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MOTION ANALYSIS OF A CRANE AS AN UNDER-ACTUATED ROBOT

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Abstract: We investigate the motion of swinging payloads connected to a cable of moving suspension point by extending the methods of robotics. In general, the suspension point is moved by a crane, but in specified applications it can also be moved by a robot. In this case the swinging load has to be capable to follow spatial target trajectories and to keep desired positions with adequate accuracy. The main purpose is to solve the inverse kinematic and dynamical problem and to provide position control of this crane structure considered as an under-actuated robot. The presented results gained from the analytical analysis of a simplified spatial model show the opportunities and the limitations of trajectory tracking and control.

Keywords: *crane, controlled pendulum, under-actuated, redundant*

1 INTRODUCTION

Cranes are pendulum-like structures that are widely used for transporting a payload to a specified position, which is usually accurately defined. Additionally, the payload sometimes has to follow a prescribed spatial trajectory, too. Since cranes are nonlinear oscillating systems, it is a complicated task to achieve a good motion control which suppresses the swinging motion of the payload. In the case of general cranes such as tower cranes, gantry cranes or overhead cranes (see Fig. 1) the position control of the payload is realized by the controlled movement of the top mounting point of the cable. The low actuating possibilities also complicate the controlling strategies. The oscillating behavior of floating and aerial cranes significantly intensifies because the position of the top mounting point is disturbed by environmental effects such as waves and wind.

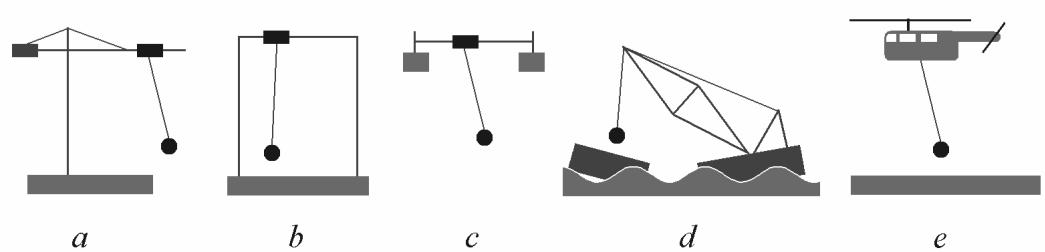

Fig. 1. Cranes: a) tower crane, b) gantry crane, c) overhead crane, d) floating crane e) aerial crane

In order to achieve high efficiency, the automation of the crane movements is significant. Several controlling strategy have been developed; most commonly feedback controllers [9]

and time delayed feedback controllers are used for oscillation suppression [5, 9, 10]. In order to attain better anti-sway control fuzzy logic controllers [8, 9] and genetic algorithm-trained neuro-controllers are also used [6].

The complexity of the task comes from the dynamics of the pendulum-like system. A rigid armed structure like a robotic arm can be actuated directly by the servomotors built in the joints. Contrarily, the actuation applied on the top/mounting point of the cable does not have direct effect on the swinging load. There are no actuators to vary the swinging angles, thus these crane systems are under-actuated. The following definition of under-actuated mechanical systems is adapted from [2, 3]. The generalized mathematical model of the dynamical system can be written as:

$$\ddot{\mathbf{q}} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q})\mathbf{u}\,, \tag{1}$$

where $\mathbf{q}$ is a vector of independent generalized coordinates, $\mathbf{f}$ is a vector field that determines the dynamics of the system, $\mathbf{G}$ the input matrix, and $\mathbf{u}$ is the vector of generalized inputs. The system is under-actuated if the rank of $\mathbf{G}$ is smaller than the dimension of $\mathbf{q}$. Thus, if the dynamical system contains less independent actuators than the degrees of freedom it has, the system is under-actuated.

During motion, the cable is not in the vertical hanging-down position. Since the cable can be winded up and down, a certain spatial position can be reached with different configurations as shown in Fig. 2, which means that beside the under-actuated nature of the system, redundancy is also present.

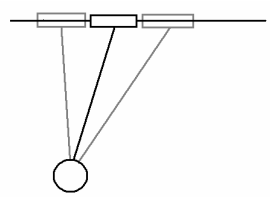

Fig. 2. Redundancy of the varied length pendulum

We examine a simplified crane model with analytic calculations. In the present paper we investigate the inverse kinematics and dynamics of the system. The solution of the inverse dynamical problem can be given by analytic expressions. Since the method of computed forces is not robust, we apply a feedback controller. The system extended by the PD controller was examined numerically.

