the Mathieu equation [1] where the stiffness parameter varies harmonically: $x''(\tau) + (\delta + \varepsilon\cos(\tau)) = 0$. Parametric excitation is usually considered as an unexpected cause of instability problems, but under certain conditions, it can also be used for stabilizing unstable processes or equilibria. The main idea is to eliminate an oscillation with the help of another oscillation. The oldest known example is the inverted pendulum which can be stabilized by the harmonic vibration of the suspension pivot point [2, 4]. If the amplitude of the vibration is large enough, the upper equilibrium can be balanced by parametric excitation. In this work, we examine whether it is possible to stabilize a normally unstable floating rigid body in a similar way, and if so, for what parameters.

## 3 Stability of the parametrically excited floating body

A floating body, like a ship, has 6 degrees of freedom (DoF), see Fig. 1. The 3 rotations are the roll, pitch and yaw and the transversal motions are the surge, sway and heave. In order to use as simple model as possible for the analytic calculations, we investigate planar motion only (see the vertical plane in Fig. 1.). The generalized coordinates are chosen to be the roll angle $\phi$ and the position coordinates $x$ (sway) and $y$ (heave) of the centre of gravity. The parameters shown in Fig. 2. are the length $l$ and the width $a$ of the body, the height $p$ of the mass centre, the mass $m$ and the moment of inertia $J_C$.

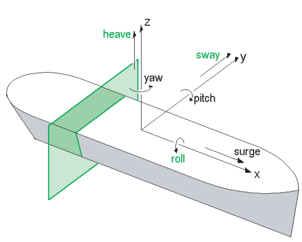


**Fig. 1** Spatial ship motions.

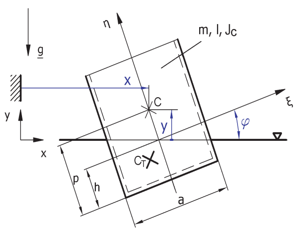


**Fig. 2** Planar mechanical model of the floating body.

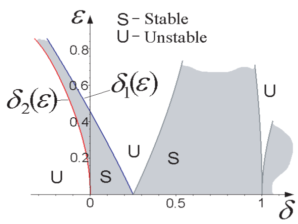


**Fig. 3** Stability chart of the Mathieu equation.

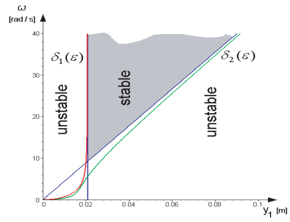


**Fig. 4** Stability chart of the parametrically excited floating body.

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The water surface is assumed to be ideally flat and steady. The damping effect of the water of density $\rho$ is completely neglected. The stability of the vertical equilibrium can be examined by the analysis of the potential function of this conservative system. The general coordinate $x$ does not appear in the Lagrangian, so $x$ is a so-called cyclic coordinate and we can fix the horizontal motion of the body. We prescribe the height of the centre of gravity by the harmonic function $y(t) = y_0 + y_1 \cos(\omega t)$ so the model is reduced to a 1 DoF system and the only generalized coordinate is the roll angle. The parameters of the parametric excitation are the amplitude $y_1$ and the angular frequency $\omega$. The equation of motion assumes the form:

$$J_C\ddot{\varphi} + mg\left(\frac{1}{2}\frac{m}{\rho a l} - p\right)\sin\varphi + \rho a l g\left[\frac{y_1^2\cos^2(\omega t) tg\varphi}{2\cos\varphi} + \frac{a^2}{24}\left(\sin\varphi + \frac{tg\varphi}{\cos\varphi}\right)\right] = 0 \tag{1}$$

The equation of motion can be linearized at the vertical position $\varphi = 0$. If we use the dimensionless time $\tau = \omega t$, the linearized equation of motion (2) is in complete correspondence with the Mathieu-equation:

$$\varphi'' + \left\{\frac{\rho a l g}{4\omega^2 J_C}\left[\frac{m}{\rho a l}\left(\frac{1}{2}\frac{m}{\rho a l} - p\right) + \frac{y_1^2}{4} + \frac{a^2}{12}\right] + \frac{\rho a l g}{4\omega^2 J_C}\frac{y_1^2}{4}\cos(\tau)\right\}\varphi = 0 \tag{2}$$

If $\varepsilon = 0$ and $\delta < 0$ then the vertical position is unstable and the boat capsizes, but it can be stable if the amplitude $y_1$ increases.

We can apply the Incze-Strutt diagram [3] (see Fig. 3) for the equation (2) of the floating body. The transformed stability chart in Fig. 4 shows the physical parameters of the excitation: the angular frequency $\omega$ and the amplitude $y_1$.

## 4 Verification by numerical simulation and experiment

The parameters of the numerical simulation fit for a canoeist. 80-85 strokes per minute is typical [5]. The forces acting between the shell and the rower and between the water and paddle blade change the resting waterline causing oscillations of 4-6 cm [6]. The chosen parameters are: $J_C = 12\,\mathrm{kgm}^2$, $m = 90\,\mathrm{kg}$, $l = 3.5\,\mathrm{m}$, $a = 0.55\,\mathrm{m}$, $p = 0.6\,\mathrm{m}$, $\omega = 6.28\,\mathrm{rad/s}$. The nonlinear equation (1) of motion was used for numerical simulation. Fig. 5. shows the time history of the roll angle $\varphi$ and Poincar sections for 3 different $y_1$ values.

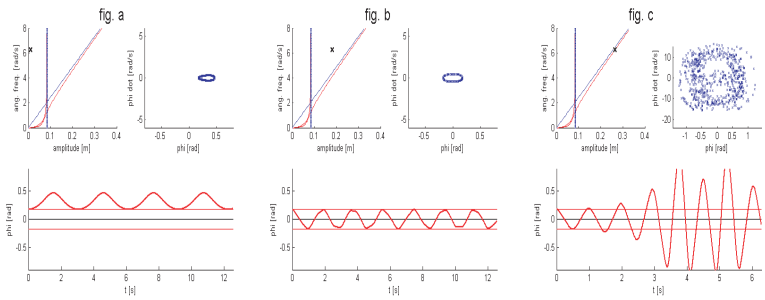


**Fig. 5** Numerical results. The simulated point on the stability map, the Poincare section and the time history of the roll angle are shown.

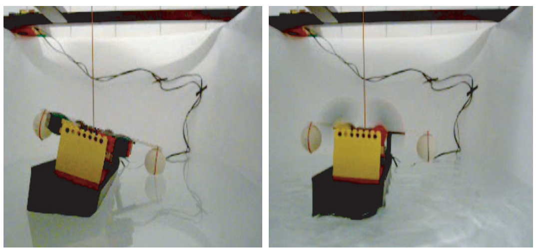

**Fig. 6** Experimentalal results. Left: stopped rotors, capsized position. Right: running rotors, stable vertical position.

A small boat was constructed and 2 eccentric counter-rotating rotors provided the parametric excitation (see Fig. 6). Due to the harmonic motion of the mass centre, the shallow dive changes periodically. The experiment clearly showed that the ship stabilization is possible: the boat floated stably with the rotors running, while it capsized when the rotors were stopped.

## 5 Conclusion

Athletes are told to keep a good rhythm of rowing and not to row faster. Realistic and finite parameter domains were found where sportsmen can stabilize their boats with this good rythm via parametric excitation.

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