2 MECHANICAL MODEL

The mounting point is moved in the $z \equiv 0$ horizontal plane by a two degree of freedom mechanical structure which can be an RR, RT (tower crane) or a TT (for example overhead crane) structure. To this end, the mounting point is attached to a simple block placed in the horizontal plane, which can be moved by any type of two degree-of-freedom (DoF) planar mechanism. The cable connecting the mounting point and the payload is assumed to be massless without any bending stiffness.

The generalized coordinates of the spatial mechanical model (see Fig. 3) are:

$$\mathbf{q} = [\vartheta \quad \psi \quad l \quad x \quad y]^T, \tag{2}$$

where ϑ is the nutation of the cable, the precession angle is denoted by ψ, l is the cable length that can be varied by the winding mechanism. The position of the mounting point is given by the planar coordinates x and y. Thus, the load as a point-mass has 3 DoF, the cable mounting point has another 2 DoF, so the system has the overall 5 DoF.

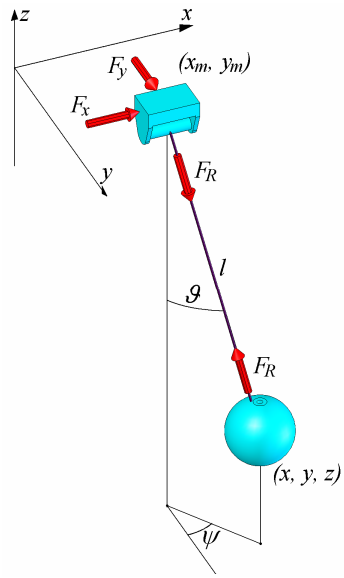

Fig. 3. Simplified mechanical model

The position of the mounting point and the centre of gravity of the load (see Fig. 3.) are given by:

$$\mathbf{r}_m = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}, \tag{3}$$

$$\mathbf{r}_l = \begin{bmatrix} x + l\sin\vartheta\cos\psi \\ y + l\sin\vartheta\sin\psi \\ -l\cos\vartheta \end{bmatrix}. \tag{4}$$

The kinetic energy assumes the form:

$$T = \frac{1}{2}m\dot{\mathbf{r}}_l^{\,2} + \frac{1}{2}m_m\dot{\mathbf{r}}_m^{\,2}, \tag{5}$$

The potential function is calculated from the height of the payload:

$$U = -mgl\cos\vartheta. \tag{6}$$

The active forces are generated by the actuators. F_x and F_y are provided to control the mounting point position. The cable force F_R acts on the block to which the cable is connected and its counterpart acts on the payload.

$$\mathbf{F}_m = \begin{bmatrix} F_x \\ F_y \\ 0 \end{bmatrix} - F_R \begin{bmatrix} -\sin\vartheta\cos\psi \\ -\sin\vartheta\sin\psi \\ \cos\vartheta \end{bmatrix}, \tag{7}$$

$$\mathbf{F}_l = F_R \begin{bmatrix} -\sin\vartheta\cos\psi \\ -\sin\vartheta\sin\psi \\ \cos\vartheta \end{bmatrix}. \tag{8}$$

The generalized forces are given by:

$$Q_k = \mathbf{F}_l \frac{\partial \mathbf{r}_l}{\partial q_k} + \mathbf{F}_m \frac{\partial \mathbf{r}_m}{\partial q_k}; \qquad q_k = \vartheta, \psi, l, x, y. \tag{9}$$

Lagrange's equations of the second kind are used to derive the equations of motion:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k} + \frac{\partial U}{\partial q_k} = Q_k; \qquad q_k = \vartheta, \psi, l, x, y. \tag{10}$$

The equations of motion are in matrix form:

$$\mathbf{M}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{Q}(\mathbf{q}). \tag{11}$$

where the matrices and vectors are:

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} ml & 0 & 0 & m\cos\vartheta\cos\psi & m\cos\vartheta\sin\psi \\ 0 & ml\sin^2\vartheta & 0 & -m\sin\vartheta\sin\psi & m\sin\vartheta\cos\psi \\ 0 & 0 & m & m\sin\vartheta\cos\psi & m\sin\vartheta\sin\psi \\ m\cos\vartheta\cos\psi & -m\sin\vartheta\sin\psi & m\sin\vartheta\cos\psi & m+m_m & 0 \\ m\cos\vartheta\sin\psi & m\sin\vartheta\cos\psi & m\sin\vartheta\sin\psi & 0 & m+m_m \end{bmatrix}, \tag{12}$$

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} 2m\dot{l}\dot{\vartheta} + mg\sin\vartheta - ml\dot{\psi}^2\sin\vartheta\cos\vartheta \\ 2m\dot{l}\dot{\psi}\sin^2\vartheta + 2ml\dot{\vartheta}\dot{\psi}\sin\vartheta\cos\vartheta \\ -mg\cos\vartheta - ml\dot{\vartheta}^2 - ml\dot{\psi}^2\sin^2\vartheta \\ m\left(2l\dot{\vartheta}\dot{\psi}\cos\vartheta\sin\psi - 2\dot{l}\left(\dot{\vartheta}\cos\vartheta\cos\psi - \dot{\psi}\sin\vartheta\sin\psi\right) + l\left(\dot{\vartheta}^2 + \dot{\psi}^2\right)\sin\vartheta\cos\psi\right) \\ m\left(-2l\dot{\vartheta}\dot{\psi}\cos\vartheta\cos\psi - 2\dot{l}\left(\dot{\vartheta}\cos\vartheta\sin\psi - \dot{\psi}\sin\vartheta\cos\psi\right) + l\left(\dot{\vartheta}^2 + \dot{\psi}^2\right)\sin\vartheta\sin\psi\right) \end{bmatrix}, \tag{13}$$

$$\mathbf{F}(\mathbf{q}) = \begin{bmatrix} 0 \\ 0 \\ -F_R \\ F_x \\ F_y \end{bmatrix}. \tag{14}$$

The equations of motion are singular at the hanging down position, where the nutation angle is zero, that is, the mass matrix is non-invertible if $\vartheta = 0$. Since the motion at and around the hanging down position is significant in this analysis, we need another approach to construct the equations of motion. We choose the coordinates of the gravity centre of the load ξ, η and ζ as generalized coordinates instead of using ϑ, ψ and l. So the new vector of the generalized coordinates is:

$$\tilde{\mathbf{q}} = \begin{bmatrix} \xi & \eta & \zeta & x & y \end{bmatrix}^T. \tag{15}$$

The position vector of the payload gravity center is given by:

$$\tilde{\mathbf{r}}_l = \begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix}. \tag{16}$$

The kinetic energy, potential energy and the generalized forces can be written as:

$$\widetilde{T} = \frac{1}{2} m \dot{\widetilde{\mathbf{r}}}_l^{\,2} + \frac{1}{2} m_m \dot{\mathbf{r}}_m^{\,2}, \tag{17}$$

$$\widetilde{U} = mg\zeta, \tag{18}$$

$$\widetilde{Q}_k = \mathbf{F}_l \frac{\partial \widetilde{\mathbf{r}}_l}{\partial q_k} + \mathbf{F}_m \frac{\partial \mathbf{r}_m}{\partial q_k}; \qquad q_k = \xi, \eta, \zeta, x, y. \tag{19}$$

The new rope force vector has to be expressed by the generalized coordinates ξ, η, ζ, x, y:

$$\widetilde{\mathbf{F}}_R = F_R \frac{1}{\sqrt{(\xi - x)^2 + (\eta - y)^2 + \zeta^2}} \begin{bmatrix} x - \xi \\ y - \eta \\ -\zeta \end{bmatrix}. \tag{20}$$

Using Newton's Second Law the equations of motion in matrix form are given by:

$$\begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & m_m & 0 \\ 0 & 0 & 0 & 0 & m_m \end{bmatrix} \begin{bmatrix} \ddot{\xi} \\ \ddot{\eta} \\ \ddot{\zeta} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} -\lambda_R(\xi - x) \\ -\lambda_R(\eta - y) \\ -mg - \lambda_R \zeta \\ F_x + \lambda_R(\xi - x) \\ F_y + \lambda_R(\eta - y) \end{bmatrix}, \tag{21}$$

where λ_R is given by:

$$\lambda_R = \frac{F_R}{\sqrt{(\xi - x)^2 + (\eta - y)^2 + \zeta^2}}. \tag{22}$$

This set of equations has no singularity and is more useful to resolve the inverse kinematical and inverse dynamical problem. In the case of equations of motion, (11) the desired generalized coordinates ϑ, ψ and l would have been calculated from (4). The system of equations (21), (22) is a differential-algebraic equation, which can be simplified to a system of ordinary differential equations after the calculation of λ_R.

3 INVERSE KINEMATICS AND DYNAMICS OF A PENDULUM-LIKE ROBOT

The simplest model of a crane is a mathematical pendulum, which is mounted on a moving point. In this case, the controller can only actuate the structure via the forces F_x, F_y and F_R transmitted by the cable. The reduced system has only five degrees of freedom: ξ, η, ζ, x and y.

The trajectory is given by ξ^d, η^d and ζ^d so the inverse kinematics is done – the coordinates x and y are optional kinematically in the redundant system. In the inverse dynamics, we have to determine the desired motion (x^d, y^d) of the mounting point and the time history of the desired actuator forces F_x^d, F_y^d, F_R^d from the equations of motion. First, the cable force can be calculated from the ζ component of equation of motion (21):

$$\lambda_R^d = -\frac{m(\ddot{\zeta}^d + g)}{\zeta^d}. \tag{23}$$

As the cable force is known, the mounting point positions can be derived from the ξ and η component of (21):

$$x^d = \xi^d - \frac{\ddot{\xi}^d \zeta^d}{\ddot{\zeta}^d + g}, \tag{24}$$

$$y^d = \eta^d - \frac{\ddot{\eta}^d \zeta^d}{\ddot{\zeta}^d + g}. \tag{25}$$

One can notice that the mounting point is not above the load, because the horizontal acceleration of the payload requires horizontal displacement of the mounting point. The forces applied on the mounting point are given by:

$$F_x^d = m_m \ddot{x}^d + m\frac{\ddot{\zeta}^d + g}{\zeta^d}(\xi - x) = m_m \frac{d^2}{dt^2}\left(\xi^d - \frac{\ddot{\xi}^d \zeta^d}{\ddot{\zeta}^d + g}\right) + m\ddot{\xi}^d, \tag{26}$$

$$F_y^d = m_m \ddot{y}^d + m\frac{\ddot{\zeta}^d + g}{\zeta^d}(\eta - y) = m_m \frac{d^2}{dt^2}\left(\eta^d - \frac{\ddot{\eta}^d \zeta^d}{\ddot{\zeta}^d + g}\right) + m\ddot{\eta}^d. \tag{27}$$

The total number of the time dependent unknowns (five generalized coordinates, and three actuator forces) is eight: and the total number of equations (three geometric equations from the trajectory definition, and five component equations of the equations of motion (21)) is the same. It is also important to notice that the number of prescribed coordinates and the number of actuators are equal.

Equations (26) and (27) show that the trajectory of the payload must be at least four times continuously differentiable in order to avoid infinitely large actuator forces. Summarizing, we gained analytic formulas to determine the desired generalized coordinates and actuator forces. These results are verified with numerical simulations. The steps of the numerical calculation are shown in Fig. 4.

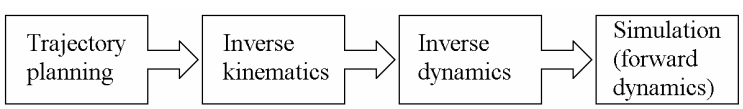

Fig. 4. Block diagram of simulations

In the numerical simulation a PD controller is used for compensating the error and keeping the payload on the desired trajectory. The real actuator forces are constructed form the desired force F^d and the force error F^e between the actual and the desired position:

$$F = F^d + F^e. \tag{28}$$

The rank of the proportional and differential coefficient matrix is less than the dimension of the system (since the system is under-actuated):

$$\begin{bmatrix} F_R^e \\ F_x^e \\ F_y^e \end{bmatrix} = \begin{bmatrix} 0 & 0 & p_{13} & 0 & 0 \\ p_{21} & 0 & 0 & p_{24} & 0 \\ 0 & p_{32} & 0 & 0 & p_{35} \end{bmatrix} \begin{bmatrix} \xi - \xi^d \\ \eta - \eta^d \\ \zeta - \zeta^d \\ x - x^d \\ y - y^d \end{bmatrix} + \begin{bmatrix} 0 & 0 & d_{13} & 0 & 0 \\ d_{21} & 0 & 0 & d_{24} & 0 \\ 0 & d_{32} & 0 & 0 & d_{35} \end{bmatrix} \begin{bmatrix} \dot{\xi} - \dot{\xi}^d \\ \dot{\eta} - \dot{\eta}^d \\ \dot{\zeta} - \dot{\zeta}^d \\ \dot{x} - \dot{x}^d \\ \dot{y} - \dot{y}^d \end{bmatrix}. \tag{29}$$

A simulation has been accomplished to confirm the validity of the analytically gained formulas. The desired spatial path defined by ξ^d, η^d, ζ^d (see light curves on Fig. 5.) of the payload was the input of the inverse kinematical calculation (see the block diagram on Fig. 4.). The desired path was generated by a polynomial interpolation algorithm.

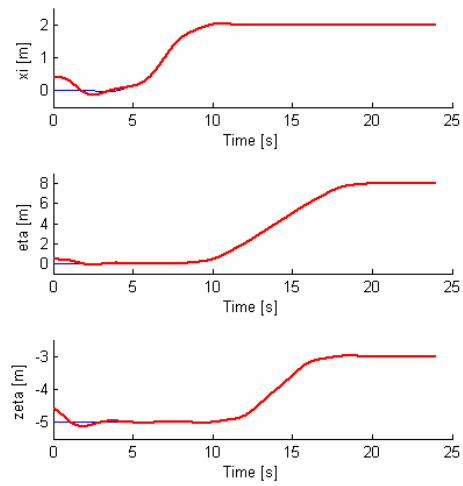

Fig. 5. The desired (light line) and the controlled coordinates (thick line) of the gravity centre of the payload

We gained the desired path of the mounting point x^d, y^d and the desired actuator forces F_R^d, F_x^d, F_y^d as a result of the inverse dynamics (see the block diagram on Fig. 4.). The desired path of the mounting point and the desired forces are shown by light lines in Fig. 6.

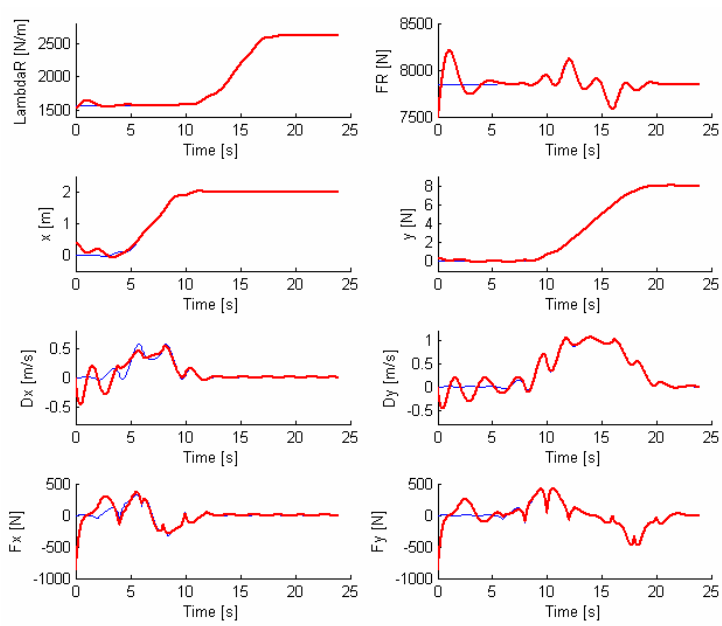

Fig. 6. The results of the inverse dynamical calculations. The light lines show the desired forces and coordinates. The thick lines are the results of the direct dynamical calculations.

The results of the forward dynamics are shown in Fig. 5. and Fig. 6. by the thick curves. The simulation of the forward dynamics was started with significant error in the initial values.

Nevertheless the system is under-actuated, fast suppression of the oscillations can be achieved.

In order to make the system fully-actuated, the actuator forces can be extended with forces applied on the swinging payload. Therefore all generalized coordinates can be prescribed arbitrarily. The inverse kinematics can be solved by the kinetic energy minimization [11], or by the minimization of the potential energy of virtual springs [12] that are fixed in the joints. Another approach to solve the inverse kinematical problem of the redundant but not under-actuated system is to minimize the joint torques required for the motion, [6, 7]. Summarizing, the payload can track any spatial trajectory in arbitrary orientation and the path of the mounting point can be determined by the inverse kinematical calculation of the redundant and fully actuated system. Since the system is not under-actuated the inverse dynamics gives the actuator forces in general way.

4 CONCLUSIONS

The singularity of the equations of motion of the pendulum-like structure has been observed at the hanging down position. In order to avoid the singularity problem we constructed the equations of motion by using an Euclidian set of generalized coordinates.

The number of prescribed coordinates of the investigated under-actuated system is equal to the number of actuators. The other generalized coordinates can be determined by the inverse dynamical calculation.

The main objective of the future work is to observe the advantages and limitations of the investigated crane models. Mainly, faster point to point motion and more accurate trajectory tracking can be achieved by the use of actuators on the payload.

If the vertical position of the mounting point can also be varied like in the case of aerial cranes, the calculations can be generalized similarly.

ACKNOWLEDGEMENTS

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